Using ACCPM in the master problem of a restricted simplicial decomposition algorithm for the traffic assignment problem

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The traffic assignment problem attempth to find the distribution of the traffic flow throughout a network of routes. It is possible to formulate the problem by means of a network model that represents the physical infrastructure and aims to compute the flows of one or more commodities on the links of the network, (each commodity being related to the flows from an origin to a destination), based on the principles of optimization, which characterize the use of the routes from origins to destinations on the corresponding network.

Whenever congestion phenomena are present, the cost functional associated with the links of the network model are nonlinear and, in most applications convex or monotone. When interactions between network links are present, the problem is known as asymmetric traffic assignment problem and it can be formulated as a variational inequality problem (Smith [8], Dafermos [3]).

The variational inequality problem, $VI(F,Y)$, solves the traffic assignment problem. It can be stated as:

$$\text{Find } y^* \in Y \text{ such that } F(y^*)^t(y - y^*) \geq 0, \quad \forall y \in Y, \tag{1}$$

where F is a continuous, pseudo-monotone mapping and Y is a nonempty, closed, convex subset of $\mathbb{R}^m$. It can be proved that for "pseudo-monotone mappings"$y^* \in Y$ solves the $VI(F,Y)$ if

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and only if $y^* \in Y$ and

$$F(y)^t(y - y^*) \geq 0 \quad \forall\, y \in Y. \tag{2}$$

This effectively means that the solution set Y^* of $VI(F, Y)$, which eventually might consist of a unique point, is defined as the intersection of all the half-spaces defined by (2). In other words, there is a convex feasibility formulation of $VI(F, Y)$, with the feasible set Y^* implicitly defined by the infinite family of cutting planes (2). (2) ensures both the convexity and closedness of Y^*, while $Y^* \subset Y$ ensures its boundness.

Generated cutting planes are considered a closer approximation to the solution set of VI. In the iterated point y_k, the half-space

$$\{y \in I\!R^m | F(y_k)^t(y_k - y) \geq 0\}$$

is added to the current solution set, and the following point y_{k+1} is chosen inside the new solution set. Goffin's, Marcotte's and Zhu's ACCPM (Analytic Center Cutting Plane Method) [5] uses an approximation of the analytic center. The computation of the analytic center is made by Newton's primal-dual method which uses this last centre as a starting point.

There are several possible ways of solving the variational inequality problem (1). One way is to apply the analytic center cutting plane method for variational inequalities directly (possibly exploiting the multicommodity structure of the problem [2]). Another approach would be to apply a simplicial decomposition technique to the problem (1).

This is the method considered in this work. We have to solve two problems at each iteration of the simplicial decomposition technique, a linearized subproblem (which is a shortest path assignment problem) and the master problem which is itself a variational inequality problem that can be solved by many effective methods, such as the projection method [1, 6] and the linear approximation methods. The analytic center cutting plane method can also be adapted to solve the variational inequality problem arising in the master problem of the simplicial decomposition scheme. We used this latter approach in this work.

The RSDVI programme (proposed by L. Montero in [7]) has been adapted to solve the variational inequality of the master problems. Originally RSDVI implemented the linear projection method proposed in [1] to find the solutions to such problems. The linear projection method has been replaced by the ACCPM in this work.

Three variants of ACCPM were programmed. The first solved the following problem master:

$$\text{Find } \lambda^* \in \Lambda \text{ such that } E^t F(E\lambda^*)(\lambda - \lambda^*) \geq 0, \quad \forall \lambda \in \Lambda$$

where E is a matrix $n \times m$ which colums are the extrem flows and n is the number of links.

$$\text{with } \Lambda = \{\lambda \mid \sum_{i=1}^{m} \lambda_i = 1, \ -\lambda_i \leq 0\},$$

where both equality and inequality constraints were considered, following the the idea of Denault and Goffin [4].

In the second variant the same problem is solved, through the following set

$$\Lambda = \{\lambda \mid \sum_{i=1}^{m} \lambda_i \leq 1, -\sum_{i=1}^{m} \lambda_i \leq -1 \ -\lambda_i \leq 0\},$$

where equality constraints are duplicated into two inequalities [5].

In the last case we used previous set and a heuristic to find the feasible initial analytic center for the variational inequality of the master problem. The heuristic attempts to find an initial ϵ-pseudo feasible point, and it can be adjusted through the ϵ-parameter

The problems that have been used for testing purposes are: Sioux Falls, Barcelona, Winnipeg and Madrid. (See Table 1).

	Nodes	Centroids	Links	OD pairs
Sioux Falls	48	24	124	528
Barcelona	930	110	2522	7922
Winnipeg	1017	154	2976	4345
Madrid	2776	490	6871	26037

Table 1. *Test networks description*

The following tables show the results obtained from asymmetric problems: artificially built by adding interactions between incoming links at intersections. The *initial gap* refers to the relative gap of the first major iteration. The *final gap* is the desired aproximation of the solution in gap terms. *Major it.* gives the necessary iterations to reach the criteria of convergence. *Minor it.* gives the average number of minor iterations for solving the master problems. $|W^{(t)}|$ is the maximum cardinality of $W^{(t)}$ i.e. the maximum number of extreme points (links space). *Global-CPU* gives the total execution time in seconds. *M.P.-CPU* is the execution time spent in the solution of the master problem, in seconds. All the runs were carried on a Sun-4, SPARC-based with a CPU of 198.3 MHz.

Table 2 shows the computational results for the asymmetric problems using the linear projection method for solving the master problem.

Problem	SIO	BCN	WIN	MAD
initial gap	.427E+02	.156E+04	.249E+03	.154E+05
final gap	.887E+00	.984E+00	.769E+00	.922E+00
major It.	12	69	11	39
minor It.	6.5	8.35	3.9	7.25
$\lvert W^{(t)}\rvert$	14	71	13	41
Global-CPU	0.2	533.1	6.6	610.8
M.P.-CPU	0.2	518.5	3.0	403.0

Table 2. *Results for TAP using linear projection method for the master problem*

Each cell of Table 3 gives the information for the first, second, and third variant of the ACCPM respectively.

Problem	SIO	BCN	WIN	MAD
initial gap	.427E+02	.156E+04	.249E+03	.154E+05
	.427E+02	.156E+04	.249E+03	.154E+05
	.412E+02	.148E+04	.240E+03	.147E+05
final gap	.887E+00	.984E+00	.769E+00	.922E+00
	.837E+00	.882E+00	.768E+00	.876E+00
	.502E+00	.910E+00	.757E+00	.775E+00
major It.	12	69	11	39
	12	66	11	41
	11	45	8	31
minor It.	89.33	379.21	86.0	238.07
	45.16	172.85	48.45	131.42
	16.1	34.87	11.25	34.19
$\|W^{(t)}\|$	14	71	13	41
	14	68	13	43
	13	47	10	33
Global-CPU	11.3	127616.9	33.8	3185.1
	3.5	6065.4	17.6	1280.5
	0.3	77.0	5.1	270.0
M.P.-CPU	11.0	127602.5	30.4	2977.2
	3.4	6050.5	14.1	1058.7
	0.3	66.7	2.1	104.7

Table 3. *Results for TAP using the three implemented vatiants of the ACCPM*

It has been observed that the method of linear projections still continues to be competitive, compared to the ACCPM. However, when the heuristic is applied, ACCPM-variant 3 seems to be more efficient as fewer cuts are made. The only disadvantage is that feasibility is lost in the traffic assignment problem, leading to an "infeasible optimal solution". In fact, there is a trade-off between feasibility and efficiency which can be controlled by the user through the ϵ-parameter.

For the separable and symmetric Winnipeg problem Table 4 shows the objective function value of the equivalent mathematical programming formulation, for several ϵ-parameters and the linear projection method. The number of both major and minor iterations are also given as well as the required CPU for the master problem.

	Winnipeg				
	$\epsilon = 10^{-2}$	$\epsilon = 10^{-4}$	$\epsilon = 10^{-6}$	$\epsilon = 10^{-7}$	RSDVI
major It.	10	16	16	16	16
minor It.	7.8	42.87	79.18	96.5	3.18
M.P.-CPU	1.8	18.2	42.1	57.7	3.3
Obj(v^*)	704207.8	705252.4	705277.1	705277.2	705277.2

Table 4. *feasibility vs. efficiency*

We also compared in statistical terms the resulting optimal solutions in the link flow space for ACCPM-variant 3 (infeasible) and for RSDVI (feasible), since the heuristic variant is competitive in time. Let wL and wD be the vectors of optimal links flows obtained with the linear projection method and ACCPM-variant 3 respectively.

For the Winnipeg problem, removing the zero link flows, which results in a reduction from 2976 to 2359 links, and using $\epsilon = 10^{-2}$ for ACCPM-variant 3, the linear regression between the wL and wD is showed in Figure 1.

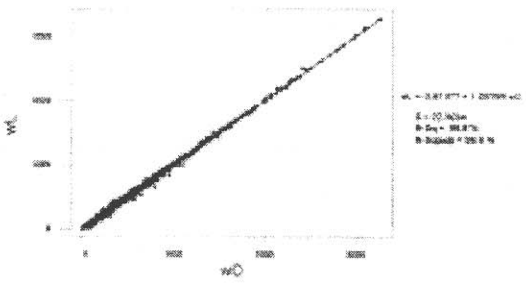

Figure 1. *Comparison of optimal link flow by regression*

In Figure 2 the difference of the two volumes is compared to the average of them.

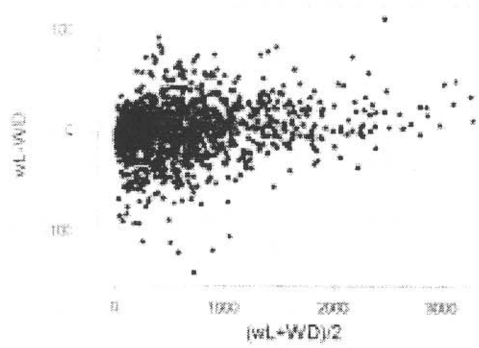

Figure 2. *Comparison of optimal links flows: RSDVI vs. ACCPM-variation 3*

At first sight, discrepancies in link flow volumes tend to decrease as link flow increases, this means, that for links with low flow, (relative) differences are expected to be high. To refine the analysis, the average of the link volumes are separated in groups and relative error $|wL - wD| / ((wL + wD)/2) * 100$, is calculated. Table 5 shows the number of observations in each group, the mean relative error, the relative error value that is only exceeded for 25% of the observations in the same group (denoted as Q3), and the relative error interval of outliers.

Group	No.	Mean	Q3	I. of Outliers
$0-500$	1330	10%	12%	$(27, 195)$
$500-1000$	514	3%	4%	$(9, 22)$
$1000-1500$	163	2%	2%	$(6, 10)$
$1500-2000$	61	1%	2%	$(4, 4)$
$2000-2359$	49	1%	1%	$(3, 5)$

Table 5. *Relative errors for link flow groups*

It is observed that the relative errors decrease when the link flow volume increases.

Conclusions

We can conclude that ACCPM-variant 3 solutions are as much *similar* to the solutions obtained with the linear projection method as ϵ-parameter is decreased.

Solving the master problem with ACCPM-variation 3 leads to an optimal user equilibrium assignment flow for TAP that in fact is related to a TAP problem with a perturbed demand OD matrix. It is possible to compare the perturbed OD matrix estimated by a bi-level scheme proposed by Spiess, and implemented in the EMME2 software, with the original OD matrix. In the estimation process, exact solutions to TAP in the link flow-space could be used as traffic counts in the adjustment macro. This is part of the additional work to be done.

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