A multistage stochastic programming model for the optimal multimarket electricity bid problem

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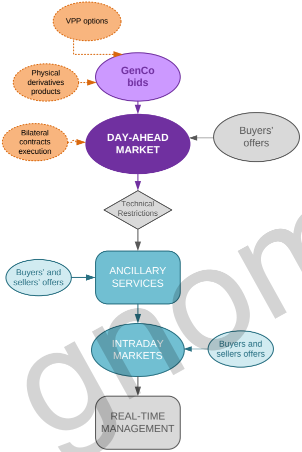
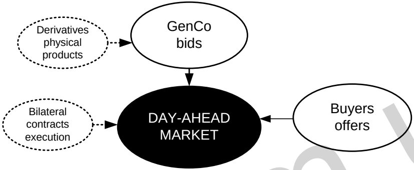
Iberian Electricity Market



- The MIBEL (created in 2007) joins Spanish and Portuguese electricity system.
- It complements the previous mechanisms of the Spanish Electricity Market with a Derivatives Market.
- It established a fully competitive framework for the generation of electricity, with a set of market mechanism centralized and managed by the *market operator*.
- It included a Day Ahead Market, a Reserve Market and a set of Intraday Markets to which the generation companies (GenCo) could submit their sell bids.

The GenCo's optimal DAM bid problem[1]

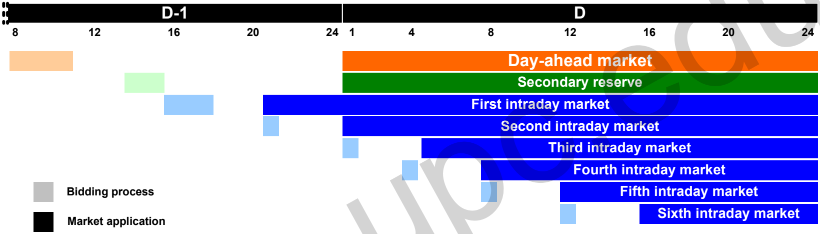
The GenCo's optimal DAM bid problem maximizes the expected profit from the DAM of a *Price-Taker* generation company with:



- A set of thermal generation units, I, with quadratic generation costs, start-up and shut-down costs and minimum operation and idle times.
- Each generation unit can submit sell bids to the 24 auctions of the DAM.
- A set of physical futures contracts, F, of energy L_j^F $j \in F$.
- A pool of bilateral contracts of energy L^B.

[1] Corchero et al. CAOR 2011, TOP 2011; Heredia et al. IEEE 2010, ANOR 2011

Sequence of markets in the MIBEL



- In the present work the reserve and intraday market sequence is integrated the in the DAM bid models
- Reserve market (RM): participants send bids to potentially increase or decrease the matched energy of the matched units in the day-ahead market.
- Intraday market (IM): similar to the DAM, except that the GenCo can participate as a buying as well as selling agent.

OMEB: Model description

OMEB: Optimal Multimarket Electricity Bid model

Max Expected profit of the markets' sequence

s.t.

- Physical futures and bilateral contract coverage
- Day-ahead market's rules
- Reserve market's rules
- Intraday market's rules
- Thermal units' operation rules
- Nonanticipativity

Mixed integer quadratic multistage stochastic program.

OMEB: Variables

First stage variables: for each period t and unit i

- The unit commitment variables: $u_{ti} \in \{0, 1\}$.
- The day-ahead market's price-acceptant bid variables: q_{ti}.
- The scheduled energy for futures contract j variables: f_{tij}.
- The scheduled energy for bilateral contract variables: b_{ti}.

Second and third stage variables: for each t, i and scenario s

- Total generation: g^s_{ti} ($g^s_{ti} = b_{ti} + p^s_{ti} + m^s_{ti}$).
- Matched energy in the day-ahead market: p^s_{ti}
- Reserve market related variables: $r^s_{ti} \in \{0, 1\}$
- Matched energy in the intraday market: m^s_{ti}

OMEB: Constraints (1/5)

Futures and Bilateral Contracts' Covering Constraints

$$\sum_{i \in I} b_{ti} = L^B \qquad \forall t \in T \tag{1}$$

$$0 \leq b_{ti} \leq \overline{P}_i u_{ti} \qquad \forall i \in I, \ \forall t \in T \tag{2}$$

$$\sum_{i \in I_j} f_{tij} = L_j^F \qquad \forall t \in T, \ \forall j \in F \tag{3}$$

$$0 \leq f_{tij} \leq \overline{P}_i u_{ti} \qquad \forall t \in T, \ \forall i \in I, \ \forall j \in F \tag{4}$$

(1),(2) The total BC energy will be covered with the contribution of all the committed units ($u_{ti} = 1$).

(3),(4) The energy of each FC j will be covered with the contribution of all the committed units ($u_{ti} = 1$) associated with that future contract j (I_j).

OMEB: Constraints (2/5)

Day-ahead market's rules

$$q_{ti} \leq p_{ti}^{s} \leq \overline{P}_i u_{ti} - b_{ti} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \qquad (5)$$

$$q_{ti} \geq \underline{P}_i u_{ti} - b_{ti} \qquad \forall i \in I,\ \forall t \in T \qquad (6)$$

$$q_{ti} \geq \sum_{\forall j \in F_i} f_{tij} \qquad \forall i \in I,\ \forall t \in T \qquad (7)$$

(5) if a unit is on, the matched energy p_{ti}^{s} will be between the price-acceptant bid q_{ti} and the total available energy not allocated to a BC.

(6) the minimum generation output of the committed units will be cleared.

(7) the contribution of the unit to the FC coverage will be included in the price-acceptant bid q_{ti}.

OMEB: Constraints (3/5)

Reserve market's rules

$$g_{ti}^s - g_{(t-1),i}^s \leq (1 - r_{ti}^s)\overline{P}_i u_{ti} \quad \forall i \in I,\ \forall t \in T,\ \forall s \in S \quad (8)$$

$$g_{ti}^s - g_{(t-1),i}^s \geq (1 - r_{ti}^s)(-\overline{P}_i u_{ti}) \quad \forall i \in I,\ \forall t \in T,\ \forall s \in S \quad (9)$$

$$r_{ti}^s \leq u_{ti} \quad \forall t \in T,\ \forall i \in I,\ \forall s \in S \quad (10)$$

(8,9) a unit participates in the RM ($r_{ti}^s = 1$) if the generation g_{ti}^s doesn't change between two consecutive intervals.

(10) uncommitted units cannot bid to the RM ($u_{ti} = 0 \Rightarrow r_{ti}^s = 1$).

OMEB: Constraints (4/5)

Total generation and intraday market's rules

$$g_{ti}^s = b_{ti} + p_{ti}^s + m_{ti}^s \qquad \forall t \in T,\ \forall i \in I,\ \forall s \in S \quad (11)$$

$$\underline{P}_i u_{ti} + \varrho_i r_{it}^s \le g_{ti}^s \le \overline{P}_i u_{ti} - \varrho_i r_{it}^s \qquad \forall t \in T,\ \forall i \in I,\ \forall s \in S \quad (12)$$

- (11) the total generation level of a given unit i, g_{ti}^s, is defined as the addition of the allocated energy to the BC, b_{ti}, plus the matched energy in the DAM and IM (p_{ti}^s and m_{ti}^s respectively)
- (12) the total generation g_{ti}^s must remain within the operational limits $\underline{P}_i$ and $\overline{P}_i$ less its fixed AGC capacity, ϱ_i(MW), if it participates in the RM (i.e., $r_{it}^s = 1$).

OMEB: Constraints (5/5)

Start-up and shut-down costs constraints:

$$c^{u}_{it} \geq c^{on}_{i}[u_{it} - u_{i,(t-1)}] \qquad t \in T \setminus \{1\}\ ,\ i \in I \qquad (13)$$

$$c^{d}_{it} \geq c^{off}_{i}[u_{i,(t-1)} - u_{it}] \qquad t \in T \setminus \{1\}\ ,\ i \in I \qquad (14)$$

Minimum up/down time constraints:

$$u \in X(t^{on}, t^{off})(unit\ commitment\ polyhedron) \qquad (15)$$

(13),(14) c^{on}_{i} and c^{off}_{i} are the constant start-up and shut-down costs, respectively, of thermal unit i.

(15) t^{on}_{i} (resp. t^{off}_{i}) is minimum operation (iddle) time.

OMEB: Objective Function

Maximization of the sequence of markets expected profits

$$\max_{p,g,r,m,u,c^u,c^d} \sum_{i \in I} \sum_{t \in T} \Big(-c^u_{ti} - c^d_{ti} + \tag{16}$$

$$+ \sum_{s \in S} P^s \Big[(\lambda^{D,s}_t p^s_{ti}) + (\lambda^{R,s}_t r^s_{ti} \varrho_i) + (\lambda^{I,s}_t m^s_{ti}) + \tag{17}$$

$$- \Big(c^b_i u_{ti} + c^l_i g^s_{ti} + c^q_i (g^s_{ti})^2 \Big) \Big] \Big) \tag{18}$$

(16) start-up and shut-down costs.

(17) day-ahead, reserve and intraday markets' incomes.

(18) thermal units' generation costs.

Perspective cuts formulation (1/2)

- The OMEB model is a Mixed-Integer Quadratic Program (MIQP), which is difficult to solve efficiently, especially for large-scale instances.
- A possibility is to use a polyhedral outer approximation of the quadratic generation cost function $f(g, u)$

$$f(g^s_{ti}, u_{ti}) = c^b_i u_{ti} + c^l_i g^s_{ti} + c^q_i (g^s_{ti})^2$$

  by means of perspective cuts (Frangioni and Gentile MP 2006), so that this problem can be solved as a Mixed-Integer Linear Program (MILP) by general-purpose MILP solvers.
- Perspective cuts shows an average speed-up factor of ten with respect to standard MIQP branch and cut methods when applied to day-ahead optimal bid problems (Corchero, Mijangos, Heredia TOP 2011 (accepted)).

Perspective cuts formulation (2/2)

- In the PCF of problem (OMEB) the quadratic generation cost function $f(g_{ti}^s, u_{ti})$ is replaced by its perspective cut approximation v_{ti}^s:

$$\max_{p,g,r,m,u,c^u,c^d} \sum_{i \in I} \sum_{t \in T} \Bigg(-c_{ti}^u - c_{ti}^d + \\ + \sum_{s \in S} P^s \left[(\lambda_t^{D,s} p_{ti}^s) + (\lambda_t^{R,s} r_{ti}^s \varrho_i) + (\lambda_t^{I,s} m_{ti}^s) - v_{ti}^s \right] \Bigg)$$

  where v_{ti}^s must satisfy the following set in inequalities (perspective cuts):

$$v_{ti}^s \geq (2c_i^q \widehat{g} + c_i^l) g_{ti}^s + (c_i^b - c_i^q \widehat{g}^2) u_{ti} \quad \widehat{g} \in \mathcal{C}_{ti}^s,\ i \in \mathcal{I},\ t \in \mathcal{T},\ s \in \mathcal{S}$$

- The elements in $\widehat{g} \in \mathcal{C}_{ti}^s \subset [\underline{P}_i, \overline{P}_i]$ are defined dynamically as the branch and cut algorithm proceeds.

Perspective cuts: Implementation

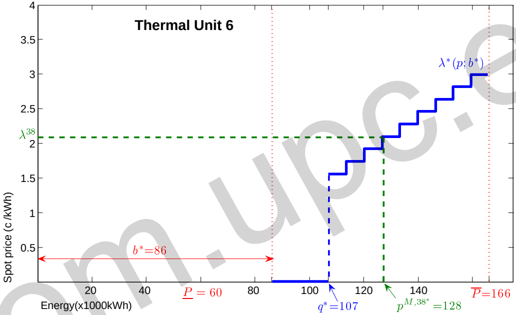
The numerical experiments solved instances of the OMEB problem with three different procedures:

- **MIQP1** The MIQP solver of Cplex 12.1
- **MIQP24** The MIQP solver of Cplex 12.1 with multithreading (24 threads).
- **PCF** The MILP solver of Cplex 12.1 were the dynamic generation of PCs was implemented by means of the `cutcallback` procedure.

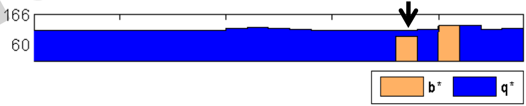
Method	Time (h)	c.v.	b.v.	Constraints	S
MIQP1	$120h30'^{*}$	145.680	48.240	381.796	200
MIQP24	$8h45'$	145.680	48.240	381.796	200
PCF	$2h58'$	261.857	48.240	641.151	200

* Execution aborted
Fuji RX200 S6 (2 x CPUs Intel Xeon X5680 Six Core / 12T 3.33 GHz, 64Gb RAM)

Results (1/3): optimal bidding curve



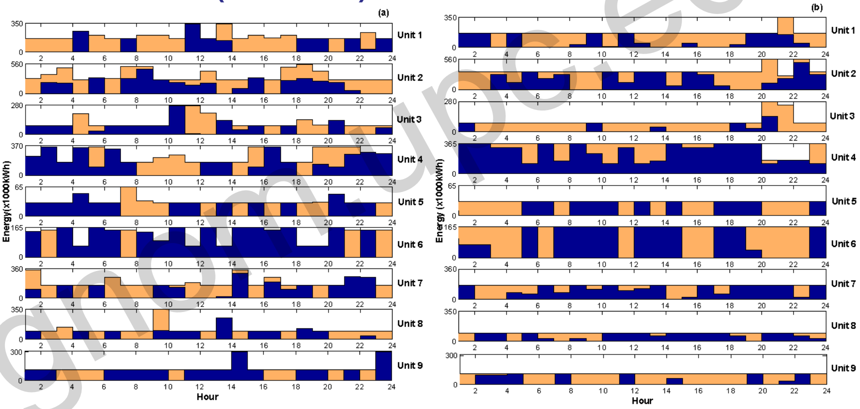
Optimal bidding curve for thermal unit 6 at interval 18



Bilateral and futures coverage of unit 6 along 24h

Results (2/3): commitment of the bilateral and future contracts

DAM + RM + IM (multimarket) **Only DAM**



Blue: bilateral contracts; orange: future contracts

Conclusions

- Model OMEB for the optimal DAM bid with Futures and Bilateral Contracts has been developed taking into account the Reserve and Intraday Market.
- The perspective cut formulation has been used to solve successfully real case instances of the OMEB model.
- The numerical experiments show how the Reserve and the Intraday Market affect both the optimal bid to the DAM and the optimal allocation of the energy of the Bilateral and Futures Contracts among the generation units.

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