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LONG-TERM HYDROTHERMAL COORDINATION OF ELECTRICITY GENERATION WITH POWER AND ENERGY CONSTRAINTS

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Abstract - Optimizing the thermal production of electricity in the long term means optimizing both the fuel procurement policies and the use of fuels for generation at each thermal unit throughout the time period under study. A fundamental constraint to be satisfied at each interval into which the long time period is subdivided, is the exact covering of its load duration curve with thermal and hydro-generation. The solution found indicates how to distribute the hydro-electric generation (cost-free) throughout the period of time and the acquisition and use of fuels for each unit at each interval, in order to minimize the cost of fuels bought, while the probabilistic demands of electric energy, expressed by the load duration curves of all intervals, are exactly matched.

Bézier curves that change with hydrogeneration are employed in the constraints to define the load duration curves to be covered by thermal units at each interval.

Keywords - Bézier Curves, Hydrothermal Coordination, Long-Term Operating Planning, Electricity Generation, Generalized Network Flows, Nonlinear Optimization

1. INTRODUCTION

Long-term hydrothermal coordination is one of the most important problems to be solved periodically in the management of a power utility with an integrated power plant, i.e: with thermal and hydraulic power stations.

The solution sought indicates how to distribute the hydroelectric generation (cost-free) in each reservoir of the reservoir system over a long period of time (e.g.: one year) , so that the fuel expenditure during the period is minimized. When some thermal units can use more than one fuel or share the same fuel contract with other units, and there are fuel limits for one or more units over the whole period or parts of it, fuel acquisition and usage must also be optimized in coordination with hydrogeneration, which leads to a bigger problem. As usual, the long time period or horizon under consideration (e.g.: one year) will be subdivided into several time intervals of shorter duration (e.g.: one month) for which optimal values of decision variables are to be found.

The fundamental difference between long term and short term hydrothermal optimization, aside from the length of the time period studied, lies in the fact that, the availability of thermal plant, the demand for electricity and the water inflows in the reservoirs are not deterministic, but only known as probability density functions. Start-up and shut-down costs of thermal units are not accounted for directly in long-term coordination.

When considering the long-term problem it is important to optimize, taking into account the whole set of probable water inflows. This has been done with a model based on nonlinear multicommodity network flows to optimize the long-term hydroscheduling (ref. 1). It assumes unlimited fuel suplies, which is much simpler than the limited supply case. A long-term hydro-optimization model using curves of expected thermal production cost with respect to hydrogeneration, derived from the load duration curve (l.d.c.) of each interval (refs. 2,3), could provide a reasonably good answer to the long-term hydro-thermal coordination problem without fuel limits.

The literature on long-term hydrothermal coordination is rich. However, only a few papers on this subject describe methods that deal with stochastic inflows and that balance thermal and hydro-generation through the l.d.c. and not just with the peak load or through the total energy demand of the interval. Sherkat et al. (ref.4) consider at each interval a staircase l.d.c. with a few load segments. These l.d.c's are peak shaved with the expected values of the generations corresponding to the releases of all reservoirs but one in turn, and optimizes with dynamic programming the releases of the remaining reservoir considering the thermal cost curves of the load segments of the l.d.c. for a series of river inflow sequences. Contaxis and Kavatza (ref.5) optimize the stored volumes of the reservoirs with dynamic programming, obtaining for each reservoir at each interval a probability distribution function of hydrogeneration from the stochastic water inflows (while satisfying the reservoir balance equations with expected values of inflows and outflows) and convolve (ref. 6) the probability distribution functions of hydrogeneration, substituting the most expensive thermal units to cover the l.d.c. of each interval. Neither the method of Sherkat et al (ref. 4) nor that of Contaxis and Kavatza (ref. 5) deals with fuel limits.

Ranjit Kumar et al. (ref. 7) optimize the long-term fuel procurement and use with fuel limits. They use probabilistic production costing methods (ref. 6) with a given priority list to determine the maximum limits on the energies generated by each unit for each interval, and then a network flow solution (ref. 8) for the entire period is used to generate a new priority list for each interval, correcting priorities according to capacity factors in the network solution. System and unit fuel limits are modified to correct the mismatches between generation and the l.d.c.. This method does not consider hydrogeneration.

The work presented here describes a new model for long-term hydrothermal coordination with fuel limits. It employs generalized network flows (ref. 8) with nonlinear side constraints to exactly cover the l.d.c's of all intervals. Hydrogeneration is also optimized because the expected value of hydroenergy at each interval contributes to fill up the l.d.c.. A model like that in ref. 1, or a simpler one, can be used together with the thermal model and the treatment of the l.d.c. described below.

The existence of reliable general purpose optimization packages allows us to make an initial check of the methodology using the objective function and constraints of the model proposed. This has been done using the "MINOS" package (ref. 9) with results which show the consistency of the model put forward.

2. PRIORITY ORDER WITH ENERGY LIMITED UNITS

The model proposed always considers a fixed priority order of thermal units, based only on unit efficiency and fuel price. Ranjit Kumar et al. (ref. 7) put forward a changeable (and optimizable) priority order when there are energy limited units, placing these units further down in the order so that on the l.d.c. they have a slice of area equal to the available energy.

This approach has not been followed because:

• given a limited fuel supply to be shared by several units over a long time period, the determination of the priority orders at the intervals which cover their l.d.c. while satisfying the fuel limits is a very hard mixed integer problem

• the presence of hydrogeneration further complicates the issue

• in one such solution, dispatching an efficient base unit in a low priority position may entail frequent start-ups and shut-downs, which is unrealistic (and expensive).

In the model with fixed priority presented, units affected by limited fuel availability behave as any other unit: at some intervals they may not generate at all, or if they do, the slice of the l.d.c. they cover cannot be wider than their maximum power capacity.

3. THE POWER-ENERGY CURVES

Given the load duration curve of a certain interval, such as the one in Fig. 1.a), one can obtain the curve shown in Fig. 1.b) through integration over the powers, which gives the relation of the power level achieved when delivering a certain energy while conforming to the shape of the load duration curve.

The curve in Fig. 1.b) will be referred to as the Power-Energy (PE) curve, and has several interesting features.

• its left part is a straight line of slope 1/T through the origin, corresponding to the energy under the base load of the load duration profile

• the durations of the load duration curve are the inverse of the derivative of the PE curve: $t = 1/(dPE/dE)$

• the far right part of the curve has infinite slope.

The procurement and the use of fuels in thermal units can be modelled as a generalized single or multicommodity network flow problem, as shown in Figure 2, where a six-interval, three thermal unit, two-fuel problem is depicted. e_{ji} is the energy

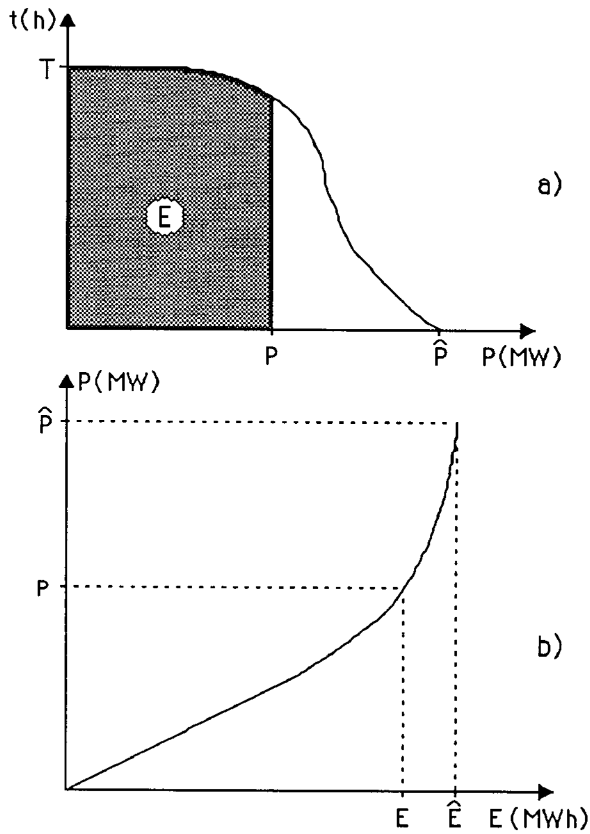

Fig. 1 a) Inverted load duration curve of a given interval
b) Power-Energy (PE) curve corresponding to the load duration curve in a)

(in MWh) generated by thermal unit "j" during interval "i", which is affected by the efficiency coefficient ε_i , r_{ji} is the remainder of fuel of unit "j" at the end of interval "i" and f^k_j is the amount of fuel of type "k" acquired for unit "j". Arcs of variables e_{ji} are "root arcs".

4. POWER AND ENERGY CONSTRAINTS

The equations of the covering of the load duration curve assume a "priority order" (natural order of thermal units 1,2,...) in loading units 1, 2,..., up to the last unit "Nu". For interval "i" we would have:

$$PE(e_{1i}) \le \overline{P}_1$$
$$PE(e_{1i}+e_{2i}) - PE(e_{1i}) \le \overline{P}_2$$
$$\cdots$$
$$PE(e_{1i}+e_{2i}+..+e_{j-1,i}+e_{ji}) - PE(e_{1i}+e_{2i}+...+e_{j-1,i}) \le \overline{P}_j \quad (1)$$
$$\cdots$$
$$PE(e_{1i} + ... + e_{ji} + ... + e_{Nui}) = \hat{P}^{i} \quad (2)$$

where $\overline{P}_j$ is the maximun capacity of unit "j" and $\hat{P}^{i}$ is the peak load in interval "i".

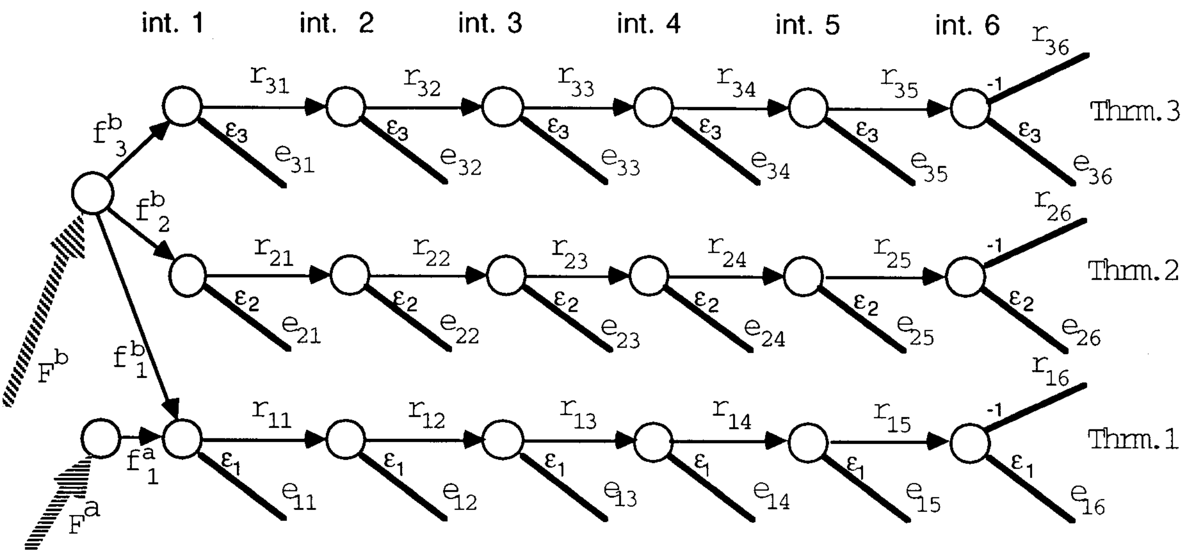

Fig. 2. Replicated generalized network for long-term fuel procurement and thermal generation optimization over a time period divided in six intervals, for three thermal units using two fuels.

This is illustrated in Figure 3. Inequality (1) expresses that the power contribution of unit "j" to cover the load-duration curve must not exceed its maximum rated capacity. Equality (2) forces the power obtained through the energies generated by all units to equal the peak load of the interval. In this way the power and energy requirements imposed by the load-duration curve of each interval are met.

Hydro-generation is used to peak-shave the load duration curves, as shown in Figures 4a) and 4b). The form of the shaving depends on the amount of hydro-energy and also on the maximun hydro-power capacity available. Fig. 4 shows two hypothetical cases with hydro-energies H_a and H_b in the same interval. The shape of the load duration curve and that of the PE curve seen by thermal units change with hydro-energy H generated in the interval. Thus the parameters defining the PE curve to be used for thermal energies are functions of H: PE[H]. The peak load seen by thermal units is also a function of H, since the maximum hydro-power generated P_H is a function of H and of the maximum hydro-power capacity $\bar{P}_H$.

Equations (1) and (2) can thus be recast as follows:

$$PE\left[H^i\right]\left(\sum_{n=1}^{j} e_{ni}\right) - PE\left[H^i\right]\left(\sum_{n=1}^{j-1} e_{ni}\right) \le \bar{P}_j \quad j=1,\dots,N_u \tag{3}$$

$$PE\left[H^i\right]\left(\sum_{n=1}^{Nu} e_{ni}\right) = \hat{P}^i - P_H{}^i \tag{4}$$

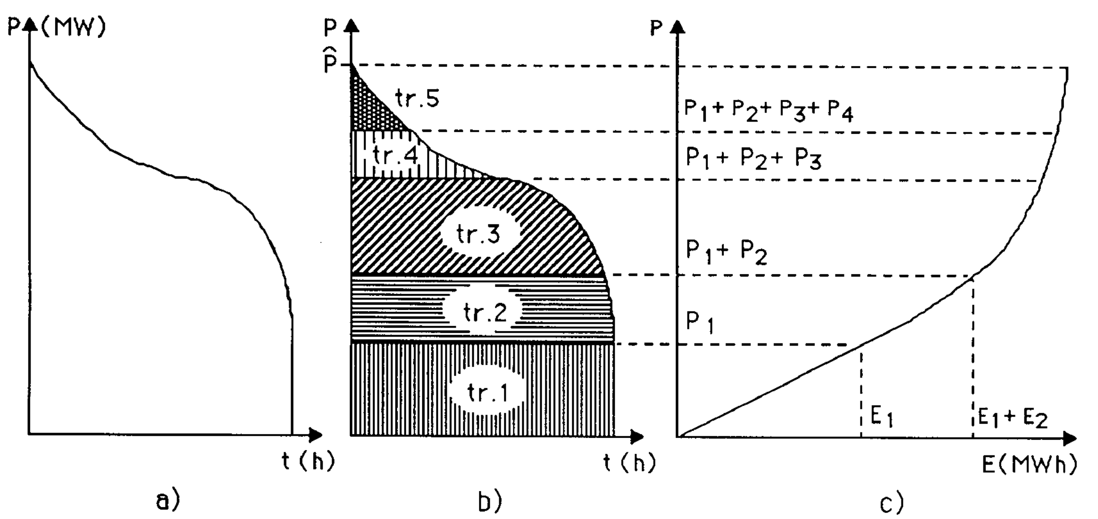

Fig. 3 a) Load duration curve
b) Load duration curve covered by the first five thermal units in merit order
c) Power-Energy curve with indications of cumulative powers and energies corresponding to the thermal units that cover the load duration curve

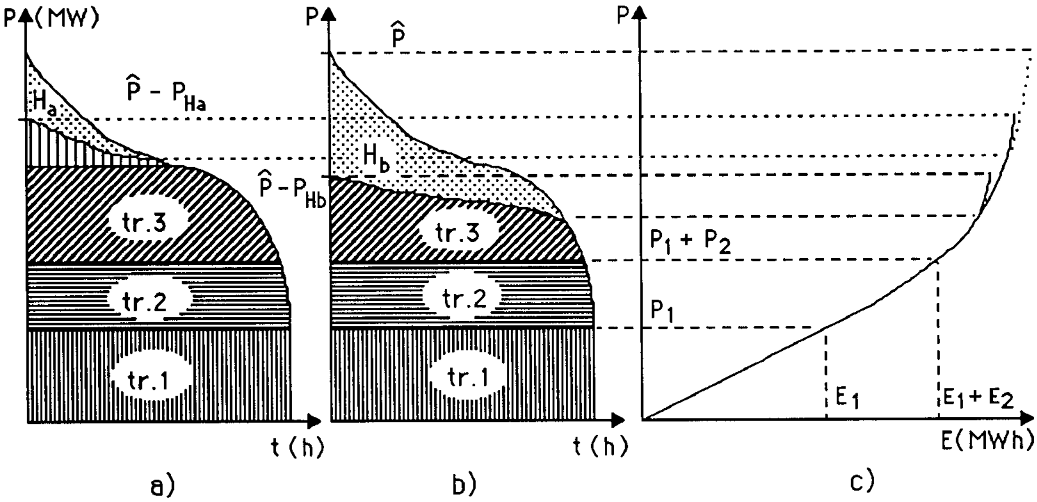

Fig. 4 a) Load-duration curve of a given interval covered by hydro-generation H_a and by thermal units
b) Load-duration curve of a given interval covered by hydro-generation $H_b > H_a$ and by thermal units
c) PE curves seen by thermal units for hydro-generations H_a and H_b.

5. OBJECTIVE FUNCTION AND NETWORK CONSTRAINTS

The objective function to minimize with respect to hydro-generation at each interval and energies generated by each thermal unit at each interval is the sum of the amounts of the Nf fuels acquired multiplied by their costs:

$$\min \sum_{k=1}^{Nf} \sum_{j=1}^{Nu} c_j^k f_j^k \tag{5}$$

Hydrogeneration H^i is supposed to be the expected value of hydro-generation at interval "i".

As implied by Fig. 2, a balance equation must be satisfied at the node corresponding to each thermal unit at each interval. For thermal unit "j" at interval "i":

$$r^k_{j,i-1} - r^k_{ji} - \varepsilon^k_j e^k_{ji} = 0\ , \quad \begin{cases} i=1,\cdots,Ni \\ j=1,\cdots,Nu \\ k=1,\cdots,Nf \end{cases} \tag{6}$$

limits have to be satisfied by flows on arcs:

$$r^k_{ji} \le \bar{r}^k_j\ ,\ e^k_{ji} \le \bar{e}^k_j\ , \quad \begin{cases} i=1,\cdots,Ni \\ j=1,\cdots,Nu \\ k=1,\cdots,Nf \end{cases} \tag{7}$$

and mutual capacity constraints with respect to the different possible fuels on each arc must be imposed

$$\sum_{k=1}^{Nf} r^k_{ji} \le \bar{r}_j\ ,\ \sum_{k=1}^{Nf} e^k_{ji} \le \bar{e}_j\ ,\ j=1,\cdots,Nu\ ,\ i=1,\cdots,Ni \tag{8}$$

6. APPROXIMATING THE POWER-ENERGY CURVE

The PE curve is approximated here by two connected segments: a straight line (S_1,B_1) through the origin and a Bézier curve (ref. 10) generated with four points (B_1, B_2, B_3 and B_4), the line uniting B_3 and B_4 being almost vertical and with B_4 defined by the peak load (as seen by the thermal units) and the total thermal energy. Points S_1,B_1 and B_2 are on the same straight line so that there is continuity in the first derivative of the curve at the linking point B_1.

Assuming that the coordinates (power and energy) of points B_i, i=1,2,3,4, in Fig. 6 are (P_{Bi} , E_{Bi}) i=1,2,3,4, the PE curve can be expressed as follows: for power between 0 and P_{B1} (base power of l.d.c.)

$$P = E/T \tag{9}$$

and for power higher than P_{B1} (or energy greater than $E_{B1} = P_{B1}*T$)

$$P = P_{B1}(1-\beta)^3 + 3P_{B2}\beta(1-\beta)^2 + 3P_{B3}\beta^2(1-\beta) + P_{B4}\beta^3$$

$$E = E_{B1}(1-\beta)^3 + 3E_{B2}\beta(1-\beta)^2 + 3E_{B3}\beta^2(1-\beta) + E_{B4}\beta^3$$

$$0 \le \beta \le 1 \tag{10}$$

which is a parametric curve in β.

Finding either P or E corresponding to a given E or P, means finding a root (between 0 and 1) of a third degree polynomial (10). In the programs developed this is done using Cardano's algorithm (refs. 11,12). Expressions of derivatives of P of (10) w.r.t. E and w.r.t. H are easy to obtain and can be found in ref. 13. Thus the derivatives of expressions (1) and (2) with respect to the thermal energy E and those of the expressions (3) and (4) with respect to E and H can be obtained without difficulty.

Changes in the PE curve due to hydro-generation are taken into account changing the extreme points of the segments (B_1 and B_4) and the position of the other Bézier points (B_2, B_3) as illustrated in Fig. 5a) and 5b). $E_{B4}(H)$ is $E_{B4}(H)=\hat{E}-H$ and the ordinate $P_{B4}(H)$ is modelled to change as

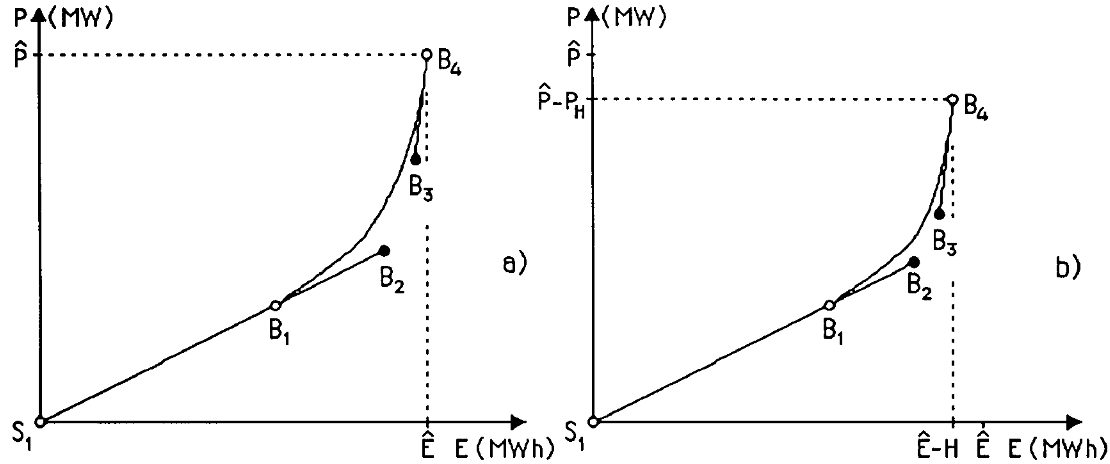

Fig. 5 a) Approximation by two segments of Power-Energy function of load duration curve
b) Approximation by two segments of Power-Energy function of load duration curve seen by thermal units when there is hydro-generation H.

$$P_{B4}(H) = \hat{P} - \bar{P}_H\left(1 - e^{-\lambda H}\right) \qquad (11)$$

B_1, B_2 and B_3 are changed as second degree polynomials of H

$$P_{Bl}(H) = b_{Pl} + l_{Pl}H + q_{Pl}H^2$$
$$l=1,2,3 \qquad (12)$$
$$E_{Bl}(H) = b_{El} + l_{El}H + q_{El}H^2$$

The coefficient λ of the exponential (11) and those of the polynomials b_{Pl}, l_{Pl}, q_{Pl}, b_{El}, l_{El}, q_{El}, l=1,2,3, of (12) are estimated beforehand by constrained least squares' fitting with a series of values of hydrogeneration H for the l.d.c. of each interval (ref. 14).

7. THE POWER-ENERGY CURVE FOR HYDROGENERATION BEYOND ITS BOUNDS

The minimization of (5) subject to the generalized network constraints (6-8) and the nonlinear side constraints (3-4) goes through an initial stage of finding a feasible point. Even though lower and upper bounds on H^i are placed: $0 \le H^i \le \overline{H}^i$ (i=1,...,Ni), in the initial stage there can be values of H^i well beyond its bounds. In this situation it is necessary for the power-energy curve generated PE(H) to keep providing sensible values of P and E. Figure 6 shows the shapes of the power-energy curve for such values of H ($H^i \le 0$ and $H^i \ge \overline{H}^i$).

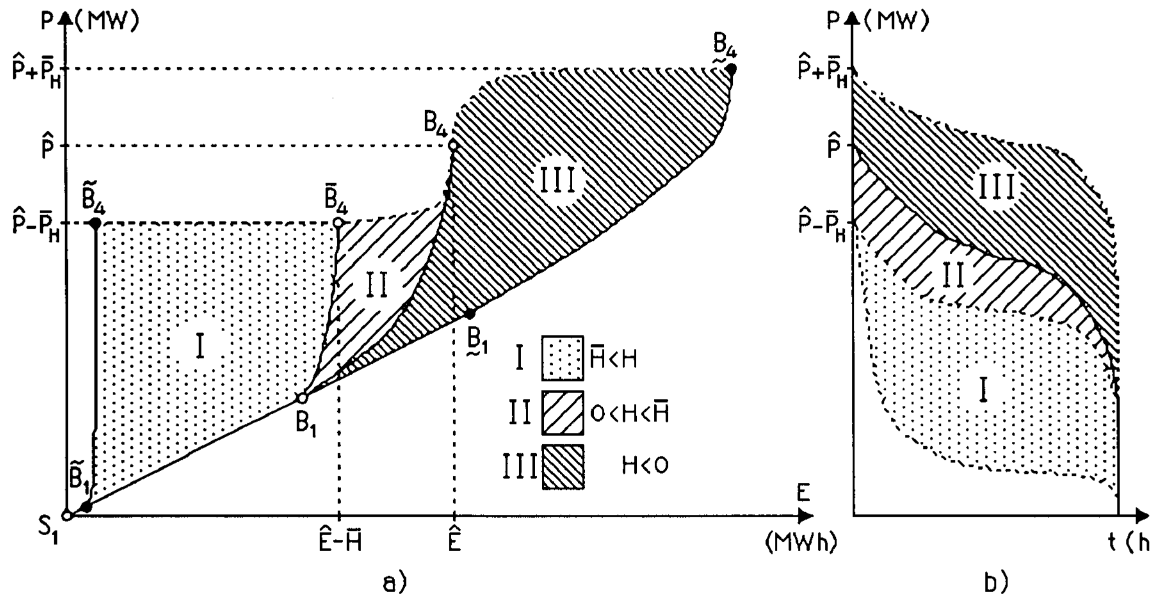

Fig. 6 a) Families of power-energy curves for hydrogeneration within (II), and out of range (I and III)
b) Families of load duration curves seen by thermal units for hydrogeneration within (II), and out of range (I and III)

PE functions for values of H^i within limits are those inside zone II in Fig. 6. When $H^i \geq \overline{H}^i$ PE functions are in zone I and the Bézier curve of the PE function recedes towards the curve $(\tilde{B}_1,\tilde{B}_4)$ as H^i increases. $(\tilde{B}_1,\tilde{B}_4)$ is a limit curve used for $H^i \geq \overline{H}^i$ increasing and tending to infinity. A curve like $(\underset{\sim}{B}_1,\underset{\sim}{B}_4)$ is the limit curve employed for $H^i<0$ decreasing and tending to minus infinity.

8. CONSIDERATION OF FORCED OUTAGE RATES

Forced outage rates of thermal units (ref.6) can be taken into account by considering that the l.d.c. to be covered by hydro and thermal generation has a different shape but the same energy $\hat{E}$. The change in shape must be such that there is a reduction in the maximum generation time T of the interval proportional to the mean availability of base units so that, as shown in Fig. 7, the maximum generation time would be $\tilde{T}$. Moreover, the peak load gets to $\tilde{P}>\hat{P}$ as in Fig. 7, and the peak units generate for a longer time than they would with the normal l.d.c. while keeping the total area under the curve.

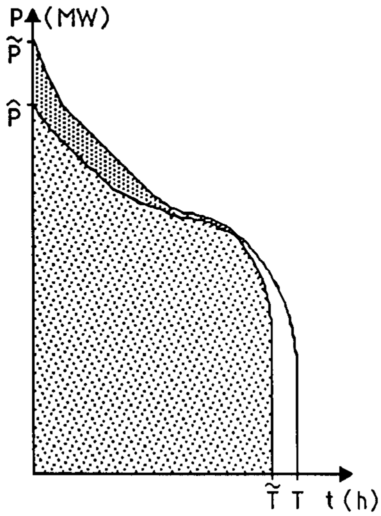

Fig. 7 Equivalent load duration curve taking into account approximately forced outage rates

9. HYDROGENERATION MODEL

Hydroenergies H^i at each interval should correspond to expected values of hydro-generation, taking into account the stochasticity of natural water inflows in the reservoir system.

Details of a hydro model that could be employed to obtain these expected values for reservoir systems with totally or partially dependent water inflows can be found in ref. 1. Using simpler hydro-models is possible too. Fig. 8 depicts a very simple model where a total hydroenergy TH will be distributed in an optimal way into the hydroenergies H^i, $i=1,...,Ni$. Either the simplified or the complete hydro model must be added to the thermal model described, so that all the thermal variables and hydroenergies H^i are optimized by the same program.

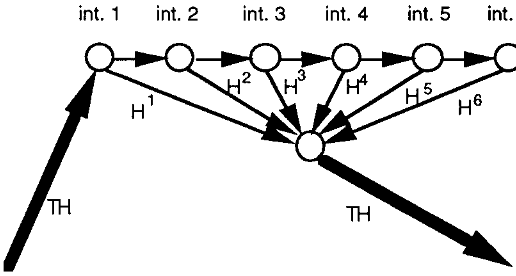

Fig. 8 Exemple of simple hydro-generation model with six intervals

10. NUMERICAL IMPLEMENTATION AND COMPUTATIONAL RESULTS

The model put forward can be solved with a general purpose constrained nonlinear optimization package, and many tests have been carried out using the MINOS package (ref.9). Some relevant characteristics of the programs developed are:

• Quite long subsidiary programs to prepare data of PE curves from load duration curves and hydroenergies must be used. These programs have also been developed.

• In general it can be said that convergence to the solution is slow, due to nonlinearities in the constraints.

Fig. 9 shows the graphical output of the results obtained using the model proposed. These results correspond to a 12 interval, 10 thermal unit, long-term hydrothermal coordination problem with fuel limits. The simplified hydro-model of Section 9 and only one fuel for each thermal unit have been considered, and fuel limits are active for units #2 and #4. The l.d.c's of the intervals correspond to a real case. There is nuch information on the picture of the l.d.c. of each interval:

- Dashed lines show the profile of the total l.d.c. and of the l.d.c. seen by thermal units.
- Peak load values are on top of the vertical axis.
- At the upper right corner of each l.d.c. picture there is the energy and maximum hydropower generated $(\overline{P}_H(1-e^{-\lambda H}))$.
- Hydrogenerations peak-shave the l.d.c's of the intervals. The optimal value at each interval is seen in Fig. 9. It should be pointed out that at interval 6 there is zero hydrogeneration, and that this is not the interval with the lowest peak load.
- Thermal units with unlimited fuel supply (all but #2 and #4) intervene at rated capacity in a given priority order at each interval. The units with fuel limits (#2 and #4) are used at some intervals at less than their rated capacity (e.g.: unit #2 at intervals 4,5,6,8,9 and 10).

The following remarks are in order:

• There are slight differences between the real and the approximated l.d.c. (using Bézier's curves). These differences are not too important given that the real l.d.c's considered are the result of transforming the l.d.c's predicted to account for forced outage rates (see Section 8.).

• At some intervals peaking units or fuel limited units may have a load at less than their minimum rated capacity (e.g.: unit #6 at interval 7 or unit #2 at interval 8). Additional techniques, described in a forthcoming paper, have been developed to avoid this type of results.

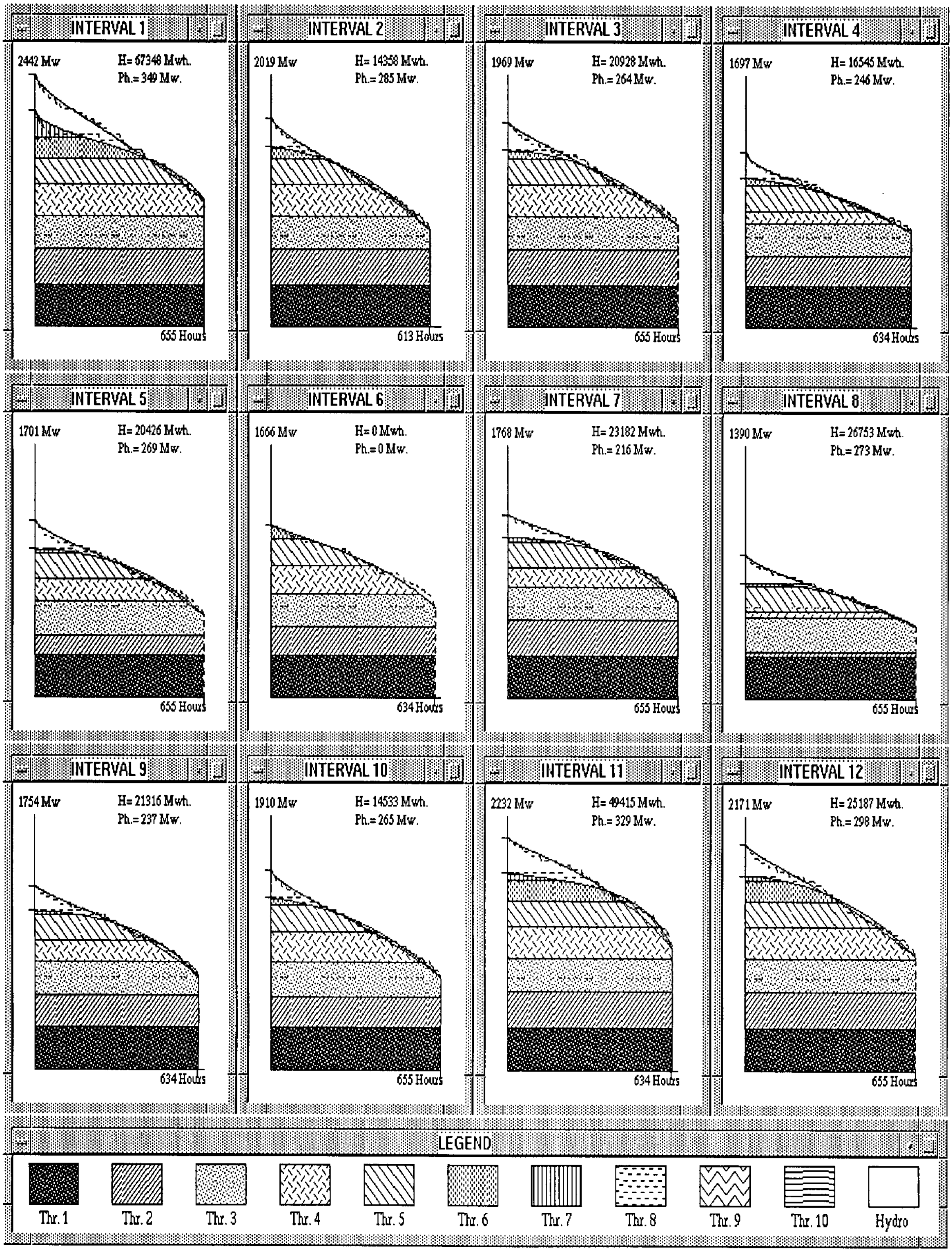

Fig. 9 Results of problem b' showing hydro and thermal generations on load duration curves of intervals

• It must be stressed that the MINOS code employed could not always find the solution to a given problem, even when one existed. In this regard the use of the techniques described in Section 7 proved to be of great help in many cases to reach a feasible point, but it was not sufficient in other problems with different data. Work is underway on devising procedures to obtain initial points from which to reach the solution, and other minimization packages are being tried.

A sample of required computation times using the MINOS package is given in the following table. The computer used is a SUN Sparc 2 Station. (Case b' corresponds to the solution described in Fig. 9.)

case	variab	network constr.	nonlinear constr.	fuel limits	iter.	CPU seconds
a	107	43	66	1	1649	46.6
b	394	164	252	-	1665	141.7
b'	394	164	252	2	3208	340.3

11. CONCLUSIONS

A model for long-term hydro-thermal coordination based on the use of the PE curves to satisfy power and energy constraints has been presented.

The results obtained using the model put forward are consistent with the operating experience of hydrothermal systems and meet the expectations. Many other extensions and refinements of the methodology proposed are possible and some are being pursued.

12. GLOSSARY OF SYMBOLS

b_{Pl}, l_{Pl}, q_{Pl} basic, linear and quadratic coefficient of polynomial expressing power coordinate of point "l" of Bézier's polygone of PE curve

b_{Pl}, l_{Pl}, q_{Pl} basic, linear and quadratic coefficient of polynomial expressing energy coordinate of point "l" of Bézier's polygone of PE curve

c_j^k cost of fuel "k" for thermal unit "j"

$E, \hat{E}$ thermal energy and total thermal energy of load-duration curve

$E_{Bl}(H)$ energy coordinate of Bézier's point B"l"

e_{ni}, e_{ni}^k thermal energy used at unit "n" in interval "i" to generate power, without and with indication of fuel "k" used

f_j^k amount of fuel "k" acquired for thermal unit "j"

H, H^i hydro-energy, hydro-energy generated in interval "i"

i (subscript or superscript) indicates interval "i"

j (subscript) indicates thermal unit "j" in merit order

k (superscript) indicates fuel "k"

l (subscript) indicates point of Bézier's polygon

l.d.c. load duration curve

n (subscript) indicates one of the thermal units

$\hat{P}, \hat{P}^i$ peak load and peak load in interval "i"

$P_{Bl}(H)$ power coordinate of Bézier's point B"l"

$PE, PE(H^i)$ power-energy curve and power-energy as a function of hydroenergy at interval "i"

$P_H, P_H^i, \bar{P}_H$ generated hydro-power, generated hydro-power at interval "i", and maximum hydor-power capacity

PE power-energy curve

$PE(H^i)$ power-energy curve as function of hydro-generation in interval "i"

r_{ni}, r_{ni}^k fuel remaining at the stockpile of unit "n" at the end of interval "i", without and with indication of fuel type "k"

T duration of interval

β parameter of Bézier curve

ε_n efficiency in power generation at unit "n"

λ constant of exponential expressing increase of hydrogeneration power with hydroenergy

$\bar{\ }$ (accent) denotes upper limit of vector or variable

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