Endogenous model for medium-term electricity generation planning in liberalized mixed markets

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Market types

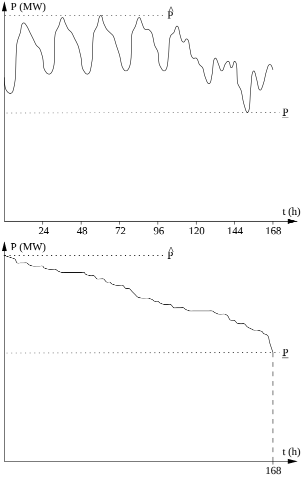
- Pure pool market: where all electricity is exchanged through a market operator.
- Pure bilateral market: where all electricity is traded directly between a given generation company and a given distribution company.
- Mixed market: where part of the load is traded as a bilateral contract and the rest is bid to the pool.

Types of agents

- Price-taker: when a company is unable to produce a change in market price
- Price-maker: when the company is able to alter market prices. The increase of market price then comes as a consequence of biding generation at prices higher than marginal production prices while having a non-negligeable share of the generation capacity of the market (above 3%)

A market where there are several price-maker generation companies is an *oligopolistic* market.

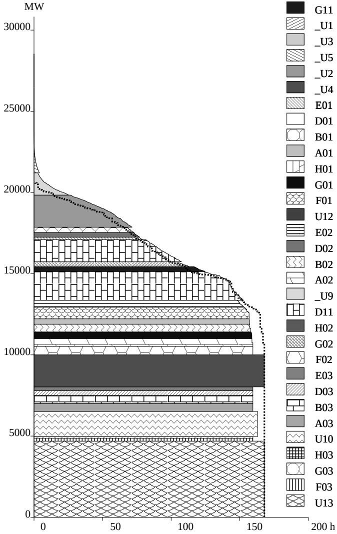
Load Duration Curve (LDC)



LDC is characterized by

- the total energy, $\hat{e}^i$
- the duration, t^i
- the base load power, $\underline{p}^i$
- the peak load power, $\widehat{p}^i$
- the shape, which is not a single parameter

Generation Duration Curve (GDC)



The *generation-duration curve* is the expected production of the thermal units over the time interval to which the LDC refers. The energy generated by each unit is the slice of area under the generation-duration curve which corresponds to the capacity of the thermal unit. The area under the LDC and the area under the generation-duration curve must coincide.

The Bloom & Gallant Formulation

$$\underset{x_j}{\text{minimize}} \quad \sum_{j=0}^{n_u} \widetilde{f}_j x_j \tag{1a}$$

$$\text{subject to:} \quad \sum_{j \in \omega} x_j \leq \hat{e} - s(\omega) \qquad \forall\, \omega \subset \Omega \tag{1b}$$

$$Cx \geq d \tag{1c}$$

$$\sum_{j=0}^{n_u} x_j = \hat{e} \tag{1d}$$

$$x_j \geq 0 \qquad j = 0, 1, \dots, n_u \tag{1e}$$

where Ω is the set of generation units of the pool, $\widetilde{f}$ is the linear generation cost and $j = 0$ is the *external energy*. Finally, $s(\omega) = T \int_0^{\widehat{p}} S_\omega(z)dz$

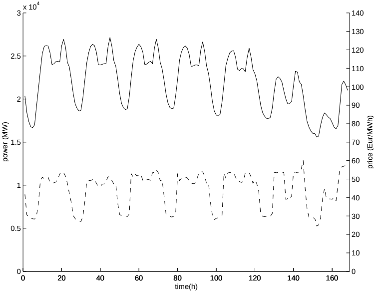
The Non-Load-Matching Constraints

- Maximum hydro generation: water availability is uncertain and the use of stored water is sometimes restricted by demands for irrigation.
- Bonus-scheme coal: some generation units which burn national coal receive a reduction on the coal cost according to a government act.
- Minimum generation time: generation units in the Spanish system are paid for their availability.
- SGC market-share: in medium-term planning it is reasonable to consider the SGC market-share.
- Other constraints

Medium-term profit maximization

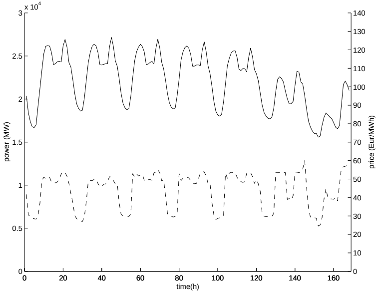
- In medium-term operation all accepted bids in a time interval (a week, or a month) must match the LDC of this interval for the whole market.
- As all generation companies pursue their maximum profit, it is natural to attempt to maximize the profit of all generation companies combined, which is the problem that will be first described. It is called the *generators' surplus (cartel)* maximization.
- The medium-term results will indicate how the SGC should program its units so that its profits be maximized while meeting all constraints.

Medium-term market price function

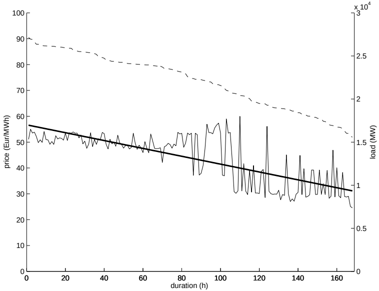


Hourly loads (continuous curve) and market prices (dashed) in a weekly interval

Medium-term market price function

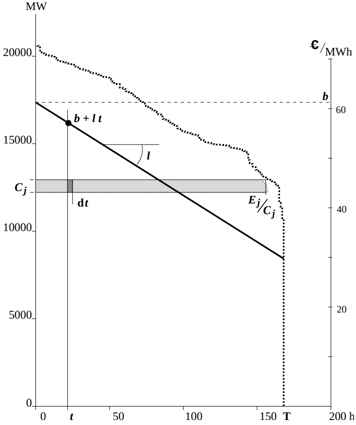


Hourly loads (continuous curve) and market prices (dashed) in a weekly interval



Market prices ordered by decreasing load power (thin continuous curve) in weekly interval,
market price linear function with respect to the load duration (thick line) and LDC (dashed)

Medium-term market price function



Medium-term price function
for a time interval and contribution of unit j

The profit (revenue minus cost) of unit j in interval i will be

$$\int_0^{\frac{x_j^i}{c_j}} c_j\{b^i + l^i t - f_j\}\, dt =$$

$$= (b^i - f_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2$$

Generators' surplus problem (*Cartel* model)

$$\underset{x_j^i}{\text{maximize}} \ \sum_{i=1}^{n_i}\left[\sum_{j\in\Omega}\left\{\left(b^i-\widetilde{f}_j\right)x_j^i+\frac{l^i}{2c_j}\left(x_j^i\right)^2\right\}-\widetilde{f}_0 x_0^i\right] \tag{2a}$$

$$\text{subject to:} \ \sum_{j\in\omega} x_j^i \le e^i - s^i(\omega) \quad \forall\,\omega\subseteq\Omega \quad \forall\, i \tag{2b}$$

$$\sum_{j\in\Omega} x_j^i + x_0^i = e^i \quad \forall\, i \tag{2c}$$

$$\sum_{i=1}^{n_i} C^i x^i \ge d \tag{2d}$$

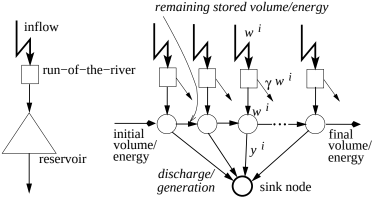
$$x_j^i \ge 0 \quad j\in\Omega \quad \forall\, i \tag{2e}$$

Constraints (2d) makes (2) non separable by periods

The representation of hydro generation

We denote with:

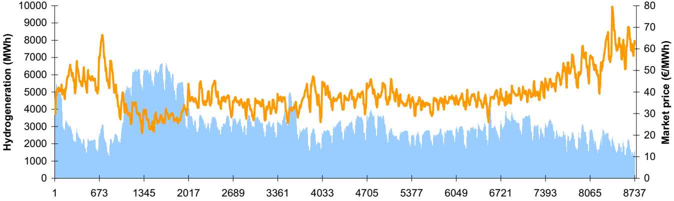
- y_h^i: the amount of water discharged
- w_h^i: the natural water inflows
- $\hat{e}^i$: the total energy
- v^i: the reservoir volume
- π^i: the market price function



Simplification of reservoir systems

$$\left.\begin{aligned} &\sum_{i=1}^{k} y_h^i \leq v_h^0 + \sum_{i=1}^{k} w_h^i \\ &\sum_{i=1}^{n_i} y_h^i + v_h^f = v_h^0 + \sum_{i=1}^{n_i} w_h^i \\ &x_h^i = y_h^i + \gamma w_h^i \end{aligned}\right\} \forall\, h \in H$$

Endogenous modification of the market price function



Weekly moving average of the market price (orange) and of hydro generation (blue area) during 2007 in the Spanish power pool

$$b^i = b_0^i - d_0^i \sum_{k \in H} x_k^i$$

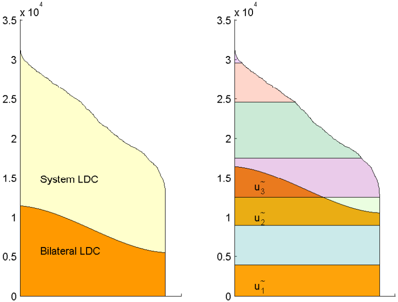
Therefore the new objective function will be

$$\sum_i^{n_i} \Big[\sum_j^{n_u} \{ (b_0^i - \tilde{f}_j) x_j^i - d_0^i \sum_{k \in H} x_k^i x_j^i + \frac{l^i}{2 c_j} {x_j^i}^2 \} - \tilde{f}_0 x_0^i \Big]$$

Bilateral Contracts: Definition and principles

- A Bilateral Contract (BC) is an agreement between two parties, normally a SGC and a big consumer or distributor, to supply or exchange electric power under a set of specified conditions such as power, energy amount, time of delivery, duration and price.
- One of the most extended electricity market types is mixed market with pool auction and BCs. In it, generation companies may have BCs to supply energy in given amounts and instants, and they bid the remaining available generation capacity to the pool Market Operator to get extra benefits.
- It will be assumed that information of total system load and of total market load are available. Subtracting the market load from the system load we get the load supplied through BCs.

The time share hypothesis in medium term power planning with BCs



LDC of the system and part corresponding to the bilateral contracts LDC (shaded part, left), optimal load-matching with production for bilateral contracts (right)

We assume that

- units are generating at full capacity
- the contribution of a unit has rectangular shape with height equal to its capacity

x is the total expected generation and $\widetilde{x}$ is the energy produced for bilateral contracts.

Profit maximization with the endogenous function and bilateral contracts

The endogenous price function for the hydro generation traded in the market is

$$\pi^i(t, g_h) = b_0^i + l^i t + d \sum_{h \in H} (x_h^i - \widetilde{x}_h^i)\,.$$

Subtracting the generation cost from the revenue we obtain the generation unit profit:

$$\begin{aligned} r_j^i(x_j^i, \widetilde{x}_j^i) = & \; c_j \int_{\frac{\widetilde{x}_j^i}{c_j}}^{\frac{x_j^i}{c_j}} (\pi^i(t, g_h) - f_j)\, dt = \\ = & \; b_0^i (x_j^i - \widetilde{x}_j^i) + d \sum_{h \in H} (x_h^i - \widetilde{x}_h^i)(x_j^i - \widetilde{x}_j^i) \\ & + \tfrac{1}{2} \tfrac{l^i}{c_j} (x_j^{i\,2} - \widetilde{x}_j^{i\,2}) - f_j x_j^i \end{aligned}$$

This function r_j^i is indefinite (and it could be decomposed as a difference of two concave functions.)

The medium-term power planning in a liberalized market with BCs

$$\underset{x_j^i,\widetilde{x}_j^i}{\text{maximize}} \sum_{i=1}^{n_i} \left\{ \sum_{j\in\widetilde{\Omega}} r_j^i(x_j^i,\widetilde{x}_j^i) + \sum_{j\in\Omega\setminus\widetilde{\Omega}} r_j^i(x_j^i,0) - f_0 x_0^i \right\} \tag{3a}$$

$$\text{subject to: } \widetilde{x}_j^i \le x_j^i \quad \forall j\in\Omega \quad \forall i \tag{3b}$$

$$\sum_{j\in\widetilde{\omega}} \widetilde{x}_j^i \le \widetilde{e}^i - s^i(\widetilde{\omega}) \quad \forall\, \widetilde{\omega}\subseteq\widetilde{\Omega} \quad \forall\, i \tag{3c}$$

$$\sum_{j\in\omega} x_j^i \le e^i - s^i(\omega) \quad \forall\, \omega\subseteq\Omega \quad \forall\, i \tag{3d}$$

$$\sum_{j\in\widetilde{\Omega}} \widetilde{x}_j^i = \widetilde{e}^i - s^i(\Omega) \quad \forall i \tag{3e}$$

$$\sum_{j\in\Omega} x_j^i + x_0^i = e^i \quad \forall\, i \tag{3f}$$

$$\sum_{i=1}^{n_i} C^i x^i \ge d \tag{3g}$$

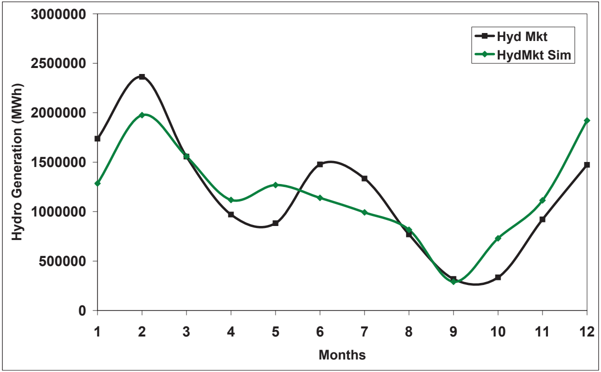
$$0 \le \widetilde{x}_j^i \quad j\in\widetilde{\Omega} \quad \forall\, i \tag{3h}$$

$$0 \le x_j^i \quad j\in\Omega \quad \forall\, i \tag{3i}$$

Hydro-to-Market Constraint

A Hydro-to-Market constraint is employed

$$\sum_{h \in H} (x_h^i - \tilde{x}_h^i) \geq \alpha w_h^i + \beta \hat{e}^i + \gamma v^i + \delta \pi^i \quad \forall\, i \in 1..n$$



Nash-Cournot equilibrium in electricity markets

- In a Nash-Cournot equilibrium we can assume either two (the SGC and the RoP), or more players (K generation companies, whose units are $\Omega_k | \Omega := \{\Omega_1, \Omega_2, ..., \Omega_K\}$).
- In the Cournot model of competition we assume that the decision (generation) of one player is conditioned by the decisions (generations) of the rest of the players and that the market price is a function of the overall decisions (total expected generation).
- In a Nash equilibrium no player can increase its revenue by unilaterally changing its decision (generation).
- We can find an equilibrium point because the endogenous revenue function relates profits to the hydro generation of each player.

Nash-Cournot equilibrium in electricity markets

For the medium-term planning to have *Cournot competition* (and a equilibrium solution) it is necessary to consider that the players mutually condition each other generations. This is so in case we consider the *endogenous model* explained, where the hydro generation of each player influences the market price.

$$\phi_k(x_k|\hat{\mathbf{x}}) = \sum_i \sum_{j\in\Omega_k^i} \Bigg\{ (f_j - b^i)x_j^i - \frac{l^i}{2c_j}(x_j^i)^2 + \\ + d\left[\sum_{l\in H_k} x_l^i x_j^i + \sum_{l\in H_{m|m\neq k}} \hat{x}_l^i x_j^i\right]\Bigg\}$$

The NIRA algorithm to obtain the Nash-Cournot equilibrium

The Nikaido-Isoda relaxation algorithm (NIRA) is an optimization-based procedure to obtain a Nash-Cournot equilibrium point.

- The Nikaido-Isoda function is

$$\Psi(\hat{\mathbf{x}},\mathbf{x}) := \sum_{k=1}^{K} (\phi_k(x_k|\hat{\mathbf{x}}) - \phi_k(\hat{\mathbf{x}}))$$

- An equivalent formulation of a Nash equilibrium point is that $\mathbf{x}^*$ is an equilibrium point if

$$\max_{x\in\mathcal{X}} \Psi(\hat{\mathbf{x}},\mathbf{x}) = 0$$

- We define the optimal response function Z as

$$Z(\hat{\mathbf{x}}) := arg \max_{x\in\mathcal{X}} \Psi(\hat{\mathbf{x}},\mathbf{x})$$

- The NIRA algorithm updating rule is

$$x^{new} \leftarrow (1-u)\hat{\mathbf{x}} + uZ(\hat{\mathbf{x}}) \;\; u\in\mathbb{R},\, 0<u<1$$

The implementation of the NIRA algorithm to obtain the Nash-Cournot equilibrium

Note that, given $\hat{\mathbf{x}}$, $\phi_k(\hat{\mathbf{x}})$ is a constant, and that $max_{\mathbf{x}\in\mathcal{X}}\Psi(\hat{\mathbf{x}},\mathbf{x})$ is equivalent to solving

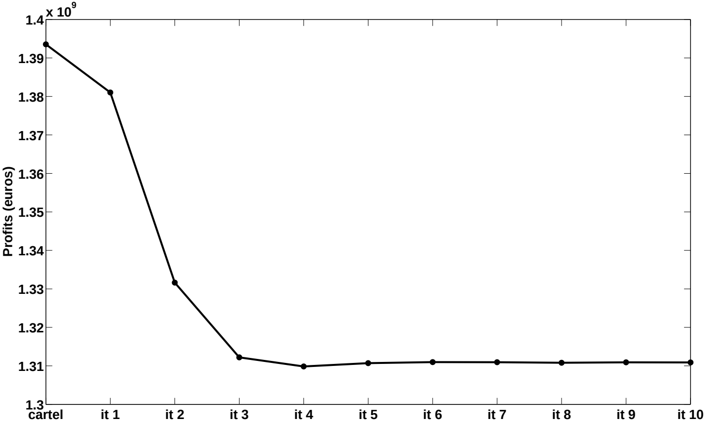
$$
\begin{aligned}
\underset{x_j^i}{\text{maximize}} \quad & \sum_{i=1}^{n_i}\sum_{k=1}^{K}\sum_{j\in\Omega_k^i}\Big\{(b^i-f_j)x_j^i+\frac{l^i}{2c_j}(x_j^i)^2+ \\
& -d[\sum_{l\in H_k} x_l^i x_j^i - \sum_{l\in H_{m|\,m\neq k}} \hat{x}_l^i x_j^i]\Big\} \\
\text{subject to:} \quad & \sum_{j\in\omega} x_j^i \le e^i - s^i(\omega) \quad \forall\,\omega\subseteq\Omega \quad \forall\, i \\
& \sum_{j\in\Omega} x_j^i + x_0^i = e^i \quad \forall\, i \\
& \sum_{i=1}^{n_i} C^i x^i \ge d \\
& x_j^i \ge 0 \quad j\in\Omega \quad \forall\, i
\end{aligned}
$$

Case study characteristics

- Real data from the Spanish Market
- First case: 9 aggregated generation units (2 hydro, 2 coal, 4 fuel/gas, 1 nuclear)
- Second case: 13 aggregated generation units (4 hydro, 4 coal, 4 fuel/gas, 1 nuclear)
- Model implemented and solved with AMPL/IPOPT

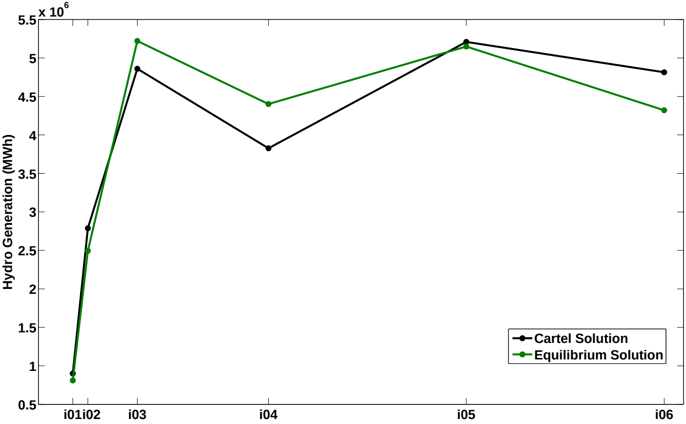
Results: 9 generation units

Evolution of the Objective Function



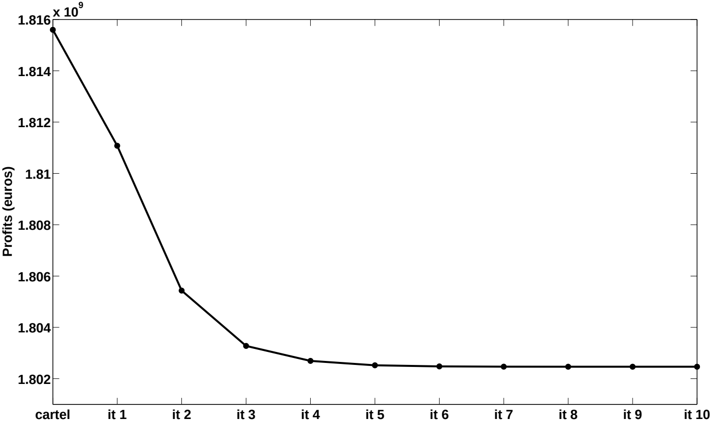
Results: 9 generation units

Variation of the Hydro Generation



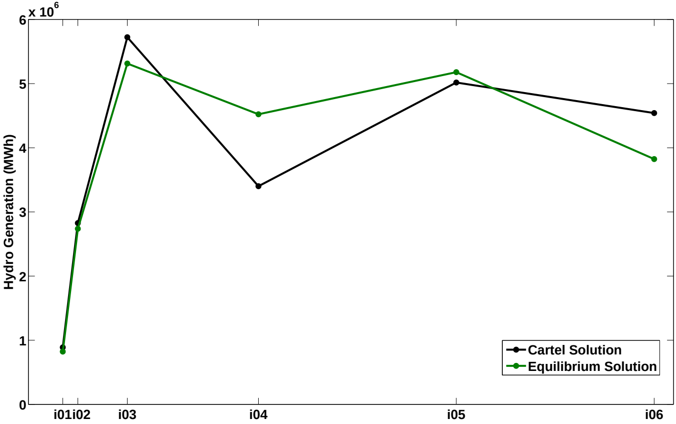
Results: 13 generation units

Evolution of the Objective Function



Results: 13 generation units

Variation of the Hydro Generation



Conclusions

- A new model for a mixed market using a time share hypothesis has been presented.
- The resulting problem has a non convex objective function.
- A Hydro-to-Market constraint is necessary.
- We found both the solution for the Cartel behaviour and Equilibrium behaviour using the Nikaido Isoda Relaxation Algorithm.
- The Equilibrium solution has profits lower than the Cartel solution, as expected.
- If not for the endogenous function due to hydro generation we would not get an equilibrium solution.
- There's a change between hydro generation over the time in the Cartel Solution and in the Equilibrium solution.

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