A model for the joint optimization of the long-term generation planning and maintenance of the units

RETOBI: Seminario sobre Decisiones Bajo Incertidumbre

en el Sector Eléctrico

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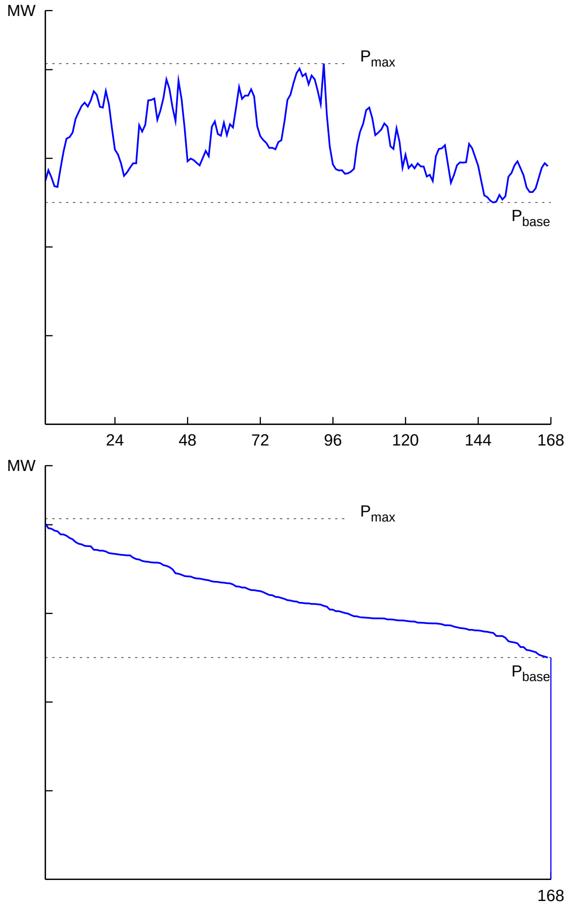
Introduction

Introduction and motivation

- Problem:
  the long-term generation planning and units' maintenance for a specific generation company participating in a liberalized market.
- Its results are used both:
  - for budgeting and planning fuel acquisitions.
  - to provide a framework for short-term generation planning.
- A long-term planning *period* is subdivided into shorter *intervals* of equal length, for which parameters are known or predicted.
- The variables are the expected energy productions of each generating unit, and its in service/on maintenance status over the intervals.

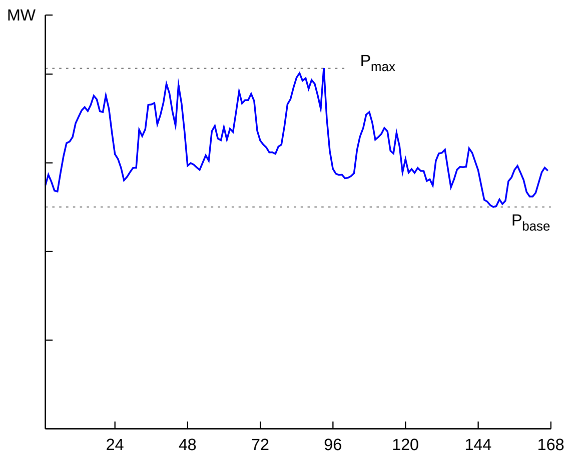
The long-term generation planning problem

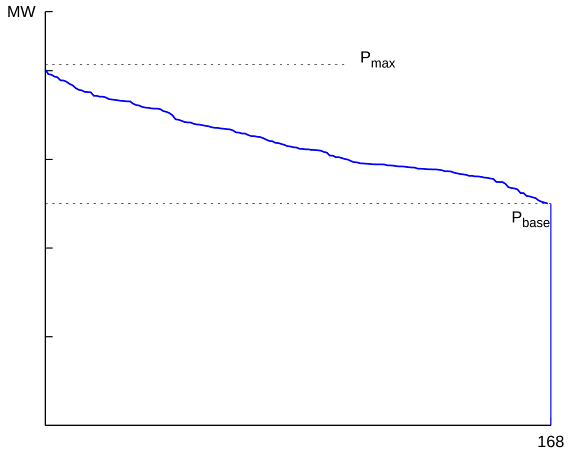
The load-duration curve & unit generators



$S_{\varnothing}(z)$: load-survival function

The load-duration curve & unit generators





$S_{\varnothing}(z)$: load-survival function

Unit characteristics:

- *power capacity*:
  (c_j) maximum power output (MW) that the unit can generate
- *outage probability*:
  (q_j) probability of a unit not being available when it is required to generate
- *linear generation cost*:
  (f_j) production cost in €/MWh
- *length of maintenance*:
  (m_j) number of intervals that the maintenance of the unit requires

Matching of the load

Suppose that ψ is a subset of units already loaded, and unit j is loaded next.

The expected energy generated by unit j is

$$x_j = t(1 - q_j) \int_0^{c_j} S_\psi(z) dz \,. \tag{1}$$

$S_\psi(z)$ is computed as proposed by Balériaux *et al.* through convolution.

This approach takes into account accurately the effects of random failures.

Bloom and Gallant established that, in order for the expected energies x_j, $j \in \Omega$, to match the LDC the linear inequality constraints, called *load-matching constraints* (lmc),

$$\sum_{j \in \psi} x_j \leq \widehat{e} - w(\psi) \qquad \forall \, \psi \subset \Omega$$

must be satisfied.

- $\widehat{e}$ is the total LDC energy
- $w(\psi)$ is the expected unsupplied load after all units in the subset ψ of thermal units are loaded

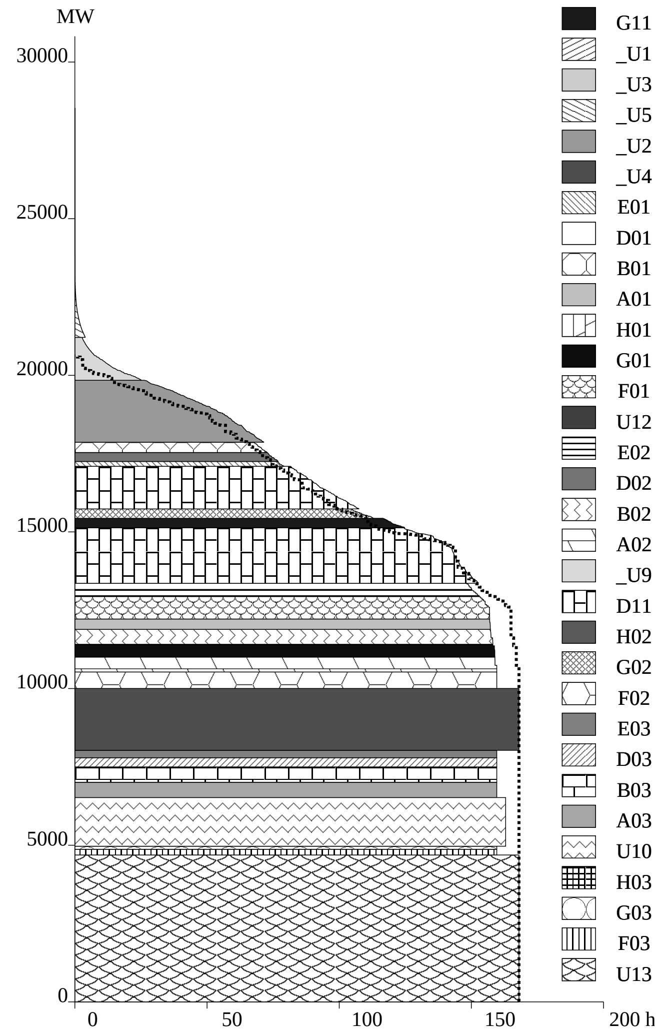
The total number of possible subsets ψ of the set Ω amounts to $2^{n_u} - 1$, $n_u = |\Omega|$.

Non load-matching constraints

Other constraints of the model (*non-lmc*), which are linear or can be linearized, are:

- Maximum hydro generation
- Bonus-schemed coal
- Minimum generation time
- Combined cycle generation
- Market-share constraints
- Maximum emission limits
- Extra constraints: take-or-pay contracts, . . .

LDC & GDC


MW
30000
25000
20000
15000
10000
5000
0
0
50
100
150
200 h
G11
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_U2
_U4
E01
D01
B01
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H01
G01
F01
U12
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D02
B02
A02
_U9
D11
H02
G02
F02
E03
D03
B03
A03
U10
H03
G03
F03
U13

Maximum profit objective function

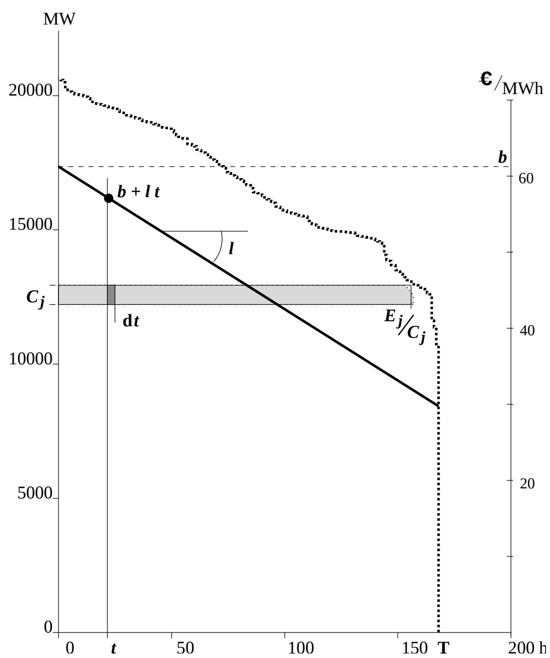
In a competitive market, the goal is to maximize the profit (revenue at market-price minus generation cost).
How do we compute the market profit?

The expected income for unit j is:

$$\int_0^{x_j^i/c_j} c_j\{b^i + l^i t - f_j\}\mathrm{d}t =$$

$$(b^i - f_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2$$

$b^i + l^i t$ is a regression of predicted market prices ordered by decreasing loads in interval i, with respect to the duration.



The Bloom and Gallant quadratic profit maximization formulation extended to n_i intervals is:

$$\begin{aligned} \underset{x_j^i}{\text{maximize}} \quad & \sum_i^{n_i} \sum_j^{n_u} \left\{ (b^i - f_j) x_j^i + \frac{l^i}{2c_j} {x_j^i}^2 \right\} \\ \text{subject to:} \quad & A\,x \geq a \\ & \sum_{j \in \psi} x_j^i \leq \widehat{e}^i - w^i(\psi) \qquad \forall \psi \subset \Omega \quad i = 1, n_i \\ & x_j^i \geq 0 \qquad \forall j \in \Omega, \quad i = 1, n_i \end{aligned} \tag{2}$$

Model to optimize the maintenance period

Optimization of the maintenance periods

Thermal units are stopped for maintenance during some weeks once a year. A Specific Generation Company (*sgc*) can choose when to stop its own units.

The goal is to place the maintenance period for each unit of the sgc and maximize the overall profit.

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Set:

- $\Phi \rightarrow$ Units whose maintenance has to be fixed ($\Phi \subset \Omega$).

Decision variables:

- $s_j^i \rightarrow$ State of the unit. It is 1 when unit j is available in interval i and 0 otherwise.
- $d_j^i \rightarrow$ It is 1 only in the interval in which unit j shuts down.
- $u_j^i \rightarrow$ It is 1 only in the interval in which unit j starts up.

Constraints

$\sum_{i=2}^{n_i} s_j^i = n_i - m_j - 1$	Unit j is available always except on the m_j intervals of maintenance
$\sum_{i=2}^{n_i - m_j} d_j^i = 1$	Unit j has only 1 maintenance period
$d_j^i + \sum_{l=i}^{i+m_j-1} u_j^l \leq 1$	Once unit j is shut down, it must be idle for m_j intervals
$d_j^i - u_j^{i+m_j} = 0$	Unit j starts up after m_j intervals
$s_j^i - s_j^{i-1} + d_j^i - u_j^i = 0$	Coordination between the binary variables
$x_j^i \leq c_j(1-q_j)t^i s_j^i$	Unit j can generate as long as it is in service

These constraints apply here only to the units of the sgc ($j \in \Phi$).

Joint model

$$\begin{aligned}
\underset{x_j^i, s_j^i, d_j^i, u_j^i}{\text{maximize}} \quad & \sum_i^{n_i} \sum_j^{n_u} \left\{ (b^i - f_j) x_j^i + \frac{l^i}{2c_j} {x_j^i}^2 \right\} \\
\text{subject to:} \quad & A\,x \geq a \\
& \sum_{j \in \psi} x_j^i \leq \hat{e}^i - w^i(\psi) \qquad \forall \psi \subset \Omega, \quad i = 1, n_i \\
& \sum_{i=2}^{n_i} s_j^i = n_i - m_j - 1 \qquad \forall j \in \Phi \\
& \sum_{i=2}^{n_i - m_j} d_j^i = 1 \qquad \forall j \in \Phi \\
& d_j^i + \sum_{l=i}^{i+m_j-1} u_j^l \leq 1 \qquad \forall j \in \Phi, \quad i = 2, n_i - m_j + 1 \\
& s_j^i - s_j^{i-1} + d_j^i - u_j^i = 0 \qquad \forall j \in \Phi, \quad i = 2, n_i \\
& x_j^i \leq c_j (1 - q_j) t^i s_j^i \qquad \forall j \in \Phi, \quad i = 1, n_i \\
& x_j^i \geq 0 \qquad \forall j \in \Omega, \quad i = 1, n_i \\
& s_j^i, d_j^i, u_j^i \in \{0, 1\} \qquad \forall j \in \Phi, \quad i = 1, n_i
\end{aligned} \tag{3}$$

Solution

Solution methods

- Direct methods:
  - need all the lmc
  - the search tree may be very large

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- Direct methods:
  - need all the lmc
  - the search tree may be very large
- Heuristic approach:
  - We relax two sets of constraints:
    - the load-matching constraints, except the all-one lmc
    - the integrity of the binary variables
  - We take advantage of the knowledge about the structure of the problem. We exploit the facts that:
    - active lmc are *nested*
    - if a unit generates it has to be in service ($s_j^i = 1$)

Heuristic outline

The relaxation in general form of the problem (3) is:

$$
\begin{aligned}
\text{minimize} \quad & h'x + \frac{1}{2}x'Hx \\
\text{subject to:} \quad & Ax \geq a \\
& B^i_{L^i} x^i \leq r^i_{L^i} \quad \forall i \\
& Cy = c \\
& 0 \leq x^i_j \leq \overline{x}^i_j s^i_j \quad \forall i \, \forall j \in \Phi \\
& 0 \leq x^i_j \leq \overline{x}^i_j \quad \forall i \, \forall j \in \Omega \setminus \Phi \\
& y^i_j = \{s^i_j, d^i_j, u^i_j\} \in [0,1] \quad \forall i \, \forall j \in \Phi
\end{aligned}
\tag{4}
$$

Sets and parameters used in the heuristic:

- $B^i_{L^i}, r^i_{L^i}$: Subset of lmc used in interval i.
  L^i is a list of lmc. Each element is a set of units which determines a unique lmc.
- ρ^i_j: Ratio between the variable x^i_j and its upper bound ($\overline{x}^i_j$).
- b^i: Set of units already nested in each interval.

Heuristic outline

Goal: To build a good loading order together with a maintenance planning.

The main steps of the heuristic are:

do $ite := 1, 2, \ldots$
- ▶ choose a subset of lmc
- ▶ fix some relaxed binary variables
- ▶ solve (4)

end

Example

Choose a subset of lmc

Iteration	
1st	$L^i = \{\Omega\} \quad \forall i$ (the all-one lmc) $b^i := \varnothing \quad \forall i$
2nd	$b^i := \{j \in \Omega \,\vert\, \rho^i_j \simeq 1\} \quad \forall i$ (set of units at upper bound) $L^i := L^i \cup \{\forall\, \xi \,\vert\, \xi \subset b^i\} \quad \forall i$ (any lmc made only of units at upper bound)
3rd, 4th, ...	Choose the pair (j, i) such that $\rho^i_j = \max_{\forall l \;\forall k \in \Omega \setminus b^l} \rho^l_k$ (the unit and int. with x^i_j closest to its upper bound) $b^i := b^i \cup \{j\}$ $L^i := L^i \cup \{b^i\}$

Back

Fix units

Iteration	
1st	Fixes no units
2nd, 3rd, ...	■ For each (j, i) such that $j \in b^i \cap \Phi$: ◆ fix s^i_j to 1. ■ If d^i_j is integer for unit j in all the intervals: ◆ fix d^i_j $\forall i$ to its current value. ◆ Then, if there are extra constraints with binary variables do preprocessing

Back

Do until ...

a planning for the maintenance is found.

(at most $n_i \cdot n_u$ iterations)

Back

Do until ...

a planning for the maintenance is found.

(at most $n_i \cdot n_u$ iterations)

Then ...

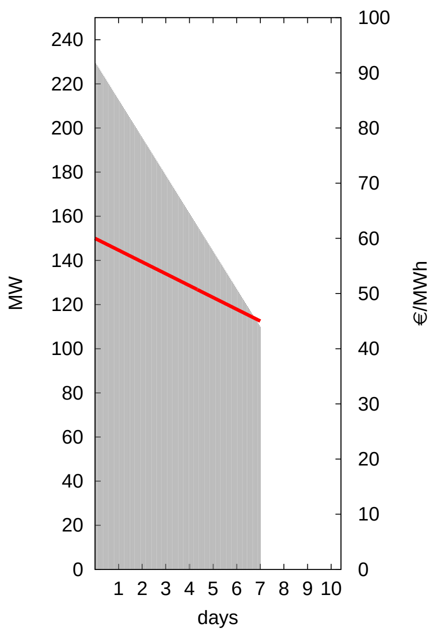
solve the full problem with the maintenance planning fixed.

Back

Example

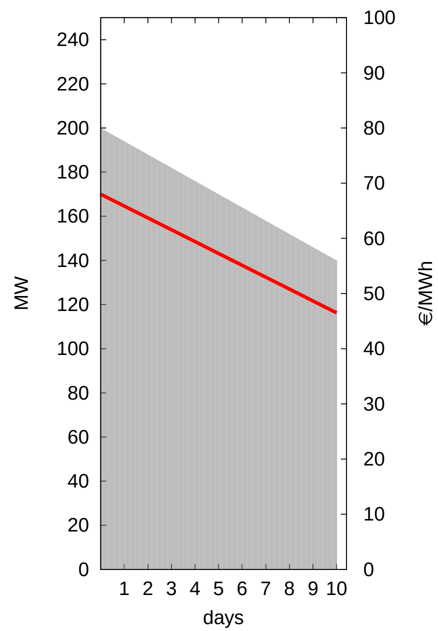
Intervals

Interval 1



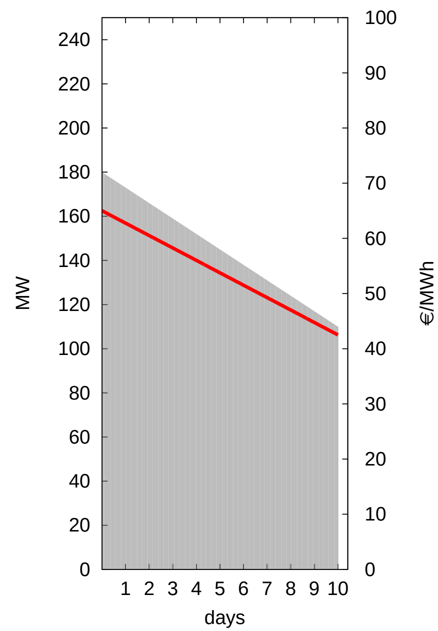
$\underline{P}$	110
$\overline{P}$	230
$\widehat{e}$	28560
t	168 h

Interval 2



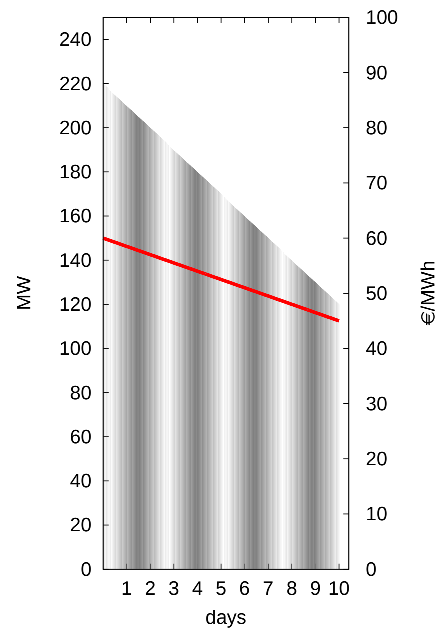
$\underline{P}$	140
$\overline{P}$	200
$\widehat{e}$	40800
t	240 h

Interval 3



$\underline{P}$	110
$\overline{P}$	180
$\widehat{e}$	34800
t	240 h

Interval 4



$\underline{P}$	120
$\overline{P}$	220
$\widehat{e}$	40800
t	240 h

Units & Constraints

- Unit characteristics:

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

- Constraints: System capacity

  In each interval, the sum of the capacity of the units in service has to be 20% higher than the peak load ($\overline{P}$).

	$\overline{P}$	$1.2\overline{P}$
i_1	230	276
i_2	200	240
i_3	180	216
i_4	220	264

Lmc: Step 1

Interval 1

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Interval 2

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Interval 3

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Interval 4

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Solution: Step 1

x	g_1	g_2	g_3	g_4	g_5
i_1	8400.0	211.6	13440.0	0.0	6424.4
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.5	19158.4	1085.2	7708.5
i_4	12000.0	1200.0	19200.0	0.0	10300.0

s	g_1	g_2
	1	1
	0.562	0.58
	0.438	0.42
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	**1**	0.0315	**1**	0.0	0.765
i_2	0.562	0.265	**1**	0.171	0.776
i_3	0.438	0.153	**0.998**	0.065	0.642
i_4	**1**	0.125	**1**	0.0	0.858

b	
	$\varnothing$
	$\varnothing$
	$\varnothing$
	$\varnothing$

Solution: Step 1

x	g_1	g_2	g_3	g_4	g_5
i_1	8400.0	211.6	13440.0	0.0	6424.4
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i_3	0.438	0.153	**0.998**	0.065	0.642
i_4	**1**	0.125	**1**	0.0	0.858

b	
	g_1, g_3
	g_3
	g_3
	g_1, g_3

Next Step

Lmc: Step 2

Interval 1

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Interval 3

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Interval 4

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Solution: Step 2

x	g_1	g_2	g_3	g_4	g_5
i_1	8106.0	342.2	13440.0	0.0	6587.8
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.4	19158.6	1085.2	7708.4
i_4	11890.0	1248.8	19200.0	0.2	10361.0

s	g_1	g_2
	1	1
	0.562	0.599
	0.438	0.401
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.965*	0.0509	*1*	0.0	0.784
i_2	0.562	0.265	*1*	0.171	0.776
i_3	0.438	0.153	*0.998*	0.0646	0.642
i_4	*0.991*	0.13	*1*	0.0	**0.863**

b	
	g_1, g_3
	g_3
	g_3
	g_1, g_3

Solution: Step 2

x	g_1	g_2	g_3	g_4	g_5
i_1	8106.0	342.2	13440.0	0.0	6587.8
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.4	19158.6	1085.2	7708.4
i_4	11890.0	1248.8	19200.0	0.2	10361.0

s	g_1	g_2
	1	1
	0.562	0.599
	0.438	0.401
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.965*	0.0509	*1*	0.0	0.784
i_2	0.562	0.265	*1*	0.171	0.776
i_3	0.438	0.153	*0.998*	0.0646	0.642
i_4	*0.991*	0.13	*1*	0.0	**0.863**

b	
	g_1, g_3
	g_3
	g_3
	g_1, g_3, g_5

Next Step

Lmc: Step 3

Interval 1

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
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1	1	1	.	.
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1	.	.	1	.
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1	1	1	.	1
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1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 2

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
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1	.	1	.	.
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1	1	1	.	.
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1	1	.	1	.
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1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
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1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 3

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
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1	1	1	.	.
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1	1	.	1	.
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1	.	1	1	.
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1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 4

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	**.**	**1**	**.**	**1**
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Solution: Step 3

x	g_1	g_2	g_3	g_4	g_5
i_1	8106.0	342.2	13440.0	0.0	6587.8
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.4	19158.6	1085.2	7708.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.596
	0.438	0.404
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.965*	0.051	*1*	0.0	**0.784**
i_2	0.562	0.265	*1*	0.171	0.776
i_3	0.438	0.153	*0.998*	0.065	0.642
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3
	g_3
	g_3
	g_1, g_3, g_5

Solution: Step 3

x	g_1	g_2	g_3	g_4	g_5
i_1	8106.0	342.2	13440.0	0.0	6587.8
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.4	19158.6	1085.2	7708.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.596
	0.438	0.404
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.965*	0.051	*1*	0.0	**0.784**
i_2	0.562	0.265	*1*	0.171	0.776
i_3	0.438	0.153	*0.998*	0.065	0.642
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b
g_1, g_3, g_5
g_3
g_3
g_1, g_3, g_5

Next Step

Lmc: Step 4

Interval 1					Interval 2					Interval 3					Interval 4				
1	.	.	.	.	1	.	.	.	.	1	.	.	.	.	1	.	.	.	.
.	1	.	.	.	.	1	.	.	.	.	1	.	.	.	.	1	.	.	.
1	1	.	.	.	1	1	.	.	.	1	1	.	.	.	1	1	.	.	.
.	.	1	.	.	.	.	1	.	.	.	.	1	.	.	.	.	1	.	.
1	.	1	.	.	1	.	1	.	.	1	.	1	.	.	1	.	1	.	.
.	1	1	.	.	.	1	1	.	.	.	1	1	.	.	.	1	1	.	.
1	1	1	.	.	1	1	1	.	.	1	1	1	.	.	1	1	1	.	.
.	.	.	1	.	.	.	.	1	.	.	.	.	1	.	.	.	.	1	.
1	.	.	1	.	1	.	.	1	.	1	.	.	1	.	1	.	.	1	.
.	1	.	1	.	.	1	.	1	.	.	1	.	1	.	.	1	.	1	.
1	1	.	1	.	1	1	.	1	.	1	1	.	1	.	1	1	.	1	.
.	.	1	1	.	.	.	1	1	.	.	.	1	1	.	.	.	1	1	.
1	.	1	1	.	1	.	1	1	.	1	.	1	1	.	1	.	1	1	.
.	1	1	1	.	.	1	1	1	.	.	1	1	1	.	.	1	1	1	.
1	1	1	1	.	1	1	1	1	.	1	1	1	1	.	1	1	1	1	.
.	.	.	.	1	.	.	.	.	1	.	.	.	.	1	.	.	.	.	1
1	.	.	.	1	1	.	.	.	1	1	.	.	.	1	1	.	.	.	1
.	1	.	.	1	.	1	.	.	1	.	1	.	.	1	.	1	.	.	1
1	1	.	.	1	1	1	.	.	1	1	1	.	.	1	1	1	.	.	1
.	.	1	.	1	.	.	1	.	1	.	.	1	.	1	.	.	1	.	1
1	.	1	.	1	1	.	1	.	1	1	.	1	.	1	1	.	1	.	1
.	1	1	.	1	.	1	1	.	1	.	1	1	.	1	.	1	1	.	1
1	1	1	.	1	1	1	1	.	1	1	1	1	.	1	1	1	1	.	1
.	.	.	1	1	.	.	.	1	1	.	.	.	1	1	.	.	.	1	1
1	.	.	1	1	1	.	.	1	1	1	.	.	1	1	1	.	.	1	1
.	1	.	1	1	.	1	.	1	1	.	1	.	1	1	.	1	.	1	1
1	1	.	1	1	1	1	.	1	1	1	1	.	1	1	1	1	.	1	1
.	.	1	1	1	.	.	1	1	1	.	.	1	1	1	.	.	1	1	1
1	.	1	1	1	1	.	1	1	1	1	.	1	1	1	1	.	1	1	1
.	1	1	1	1	.	1	1	1	1	.	1	1	1	1	.	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Solution: Step 4

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.5	19158.3	1085.3	7708.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.598
	0.438	0.402
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.0443	*0.66*
i_2	0.562	0.265	*1*	0.171	**0.776**
i_3	0.438	0.153	*0.998*	0.0646	0.642
i_4	*0.981*	0.205	*1*	0.0712	*0.714*

b	
	g_1, g_3, g_5
	g_3
	g_3
	g_1, g_3, g_5

Solution: Step 4

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.5	19158.3	1085.3	7708.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.598
	0.438	0.402
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.0443	*0.66*
i_2	0.562	0.265	*1*	0.171	**0.776**
i_3	0.438	0.153	*0.998*	0.0646	0.642
i_4	*0.981*	0.205	*1*	0.0712	*0.714*

b	
	g_1, g_3, g_5
	g_3, g_5
	g_3
	g_1, g_3, g_5

Next Step

Lmc: Step 5

Interval 1

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 2

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
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1	1	.	1	.
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1	.	1	1	.
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1	1	1	1	.
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1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
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1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
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1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 3

1	.	.	.	.
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1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
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1	1	1	1	.
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1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
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.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 4

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
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1	1	.	1	.
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1	.	.	.	1
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1	1	.	.	1
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1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Solution: Step 5

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.5	19158.3	1085.3	7708.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.598
	0.438	0.402
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	0.562	0.265	*1*	0.171	*0.776*
i_3	0.438	0.153	*0.998*	0.065	**0.642**
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_3, g_5
	g_3
	g_1, g_3, g_5

Solution: Step 5

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.5	19158.3	1085.3	7708.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.598
	0.438	0.402
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	0.562	0.265	*1*	0.171	*0.776*
i_3	0.438	0.153	*0.998*	0.065	**0.642**
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_3, g_5
	g_3, g_5
	g_1, g_3, g_5

Next Step

Lmc: Step 6

Interval 1

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 2

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 3

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
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1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 4

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Solution: Step 6

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.4	19158.7	1085.1	7708.4
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	0.562	0.582
	0.438	0.418
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	**0.562**	0.265	*1*	0.171	*0.776*
i_3	0.438	0.153	*0.998*	0.065	*0.642*
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_3, g_5
	g_3, g_5
	g_1, g_3, g_5

Solution: Step 6

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	6745.6	2539.4	19200.0	2881.2	9313.8
i_3	5254.4	1473.4	19158.7	1085.1	7708.4
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	1	0.582
	0.438	0.418
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	**0.562**	0.265	*1*	0.171	*0.776*
i_3	0.438	0.153	*0.998*	0.065	*0.642*
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_1, g_3, g_5
	g_3, g_5
	g_1, g_3, g_5

Fix s_1^2 to 1.

Next Step

Lmc: Step 7

Interval 1

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 2

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	**.**	**1**	**.**	**1**
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 3

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 4

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Solution: Step 7

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	10403.3	1625.0	19200.0	1280.9	8170.8
i_3	0.0	2776.7	19200.0	3365.8	9337.5
i_4	11769.9	1963.6	19200.0	1196.4	8570.1

s	g_1	g_2
	1	1
	1	0.169
	0	0.831
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	*0.867*	0.169	*1*	0.076	*0.681*
i_3	0	**0.289**	*1*	0.2	*0.778*
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_1, g_3, g_5
	g_3, g_5
	g_1, g_3, g_5

Solution: Step 7

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	10403.3	1625.0	19200.0	1280.9	8170.8
i_3	0.0	2776.7	19200.0	3365.8	9337.5
i_4	11769.9	1963.6	19200.0	1196.4	8570.1

s	g_1	g_2
	1	1
	1	0.169
	0	**1**
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	*0.867*	0.169	*1*	0.076	*0.681*
i_3	0	**0.289**	*1*	0.2	*0.778*
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_1, g_3, g_5
	g_2, g_3, g_5
	g_1, g_3, g_5

Fix s_2^3 to 1.

Solution: Step 7

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	10403.3	1625.0	19200.0	1280.9	8170.8
i_3	0.0	2776.7	19200.0	3365.8	9337.5
i_4	11769.9	1963.6	19200.0	1196.4	8570.1

s	g_1	g_2
	1	1
	1	0.169
	0	**1**
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	*0.867*	0.169	*1*	0.076	*0.681*
i_3	0	**0.289**	*1*	0.2	*0.778*
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b
g_1, g_3, g_5
g_1, g_3, g_5
g_2, g_3, g_5
g_1, g_3, g_5

Fix s_2^3 to 1.

Preprocess: g_1 has an all-integer solution.
To be feasible s_2^3 has to be one.

Next Step

Lmc: Step 8

Interval 1

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 2

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 3

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	**1**	**1**	**.**	**1**
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Interval 4

1	.	.	.	.
.	1	.	.	.
1	1	.	.	.
.	.	1	.	.
1	.	1	.	.
.	1	1	.	.
1	1	1	.	.
.	.	.	1	.
1	.	.	1	.
.	1	.	1	.
1	1	.	1	.
.	.	1	1	.
1	.	1	1	.
.	1	1	1	.
1	1	1	1	.
.	.	.	.	1
1	.	.	.	1
.	1	.	.	1
1	1	.	.	1
.	.	1	.	1
1	.	1	.	1
.	1	1	.	1
1	1	1	.	1
.	.	.	1	1
1	.	.	1	1
.	1	.	1	1
1	1	.	1	1
.	.	1	1	1
1	.	1	1	1
.	1	1	1	1
1	1	1	1	1

Solution: Step 8

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	10881.3	0.0	19200.0	1950.0	8648.7
i_3	0.0	2776.7	19200.0	3365.8	9337.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	1
	1	0
	0	1
	1	1

ρ	g_1	g_2	g_3	g_4	g_5
i_1	0.926	0.178	1	0.044	0.66
i_2	0.907	0	1	0.116	0.721
i_3	0	0.289	1	0.2	0.778
i_4	0.981	0.205	1	0.071	0.714

b	
	g_1, g_3, g_5
	g_1, g_3, g_5
	g_2, g_3, g_5
	g_1, g_3, g_5

Solution: Step 8

x	g_1	g_2	g_3	g_4	g_5
i_1	7780.5	1193.8	13440.0	521.2	5540.5
i_2	10881.3	0.0	19200.0	1950.0	8648.7
i_3	0.0	2776.7	19200.0	3365.8	9337.5
i_4	11770.0	1963.6	19200.0	1196.4	8570.0

s	g_1	g_2
	1	**1**
	1	**0**
	0	**1**
	1	**1**

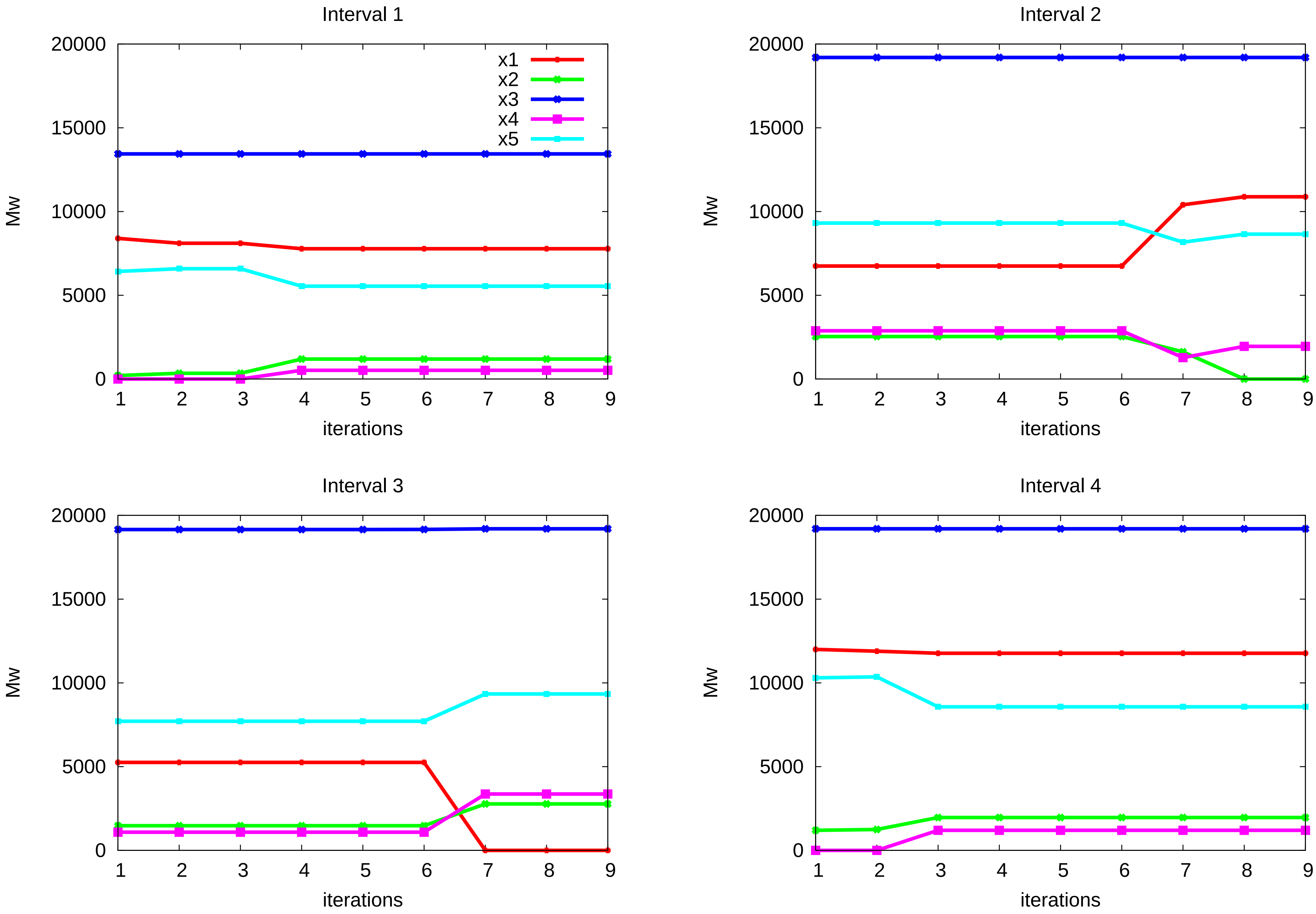
ρ	g_1	g_2	g_3	g_4	g_5
i_1	*0.926*	0.178	*1*	0.044	*0.66*
i_2	*0.907*	0	*1*	0.116	*0.721*
i_3	0	*0.289*	*1*	0.2	*0.778*
i_4	*0.981*	0.205	*1*	0.071	*0.714*

b	
	g_1, g_3, g_5
	g_1, g_3, g_5
	g_2, g_3, g_5
	g_1, g_3, g_5

Variable s is integer.
Fix this planning and solve the full problem.

Next Step

Step 9: Solution



Computational Results

Test cases

case	n_u	n_i	days	$=$ const	$\geq$ const	n_m
ltp_wm_1	13	27	14	4	21	6
ltp_wm_2	14	27	14	4	20	6
ltp_wm_3	12	27	14	4	21	5
ltp_wm_4	16	27	14	4	20	8
ltp_wm_5	18	27	14	4	13	10
ltp_wm_6	34	27	14	4	21	21

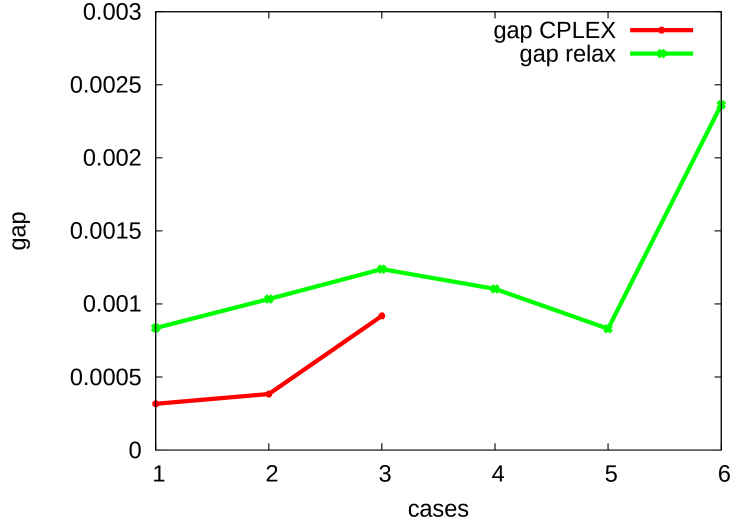
Non-lmc: max hydro, max nuclear, min special regime, min market share

Extra constraints: min system capacity, min company capacity

Realistic cases. Data obtained from public web pages: www.omel.es and www.ree.es

Solution Gap

	Obj. Fun.	gap CPLEX	gap relax.
ltp_wm_1	4 975 115 139	0.00032	0.00084
ltp_wm_2	4 744 291 421	0.00038	0.00103
ltp_wm_3	6 034 971 646	0.00092	0.00124
ltp_wm_4	7 063 780 400		0.00110
ltp_wm_5	4 707 261 520		0.00083
ltp_wm_6	7 022 827 384		0.00236



Time & Iterations

	time	iterations		lmc	
		heur.	sol.	heur.	sol.
ltp_wm_1	13’40”	150	8	175	249
ltp_wm_2	20’05”	239	8	264	357
ltp_wm_3	15’15”	241	6	266	314
ltp_wm_4	24’43”	292	13	317	412
ltp_wm_5	28’54”	326	16	351	460
ltp_wm_6	83’04”	467	19	492	839

The heuristic is coded in AMPL and uses CPLEX 9.1 to solve the subproblems. The computation of the right-hand side (rhs) of the lmc are also coded with AMPL. AMPL is inefficient doing long calculations. The solution time for the small cases with the rhs already computed is reduced to seconds.

Conclusions and Current Work

Conclusions and Current Work

- Conclusions
  - The model for the joint optimization of the long term generation planning and units' maintenance is binary quadratic with an exponential number of lmc. It can be easily extended to multi-periods of maintenance.
  - Characteristics of the heuristic proposed:
    - uses a small subset of lmc
    - binary variables are fixed in successive steps
  - The prototype coded in AMPL gives good solutions in moderated time.
- Current work
  - To decide in which interval a unit is stopped (variable d) is typically modelled as a Special Ordered Set of type 1. A Branch and Bound algorithm which exploits the SOS1 structure is being developed. The heuristic is used to find a good incumbent.

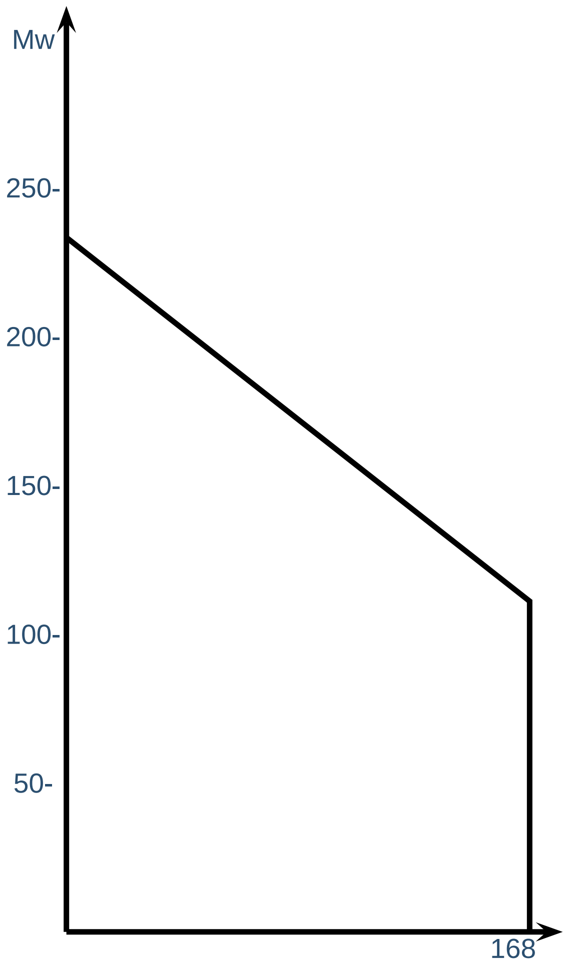
Thanks for your attention!

Extras

Lmc for a given order

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

$$\Omega_o = \{g_3, g_1, g_5, g_2, g_4\}$$

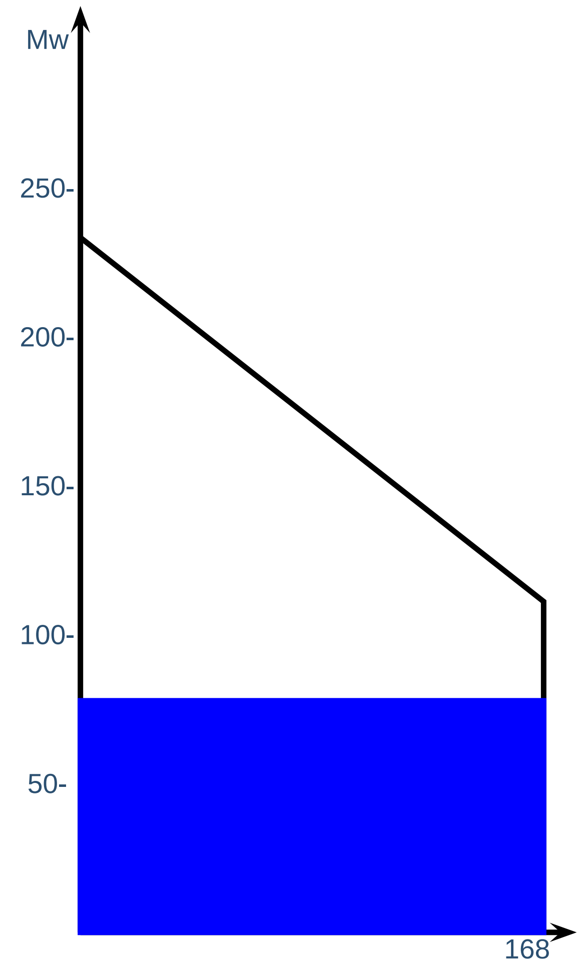


Back

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

$$\Omega_o = \{g_3, g_1, g_5, g_2, g_4\}$$

. . 1 . . ≤ 13440



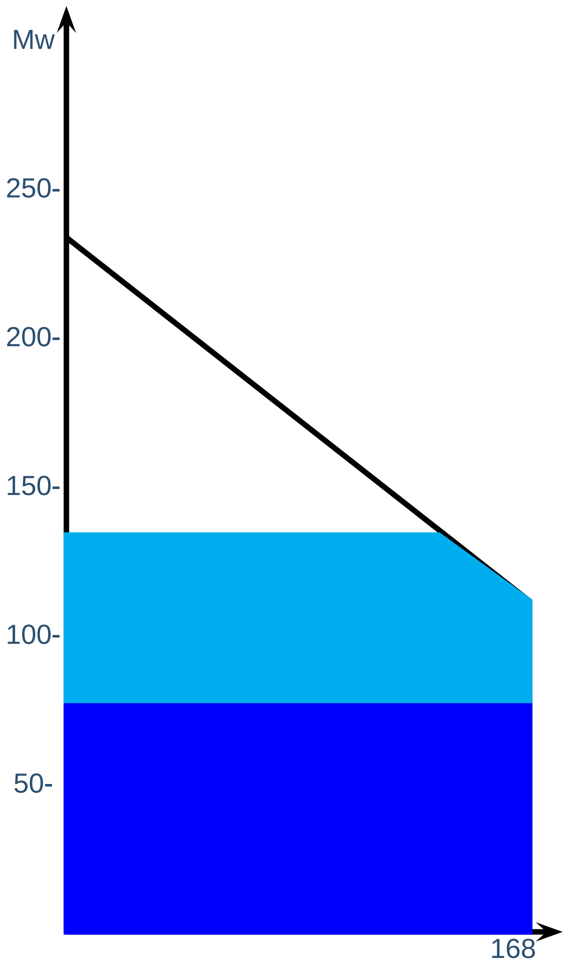
Back

Lmc for a given order

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

$$\Omega_o = \{g_3, g_1, g_5, g_2, g_4\}$$

.	.	1	.	.	≤ 13440
1	.	1	.	.	≤ 21546



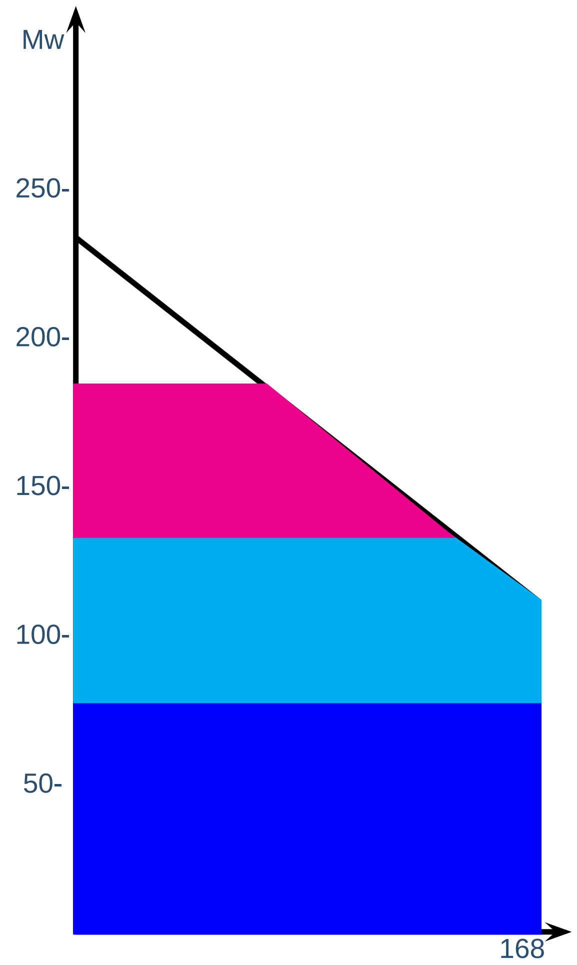
Back

Lmc for a given order

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

$$\Omega_o = \{g_3, g_1, g_5, g_2, g_4\}$$

.	.	1	.	.	≤ 13440
1	.	1	.	.	≤ 21546
1	.	1	.	1	≤ 26761



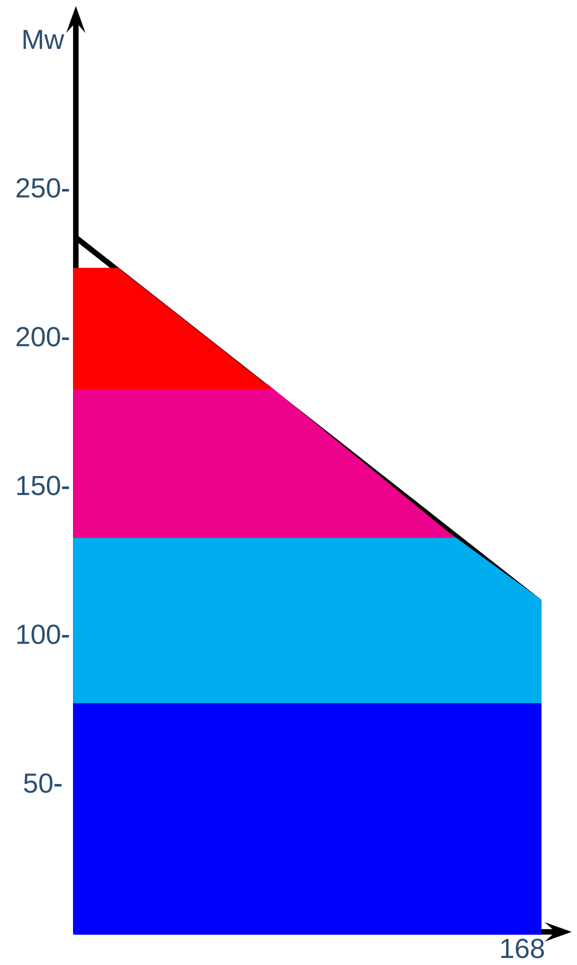
Back

Lmc for a given order

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

$$\Omega_o = \{g_3, g_1, g_5, g_2, g_4\}$$

.	.	1	.	.	≤ 13440
1	.	1	.	.	≤ 21546
1	.	1	.	1	≤ 26761
1	1	1	.	1	≤ 28413



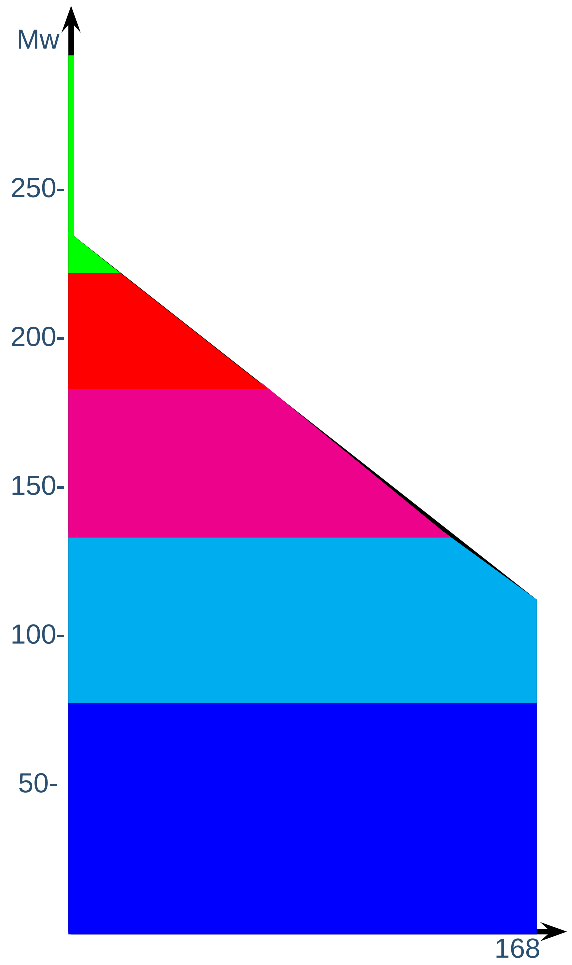
Back

Lmc for a given order

	g_1	g_2	g_3	g_4	g_5
c_j	50	40	80	70	50
p_j	1	1	1	1	1
f_j	5	20	1	22	9
m_j	1	1	-	-	-

$$\Omega_o = \{g_3, g_1, g_5, g_2, g_4\}$$

.	.	1	.	.	≤ 13440
1	.	1	.	.	≤ 21546
1	.	1	.	1	≤ 26761
1	1	1	.	1	≤ 28413
1	1	1	1	1	≤ 28476



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