

# A mixed-integer stochastic programming model for the day-ahead and futures energy markets coordination

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## Introduction

Introduction and motivation

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## MIBEL Futures Market

MIBEL Futures Contracts

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# Introduction and motivation

- ▶ The recent creation of short term futures markets in the MIBEL and its particular rules
- ▶ The existence of futures market in most of the liberalized power markets around the world
- ▶ The fact that coordination between short term futures and spot markets is necessary for a GENCO
- ▶ Analyze hedging in electricity markets and interaction between physical production and electricity futures contracts

# Electric Energy Iberian Market: MIBEL



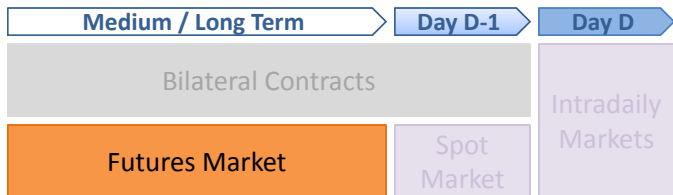
# Electric Energy Iberian Market: MIBEL



## Main characteristics of bilateral contracts:

- ▶ Non organized market
- ▶ Physical bilateral contracts
- ▶ Minimum contract duration one year

# Electric Energy Iberian Market: MIBEL



## OMIP's main characteristics:

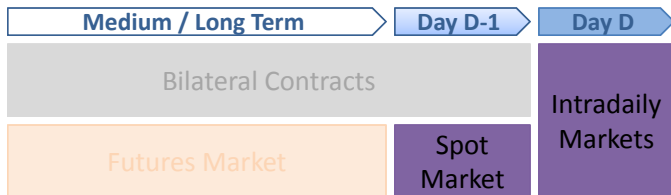
### Physical Contracts

Physical Settlement
Positions are sent to OMEL's Mercado Diario for physical delivery
Financial Settlement
OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

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# Electric Energy Iberian Market: MIBEL



## OMEL's main characteristics:

- ▶ Organized markets
- ▶ Spot market:
  - ▶ The matching procedure takes place 24h before the delivery period
  - ▶ Hourly auction

# MIBEL Futures Contracts

## Main characteristics:

- ▶ Base load
- ▶ Physical or financial settlement.
- ▶ Delivery period: years, quarters, months and weeks.



# MIBEL Futures Contracts

## Main characteristics:

- ▶ Base load
- ▶ Physical or financial settlement.
- ▶ Delivery period: years, quarters, months and weeks.

## Definition:

- ▶ A *Base Load Futures Contract* consists in a pair  $(L^f, \lambda^f)$ 
  - ▶  $L^f$ : amount of energy (MW) to be procured each interval of the delivery period.
  - ▶  $\lambda^f$ : price of the contract (€/MW).

# Physical Base Load Futures Contracts

## Market physical settlement rules:<sup>1</sup>

- ▶ *At least two days prior to the physical delivery day, physical delivery futures contracts are entered as orders at 'acceptance price' in the call auction of OMEL's Mercado Diario*
- ▶ *Before the call auction each Physical Settlement Agent must specify which production/consumption units are to be allocated to the orders.*

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<sup>1</sup>Omicp/Omiclear Operational Guide

# Problems associated to the Futures Market

## Optimal bidding at futures market:

- ▶ During the trading period the GENCO could send bids for all products opened in the Futures Market.

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## **Futures contract energy allocation:**

- ▶ Given the portfolio of futures contracts with physical-delivery the GENCO has to decide how to allocate the energy among the offer to the spot market.

## Characteristics of the study

- ▶ The model currently developed is restricted to:
  - ▶ A *Price Taker* generation company
  - ▶ A set of thermal generation units,  $T$
  - ▶ An optimization horizon of 24h,  $I$
  - ▶ A fan of spot market price scenarios,  $S$
- ▶ It has been implemented with AMPL, without exploiting the structure of the problem, and it has been solved with CPLEX.
- ▶ The main objective of the computational tests is to evaluate the coherence of the proposed methodology.

## Optimal bid curve for thermal unit $t$ (I/II)

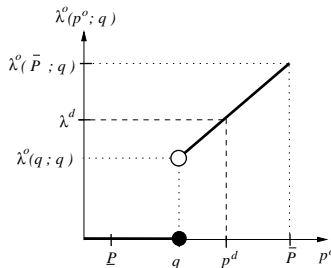
- ▶ Let  $q_i^t$  be the generation of thermal  $t$  at time  $i$  allocated to all the physical contracts of the portfolio.
- ▶ The market rules forces each generator to send the amount  $q_i^t$  to the day-ahead market through an *instrumental price bid* (bid at zero price).
- ▶ For a given value  $q_i^t$ , the *optimal bid curve* is the function  $\lambda_i^{o,t}(p_i^{o,t}; q_i^t)$  that provides the energy-price pairs  $(p_i^{o,t}, \lambda_i^{o,t})$  that maximize the benefit function for any given spot price  $\lambda_i^d$

## Optimal bid curve for thermal unit $t$ (II/II)

- ▶ The expression of the optimal bid curve for thermal unit  $t$  at time interval  $i$ , for a given  $q_i^t$ , is:

$$\lambda_i^{o,t}(p_i^{o,t}; q_i^t) = \begin{cases} 0 & \text{if } 0 \leq p_i^{o,t} \leq q_i^t \\ 2c_q^t p_i^{o,t} + c_t^t & \text{if } q_i^t < p_i^{o,t} \leq \bar{P}^t \end{cases} \quad (1)$$

graphically:





## Matched energy (I/II)

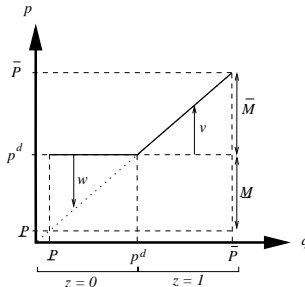
- ▶ Given a spot price  $\lambda_i^{d,s}$ , corresponding to scenario  $s$ , and a value  $q_i^t$ , the *matched energy*  $p_i^{ts}$  is completely determined through expression (1), and depends on the comparison between  $q_i^t$  and  $p_i^{d,ts}$ :

$$p_i^{ts} = \begin{cases} q_i^t & \text{if } q_i^t \geq p_i^{d,ts} \\ p_i^{d,ts} & \text{otherwise} \end{cases} \quad (2)$$

where the constant  $p_i^{d,ts}$  is the generation that maximizes the benefit function for a given spot-price  $\lambda_i^{d,s}$ .

## Matched energy (II/II)

- ▶ Expression (2) defines the matched energy  $p_i^{ts}$  as a piece-wise linear function of the zero priced bid  $q_i^t$



- ▶ This non-differential expression can be conveniently expressed through an equivalent mixed-linear formulation.

# Two-stage stochastic program formulation

- ▶ **Scenarios:** spot prices  $\lambda^{d,s} \in \mathbb{R}^{|I|}$ ,  $s \in \mathcal{S}$
- ▶ **First stage variables:**  $\forall t \in \mathcal{T}$ ,  $\forall i \in I$ 
  - ▶ **Instrumental price offer bid :**  $q_i^t$
  - ▶ **Scheduled energy for contract  $j$ :**  $f_{ij}^t$ ,  $\forall j \in F$
  - ▶ **Unit commitment:**  $u_i^t$ ,  $a_i^t$ ,  $e_i^t$ ,  $\forall i \in I$ ,  $\forall t \in \mathcal{T}$
- ▶ **Second stage variables:**  $\forall t \in \mathcal{T}$ ,  $\forall i \in I$ ,  $\forall s \in \mathcal{S}$ :
  - ▶ **Matched energy:**  $p_i^{ts}$
  - ▶ **Auxiliary variables:**  $z_i^{ts}$ ,  $v_i^{ts}$ ,  $w_i^{ts}$

# Objective function

$$\min_{q,f,u,a,e,p,z,v,w} \sum_{\forall i \in I} \sum_{\forall t \in T} c_{on}^t e_i^t + c_{off}^t a_i^t + c_b^t u_i^t + \sum_{s \in S} P^s \left[ (c_l^t - \lambda_i^{d,s}) p_i^{ts} + c_q^t (p_i^{ts})^2 \right] \quad (3)$$

Associated constants:  $c_{on}^t, c_{off}^t, c_b^t, c_l^t, c_q^t, P^s, \lambda_i^{d,s}$

## Physical Future contracts constraints

**Physical future contract covering:**

$$\sum_{t \in T} f_{ij}^t = L_j, \forall j \in F \quad (4)$$

**Instrumental price bid:**

$$q_i^t \geq \sum_{j \in F} f_{ij}^t, \forall t \in T, \forall i \in I \quad (5)$$

Associated variables:  $q_i^t, f_{ij}^t \in 0 \cup [\underline{P}^t, \overline{P}^t]$

Associated constants:  $L_j$

Start-up/Shut-down constraints:  $\forall i \in I, \forall t \in T$

$$a_i^t + e_i^t \leq 1 \quad (6)$$

$$u_i^t - u_{i-1}^t - e_i^t + a_i^t = 0 \quad (7)$$

$$a_i^t + \sum_{j=i+1}^{i+\min_{off}} e_j^t \leq 1 \quad (8)$$

$$e_i^t + \sum_{j=i+1}^{i+\min_{on}} a_j^t \leq 1 \quad (9)$$

Associated variables:  $u_i^t, a_i^t, e_i^t \in \{0, 1\} \cap \mathcal{U}^t$

Definition of the matched energy:  $\forall s \in S, \forall i \in I, \forall t \in T$

$$p_i^{ts} = p_i^{d,ts} u_i^t + v_i^{ts} \quad (10)$$

$$v_i^{ts} - w_i^{ts} = q_i^t - p_i^{d,ts} u_i^t \quad (11)$$

$$v_i^{ts} \leq \overline{M}^{ts} z_i^{ts}, w_i^{ts} \leq \underline{M}^{ts} (1 - z_i^{ts}) \quad (12)$$

$$\underline{P}^t u_i^t \leq p_i^{ts} \leq \overline{P}^t u_i^t \quad (13)$$

$$p_i^{d,ts} u_i^t + \underline{M}^{ts} (z_i^{ts} - 1) \leq q_i^t \leq p_i^{d,ts} u_i^t + \overline{M}^{ts} z_i^{ts} \quad (14)$$

$$\sum_{s \in S} z_i^{ts} \leq |S| u_i^t \quad (15)$$

Associated variables:  $p_i^{ts} \in 0 \cup [\underline{P}^t, \overline{P}^t], z_i^{ts} \in \{0, 1\}, v_i^{ts}, w_i^{ts} \geq 0$


Associated constants:  $p_i^{d,ts}, \overline{M}_i^{ts} = \overline{P}^t - p_i^{d,ts}, \underline{M}_i^{ts} = p_i^{d,ts} - \underline{P}^t$

## Price scenario generation

- ▶ Price Spot Market,  $\lambda_j^{d,s}$ , is a stochastic variable, in particular, a time serie.
- ▶ Time series study results in a ARIMA model:  
ARIMA (23, 1, 13)(14, 1, 21)<sub>24</sub>(0, 1, 1)<sub>168</sub><sup>2</sup>
- ▶ Price scenario construction:
  - ▶ Generation of 350 scenarios by time series simulation
  - ▶ Reduction of the number of scenarios <sup>3</sup>

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<sup>2</sup> Amell et Bernáldez *Previsió de preus i planificació de la producció al MIBEL*

<sup>3</sup> Gröwe-Kuska et al. *Scenario Reduction and Scenario Tree Construction for Power Management Problems* 



## Case study characteristics

- ▶ October, 24th and 25th 2006
- ▶ 10 thermal generation units (7 coal, 3 fuel) from a generation company with daily bidding to the MIBEL

$[\bar{P} - \underline{P}]$ (MW)	160-243	250-550	80-260	160-340	30-70
$min_{on/off}$ (h)	3	3	3	4	4

$[\bar{P} - \underline{P}]$ (MW)	60-140	160-340	90-340	110-157	110-157
$min_{on/off}$ (h)	3	3	4	4	4

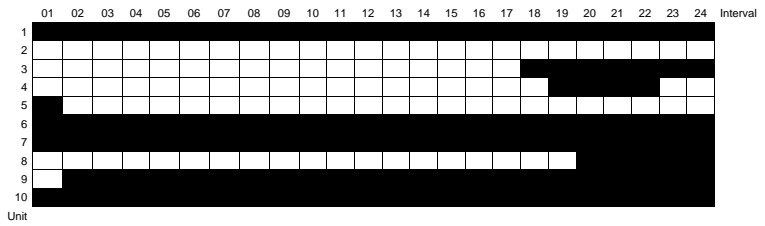
- ▶ 6 physical futures contracts

$L_f$ (MW)	20	150	320	50	200	150
$\lambda_f$ (c€/KW)	5.12	4.96	6.60	5.35	5.09	5.00

- ▶ 10 spot-market price scenarios

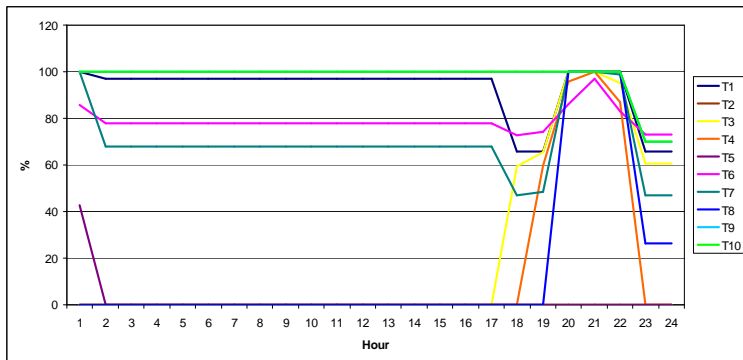
# Results (I/IV)

## Unit commitment



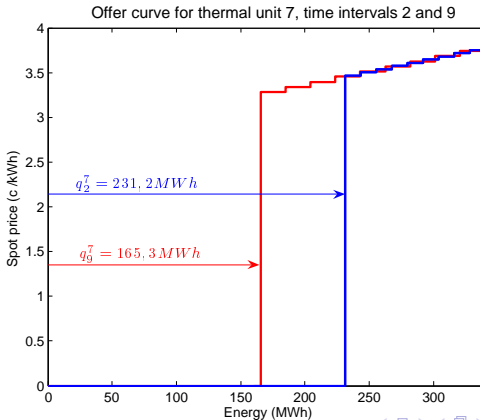
## Results (II/IV)

### Procurement of physical-delivery contracts



## Results (III/IV)

### Optimal bid



## Results (IV/IV)

### Futures contracts covering

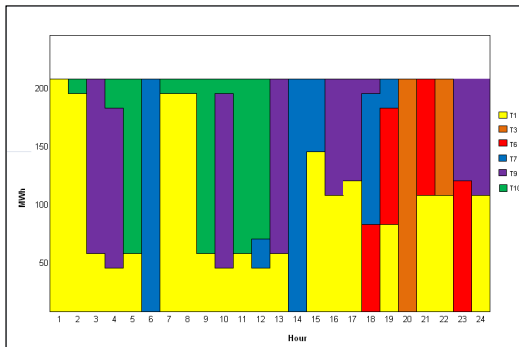


Figure: Futures contract #6

## Conclusions (I/II)

- ▶ It has been build an Optimal Bidding Model for a *price-taker* GENCO following in detail the MIBEL rules.
- ▶ The stochasticity of the spot price has been took into account and it has been fully represented by the scenario tree.
- ▶ The model developed gives the GENCO:
  - ▶ Optimal bid for the spot market: quantity at 0€/MWh and the rest of the power capacity at the unit's marginal cost
  - ▶ Unit commitment
  - ▶ Optimal allocation of the physical futures contracts among the thermal units

## Conclusions (II/II)

- ▶ Further developments:
  - ▶ Exploitation of the problem structure
  - ▶ Coordination with mid-term strategies
  - ▶ Inclusion of hydro units
  - ▶ Inclusion of emissions rights trading
  - ▶ Introduction of risk terms

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