# Unit Commitment by Augmented Lagrangean Relaxation

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## Abstract

One of the main drawbacks of the Augmented Lagrangean Relaxaton (ALR) method is that the quadratic term introduced by the augmented Lagrangean is not separable. We compare, empirically and theoretically, two methods designed to cope with the nonseparability of the Lagrangean function: the Auxiliary Problem principle (APP) method and the Block Coordinated Descent (BCD) method.

# Introduction

- A simple example of unit commitment:
  - Solution methodology.
  - Computational test and theoretical insight.
- Short-term hydrothermal coordination.
  - Solution algorithm.
  - Coputational results.
- Conclusions.

# Example of Unit Commitment

- Initial problem:

$$\left.\begin{array}{ll} \min & 2x^2 + C_{on}(x) + 2y^2 + C_{on}(y) \\ s.t. & x + y = 3 \\ & x \in \{0\} \cup [1,3] \\ & y \in \{0\} \cup [1,3] \end{array}\right\} \quad (1)$$

- Duplicated variables:

$$\left.\begin{array}{ll} \min & x^2 + y^2 + \widetilde{x}^2 + C_{on}(\widetilde{x}) + \widetilde{y}^2 + C_{on}(\widetilde{y}) \\ s.t. & x + y = 3 \\ & \widetilde{x} \in \{0\} \cup [1,3] \\ & \widetilde{y} \in \{0\} \cup [1,3] \\ & x = \widetilde{x}, \;\; y = \widetilde{y} \end{array}\right\} \quad (2)$$

- Primal problem:

$$\left.\begin{array}{ll} \min & f(x,y) + f(\widetilde{x},\widetilde{y}) \\ s.t. & (x,y) \in \mathcal{D} \\ & (\widetilde{x},\widetilde{y}) \in \widetilde{\mathcal{D}} \\ & (x,y) = (\widetilde{x},\widetilde{y}) \end{array}\right\} \quad (3)$$

# Augmented Lagrangean

- Primal problem:

$$\left.\begin{array}{ll} \text{min} & f(x) + f(\tilde{x}) \\ s.t. & x \in \mathcal{D},\ \tilde{x} \in \tilde{\mathcal{D}} \\ & x - \tilde{x} = 0 \end{array}\right\} \tag{4}$$

- Dual problem:

$$\begin{array}{c} \text{max} \\ \lambda \in R^n \end{array} \left\{ \min_{\substack{x \in \mathcal{D} \\ \tilde{x} \in \tilde{\mathcal{D}}}} f(x) + f(\tilde{x}) + \lambda'(x - \tilde{x}) + \frac{c}{2}\|x - \tilde{x}\|^2 \right\} \tag{5}$$

- Equivalently:

$$\begin{array}{c} \text{max} \\ \lambda \in R^n \end{array} \left\{ \min_{\substack{x \in \mathcal{D} \\ \tilde{x} \in \tilde{\mathcal{D}}}} L_c(x, \tilde{x}, \lambda) \right\} \tag{6}$$

- For short:

$$\begin{array}{c} \text{max} \\ \lambda \in R^n \end{array} \left\{ q_c(\lambda) \right\} \tag{7}$$

# Compared Methods

$$\min_{\substack{x \in \mathcal{D} \\ \tilde{x} \in \tilde{\mathcal{D}}}} f(x) + f(\tilde{x}) + \lambda'(x - \tilde{x}) + \frac{c}{2}\|x - \tilde{x}\|^2 \qquad (8)$$

- Auxiliary Problem Principle (APP):

$$\min_{x \in \mathcal{D}} \; f(x) + {\lambda_n}' x + c(\mathbf{x_n} - \widetilde{\mathbf{x_n}})' x + \frac{b}{2}\|x - \mathbf{x_n}\|^2 \qquad (9)$$

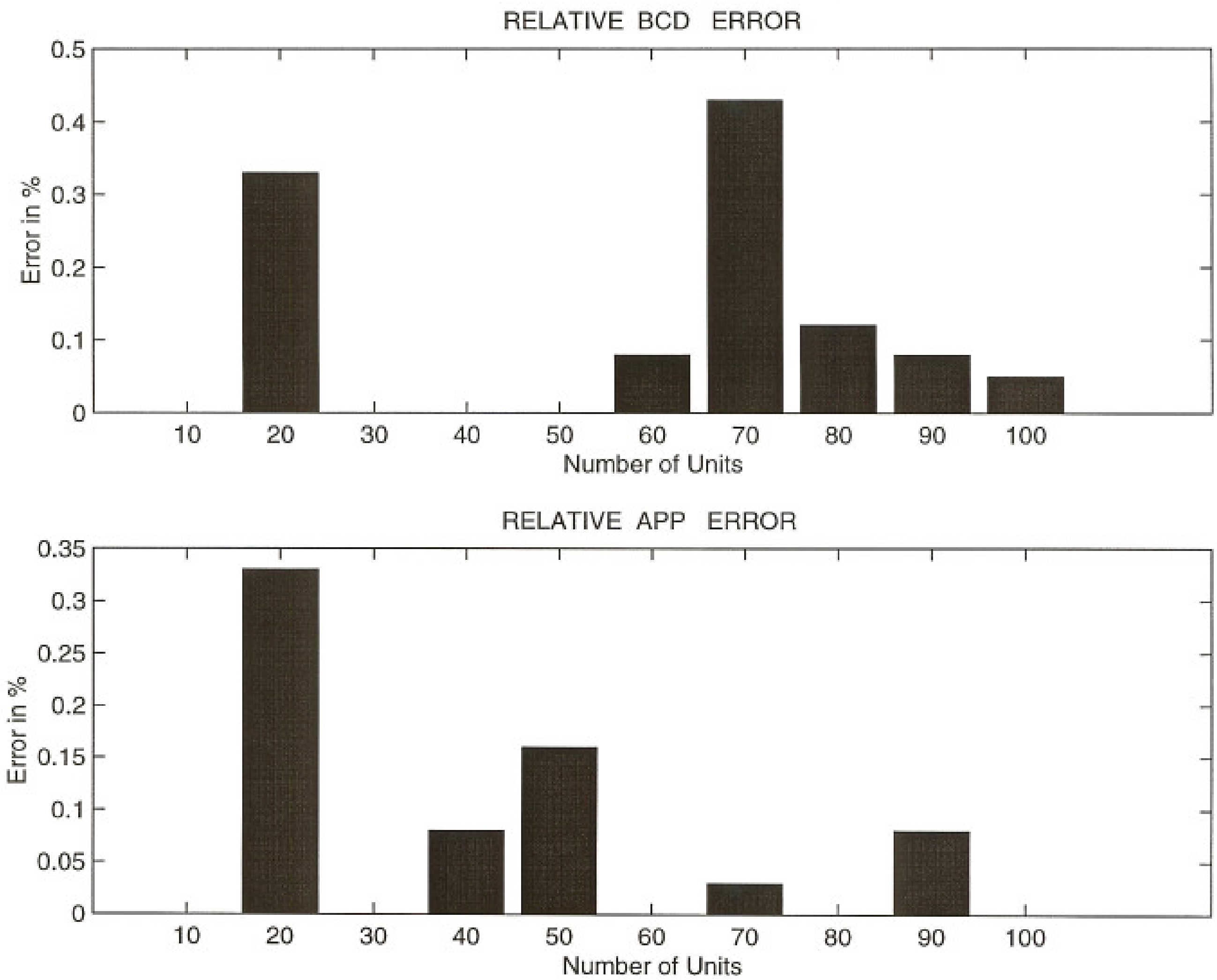
$$\min_{x \in \tilde{\mathcal{D}}} \; \tilde{f}(\tilde{x}) - {\lambda_n}' \tilde{x} - c(\mathbf{x_n} - \widetilde{\mathbf{x_n}})' \tilde{x} + \frac{b}{2}\|\tilde{x} - \widetilde{\mathbf{x_n}}\|^2 \qquad (10)$$

- Block Coordinated Descent (BCD):

$$\min_{x \in \mathcal{D}} \; f(x) + {\lambda_n}' x + \frac{c}{2}\|x - \widetilde{\mathbf{x_n}}\|^2 \qquad (11)$$

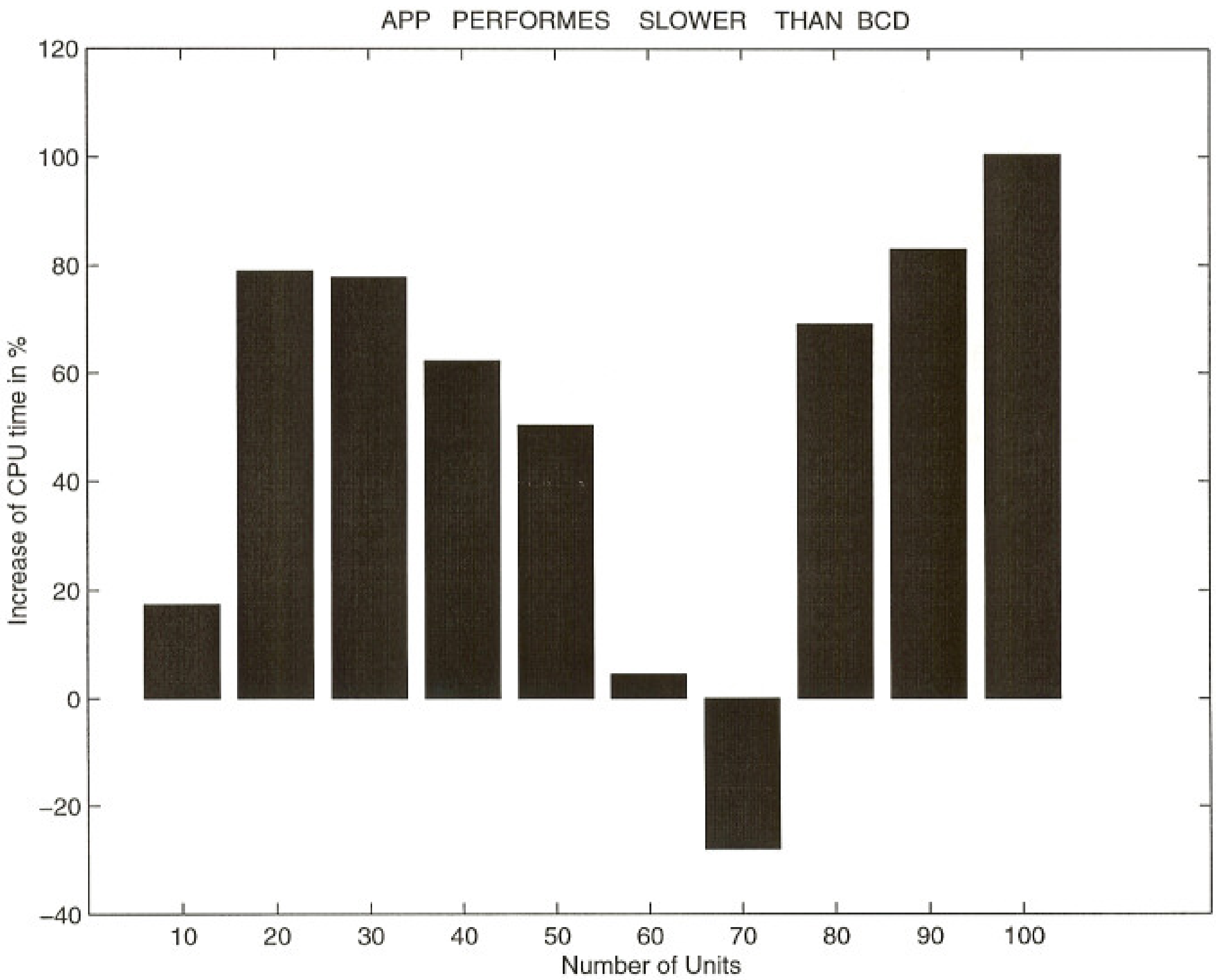
$$\min_{x \in \tilde{\mathcal{D}}} \; \tilde{f}(\tilde{x}) - {\lambda_n}' \tilde{x} + \frac{c}{2}\|\mathbf{x_{n+1}} - \tilde{x}\|^2 \qquad (12)$$

# Computational Test I (a)




We obtain the global optimum or a high quality local optimum (relative error under 0.5%.)

# Computational Test I (b)




In average, the APP method has performed 51.5% slower than the BCD method.

# Proposition

For $b = c$ the APP method can be recasted as:

$$\min_{x \in \mathcal{D}} \; f(x) + \lambda_n' x + \frac{c}{2}\|x - \widetilde{\mathbf{x_n}}\|^2 \;\; \rightsquigarrow \;\; \mathbf{x_{n+1}} \tag{13}$$

$$\min_{x \in \widetilde{\mathcal{D}}} \; \widetilde{f}(\widetilde{x}) - \lambda_n' \widetilde{x} + \frac{c}{2}\|\mathbf{x_n} - \widetilde{x}\|^2 \;\; \rightsquigarrow \;\; \widetilde{\mathbf{x}}_{\mathbf{n+1}} \tag{14}$$

The BCD method:

$$\min_{x \in \mathcal{D}} \; f(x) + \lambda_n' x + \frac{c}{2}\|x - \widetilde{\mathbf{x_n}}\|^2 \;\; \rightsquigarrow \;\; \mathbf{x_{n+1}} \tag{15}$$

$$\min_{x \in \widetilde{\mathcal{D}}} \; \widetilde{f}(\widetilde{x}) - \lambda_n' \widetilde{x} + \frac{c}{2}\|\mathbf{x_{n+1}} - \widetilde{x}\|^2 \;\; \rightsquigarrow \;\; \widetilde{\mathbf{x}}_{\mathbf{n+1}} \tag{16}$$

# The Short-Term Hydrothermal Coordination (STHC) Problem

# Modeling the STHC problem

# Formulation of the STHC problem

# ALR + APP Algorithm

**S-1** [Check the stopping criterion.] If the gradient of the dual function $(x_n - \widetilde{x}_n) = 0$ then stop. $(x_n, \widetilde{x}_n, \lambda_n)$ is a solution.

**S-2** [Compute $x_{n+1}$ - Nonlinear Network Flow.]

$$\min_{x \in \mathcal{D}_{htd}} \quad C_{htd}(x) + \lambda_n{}'x + c_n(\mathbf{x_n} - \widetilde{\mathbf{x_n}})'x + \frac{b_n}{2}\|x - \mathbf{x_n}\|^2$$

**S-3** [Compute $\widetilde{x}_{n+1}$ - Dynamic Programming.]

$$\min_{x \in \mathcal{D}_m} \quad C_m(\widetilde{x}) - \lambda_n{}'\widetilde{x} - c_n(\mathbf{x_n} - \widetilde{\mathbf{x_n}})'\widetilde{x} + \frac{b_n}{2}\|\widetilde{x} - \widetilde{\mathbf{x_n}}\|^2$$

**S-4** [Dual variable updating]

$$\lambda_{n+1} = \lambda_n + c_n \cdot (x_{n+1} - \widetilde{x}_{n+1})$$

# ALR + BCD Algorithm

**S-1** [Check the stopping criterion.] If the gradient of the dual function $(x_n - \tilde{x}_n) = 0$ then stop. $(x_n, \tilde{x}_n, \lambda_n)$ is a solution.

**S-2** [Compute $x_{n+1}$ - Nonlinear Network Flow.]

$$\min_{x \epsilon \mathcal{D}_{htd}} \left[ C_{htd}(x) + \lambda_n' x + \frac{c_n}{2} ||x - \tilde{\mathbf{x}}_\mathbf{n}||^2 \right]$$

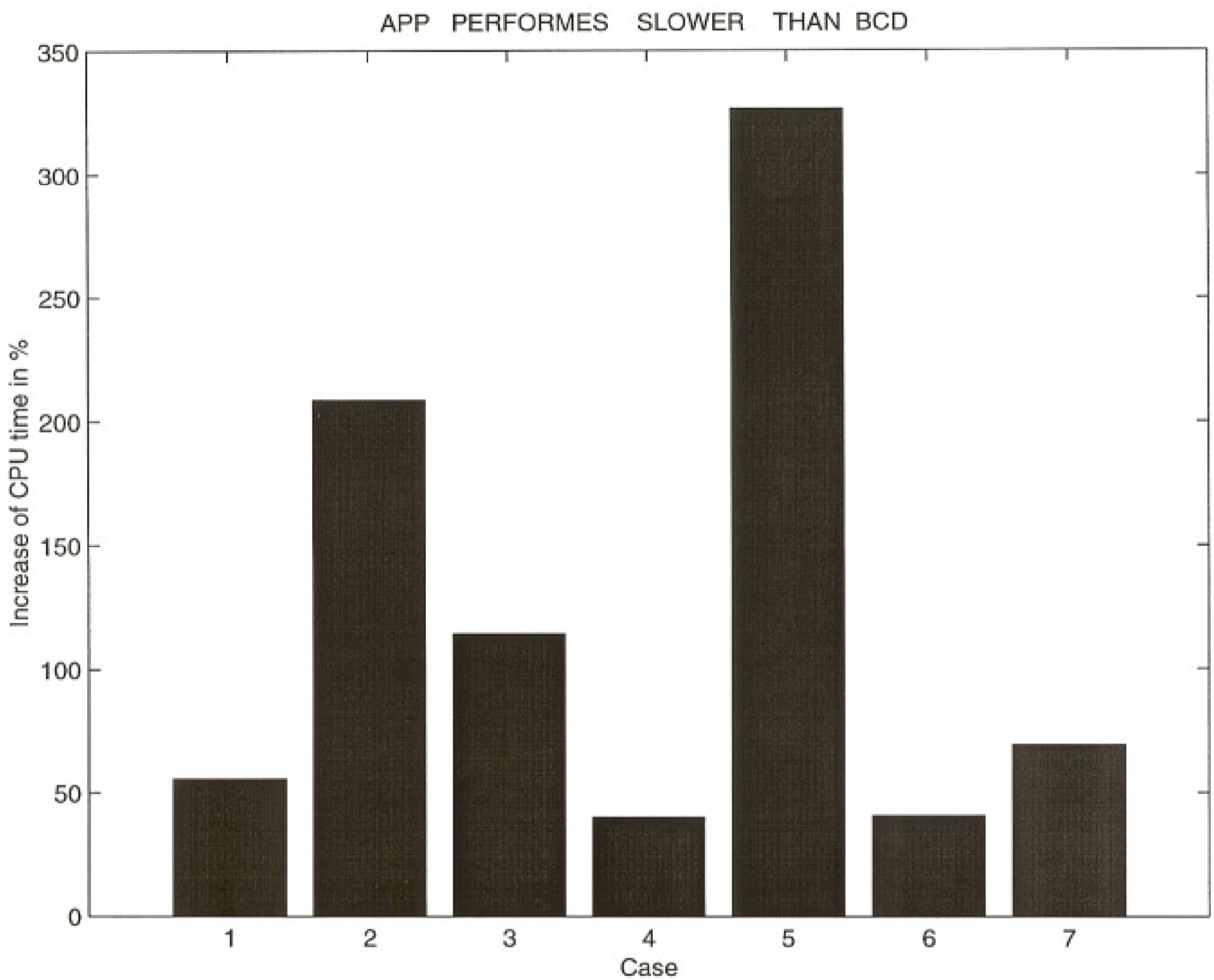
**S-3** [Compute $\tilde{x}_{n+1}$ - Dynamic Programming.]

$$\min_{\tilde{x} \epsilon \mathcal{D}_m} \left[ C_m(\tilde{x}) - \lambda_n' \tilde{x} + \frac{c_n}{2} ||\mathbf{x}_{\mathbf{n+1}} - \tilde{x}||^2 \right]$$

**S-4** [Dual variable updating]

$$\lambda_{n+1} = \lambda_n + c_n \cdot (x_{n+1} - \tilde{x}_{n+1})$$

# Test II (STHC Problem)




In average, the APP method has performed 85.5% slower than the BCD method.

# Conclusions

- The Short-Term Hydrothermal Coordination problem has been succesfully solved using:
  - Variable Duplication Mehod.
  - Augmented Lagrangean Relaxation.

- The Block Coordinate Descend Method shows to be faster than the Auxiliary Problem Principle Method to deal with the non separable Lagrangean.

- With the above methodology one obtains a local or a global optimizer. Further research on this subject is needed.