Joint solution to the long-term power generation planning and maintenance scheduling

23rd IFIP TC7 – Krakow

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Introduction

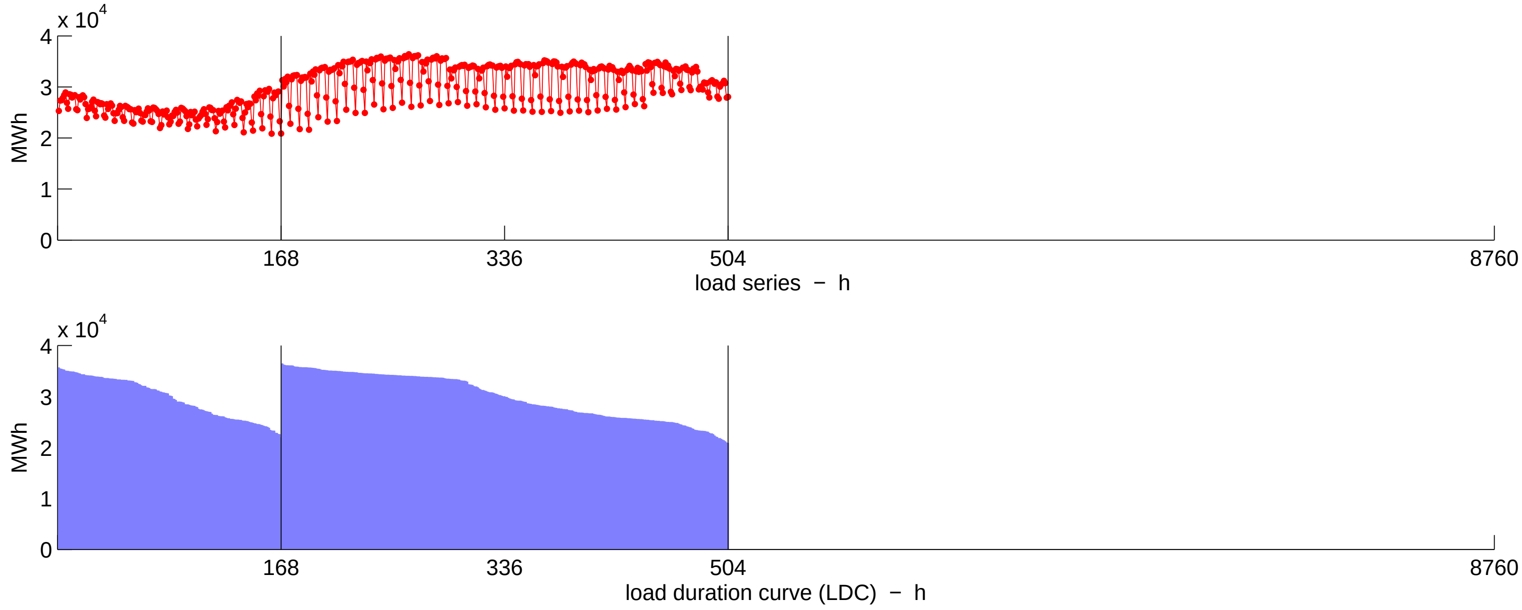
Introduction and motivation

- Problem:
  the long-term generation planning and units' maintenance for a specific generation company participating in a liberalized market.
- Its results are used both:
  - for budgeting and planning fuel acquisitions.
  - to provide a framework for short-term generation planning.

- A long-term planning *period* is subdivided into shorter *intervals* of equal length, for which parameters are known or predicted.
- The variables are the expected energy productions of each generating unit, and its in service/on maintenance status over the intervals.

Long-term generation planning problem

Intervals & Units



The LDC rotated and scaled by the length of the interval, t^i, corresponds to the load-survival function: $S_{\varnothing}(x)$.

Unit characteristics:

- c_j: Capacity
- p_j: Probability of being in service
- f_j: Linear generation cost
- m_j: Number of intervals required for maintenance

Description of the LTGP problem

Variables: Expected generation of each unit in each interval: x_j^i.

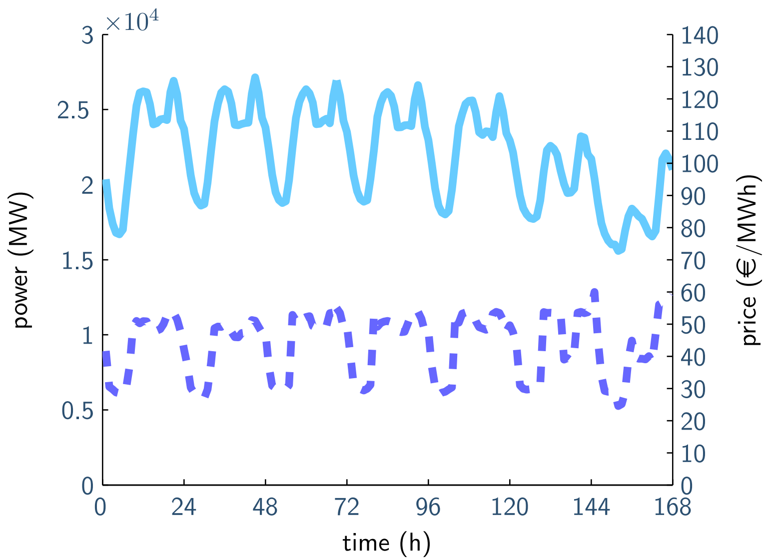
Quadratic objective function: Maximization of the generators surplus.

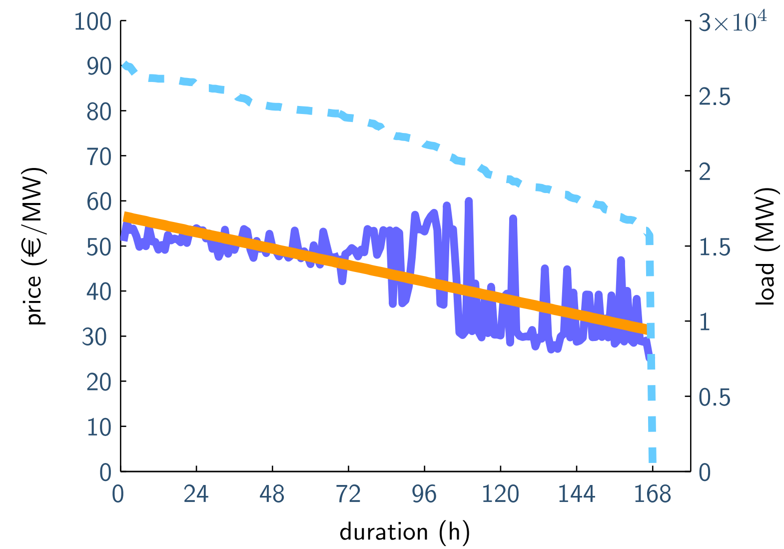
Linear constraints:

- Power balance (simplification of the Bloom and Gallant formulation)
- Maximum hydro generation
- Bonus-schemed coal
- Minimum generation time
- Market-share constraints
- Maximum emission limits
- Extra constraints: take-or-pay contracts, ...

Market price in terms of load duration

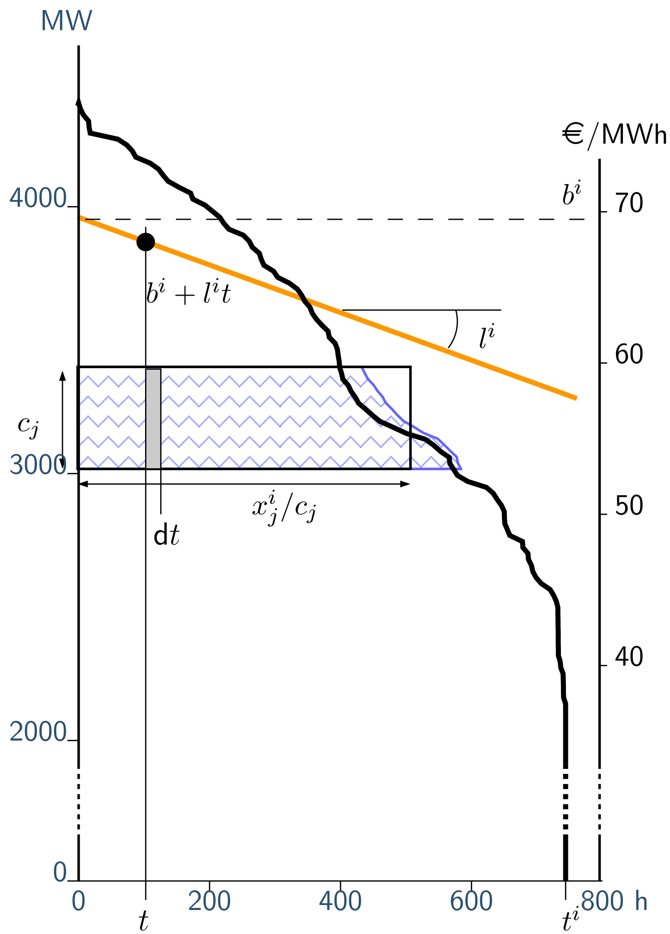
In a competitive market the goal is to maximize the profit of each unit from the pool. The profit is the revenue at market price minus generation cost. We need to relate the market price with the load duration:





Maximum generation surplus objective function

How do we compute the market profit?



Simplification: The contribution of unit j is rectangular.

The expected profit for unit j is:

$$\int_0^{x_j^i/c_j} c_j\{b^i + l^i t - f_j\}dt =$$

$$(b^i - f_j)x_j^i + \frac{1}{2}\frac{l^i}{c_j}{x_j^i}^2.$$

$b^i + l^i t$ is a regression of predicted market prices ordered by decreasing loads in interval i.

The long-term generation planning problem

The generator surplus maximization model extended to n_i intervals is:

$$\underset{x}{\text{maximize}} \sum_{i}^{n_i} \sum_{j}^{n_u} \{(b^i - \widetilde{f}_j) x_j^i + \frac{1}{2} \frac{l^i}{c_j} {x_j^i}^2\} \tag{1a}$$

$$\text{subject to: } Ax \geq a \tag{1b}$$

$$x_j^i \geq 0 \qquad \forall i \quad \forall j \in \Omega \tag{1c}$$

Long-term maintenance planning problem

Optimization of the maintenance schedule

The goal is to provide a maintenance schedule for each thermal unit of our SGC: $\Phi \subset \Omega$.

Binary variables:

- s^i_j : **S**tate of the unit. It is 1 when unit j is available in interval i and 0 otherwise.
- d^i_j : It is 1 only in the interval in which unit j shuts **d**own.
- u^i_j : It is 1 only in the interval in which unit j starts **u**p.

Objective function: We do not consider any maintenance cost because it would appear as a constant in the objective function.

Linear constraints:

- Maintenance schedule
- Minimum system and company capacity

LTMP mathematical model

$$\max_{s_j^i, d_j^i, u_j^i} \quad 0 \tag{2a}$$

$$\text{s.t.:} \quad \sum_{j \in \Omega \setminus \Phi} c_j + \sum_{j \in \Phi} c_j s_j^i \geq \kappa_s \overline{p}^{\,i} \qquad \forall\, i \tag{2b}$$

$$\sum_{j \in \Phi} c_j s_j^i \geq \kappa_c \widehat{p}_c \qquad \forall\, i \tag{2c}$$

$$\sum_{i=2}^{n_i - m_j} d_j^i = 1 \qquad \forall j \in \Phi \tag{2d}$$

$$d_j^i + \sum_{l=i}^{i+m_j-1} u_j^l \leq 1 \qquad \forall j \in \Phi, \quad i = 2, n_i - m_j + 1 \tag{2e}$$

$$s_j^i - s_j^{i-1} + d_j^i - u_j^i = 0 \qquad \forall j \in \Phi, \quad i = 2, n_i \tag{2f}$$

$$\sum_{i=2}^{n_i} s_j^i = n_i - m_j - 1 \qquad \forall j \in \Phi \tag{2g}$$

$$s_j^i, d_j^i, u_j^i \in \{0, 1\} \qquad \forall j \in \Phi, \quad i = 1, n_i \tag{2h}$$

Long-term generation and maintenance planning problem

LTGP ∪ LTMP → LTGMP

$$\max_{x,s,d,u} \sum_{i}^{n_i} \sum_{j}^{n_u} \{(b^i - \widetilde{f}_j)x_j^i + \frac{1}{2}\frac{l^i}{c_j}\, {x_j^i}^{\,2}\} \tag{3a}$$

$$\text{s.t.: LTGP constraints} \tag{3b}$$

$$\text{LTMP constraints} \tag{3c}$$

$$0 \le x_j^i \le \overline{x}_j^i s_j^i \qquad \forall\, i \quad \forall\, j \in \Phi \tag{3d}$$

$$0 \le x_j^i \le \overline{x}_j^i \qquad \forall\, i \quad \forall\, j \in \Omega \setminus \Phi \tag{3e}$$

$$s_j^i, d_j^i, u_j^i \in \{0, 1\} \qquad \forall\, i \quad \forall\, j \in \Phi \tag{3f}$$

Solution to the long-term generation and maintenance planning problem

LTGMP problem

The LTGMP problem (3) can be recast as:

$$\text{minimize } h'x + \frac{1}{2}x'Hx \tag{4a}$$

$$\text{subject to: } Ax \geq b \tag{4b}$$

$$Cy = d \tag{4c}$$

$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \quad \forall\, i,\, j \tag{4d}$$

$$y_j^i \in \{0, 1\} \quad \forall\, i,\, j \tag{4e}$$

Objective:

Solve our Quadratic Mixed Binary Problem (4) using global optimization concepts together with interior-point methods.

QMB problem ➜ Equivalent continuous problem ➜ Relaxed problem

Equivalent continuous problem

$$\min \quad h'x + \frac{1}{2}x'Hx \qquad (5a)$$

$$\text{s.t.:} \quad Ax \geq b \quad Cy = d \qquad (5b)$$

$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \qquad (5c)$$

$$\mathbf{0 \leq y_j^i \leq 1} \qquad (5d)$$

$$\mathbf{y_j^i(y_j^i - 1) \geq 0} \qquad (5e)$$

QMB problem ➜ Equivalent continuous problem ➜ Relaxed problem

Equivalent continuous problem

$$\min \quad h'x + \frac{1}{2}x'Hx \tag{5a}$$
$$\text{s.t.:} \quad Ax \geq b \quad Cy = d \tag{5b}$$
$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \tag{5c}$$
$$\mathbf{0} \leq \mathbf{y_j^i} \leq \mathbf{1} \tag{5d}$$
$$\mathbf{y_j^i}(\mathbf{y_j^i} - \mathbf{1}) \geq \mathbf{0} \tag{5e}$$

Lagrangian relaxed problem

$$\min \quad h'x + \frac{1}{2}x'Hx - \lambda\mathbf{y}'(\mathbf{y} - \mathbf{e}) \tag{6a}$$
$$\text{s.t.:} \quad Ax \geq b \quad Cy = d \tag{6b}$$
$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \tag{6c}$$
$$0 \leq y_j^i \leq 1 \tag{6d}$$

with $\lambda >= 0$.

QMB problem → Equivalent continuous problem → Relaxed problem

Equivalent continuous problem

$$\min \quad h'x + \frac{1}{2}x'Hx \quad (5a)$$
$$\text{s.t.}: \quad Ax \geq b \quad Cy = d \quad (5b)$$
$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \quad (5c)$$
$$\mathbf{0 \leq y_j^i \leq 1} \quad (5d)$$
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$$\min \quad h'x + \frac{1}{2}x'Hx - \lambda\mathbf{y}'(\mathbf{y} - \mathbf{e}) \quad (6a)$$
$$\text{s.t.}: \quad Ax \geq b \quad Cy = d \quad (6b)$$
$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \quad (6c)$$
$$0 \leq y_j^i \leq 1 \quad (6d)$$

with $\lambda >= 0$.

DC decomposition

$$\min \quad h'x + \frac{1}{2}x'Hx - \mathbf{t} \quad (7a)$$
$$\text{s.t.}: \quad Ax \geq b \quad Cy = d \quad (7b)$$
$$\lambda\mathbf{y}'(\mathbf{y} - \mathbf{e}) - \mathbf{t} \geq \mathbf{0} \quad (7c)$$
$$0 \leq x_j^i \leq \overline{x}_j^i y_j^i \quad (7d)$$
$$0 \leq y_j^i \leq 1 \quad (7e)$$

with $t \leq 0$.

Reverse convex cut

We consider the reverse convex constraint linearizing it at the current point $\widetilde{x}$, $\widetilde{y}$ and $\widetilde{t}$ (where $\widetilde{t} = \lambda\widetilde{y}'(\widetilde{y} - e)$):

$$\lambda(2\widetilde{y}-e)'(y-\widetilde{y}) \geq t-\widetilde{t} \quad (8)$$

In the computational trials we have used a relaxed cut:

$$\lambda(2\widetilde{y}-e)'(y-\widetilde{y}) \geq t-\beta\widetilde{t} \quad (9)$$

with $\beta < 1$.

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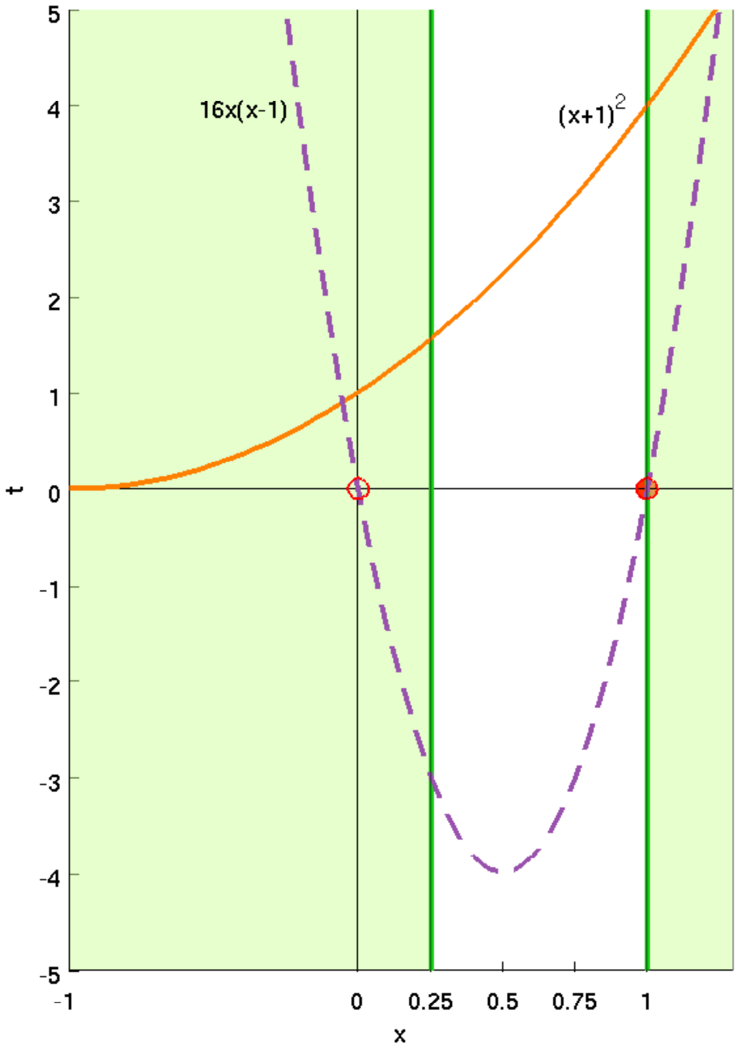
$$\min \ (x+1)^2$$
$$\text{s.t.:} \ \tfrac{1}{4} \leq x \leq 1 \quad x \in \mathbb{B}$$



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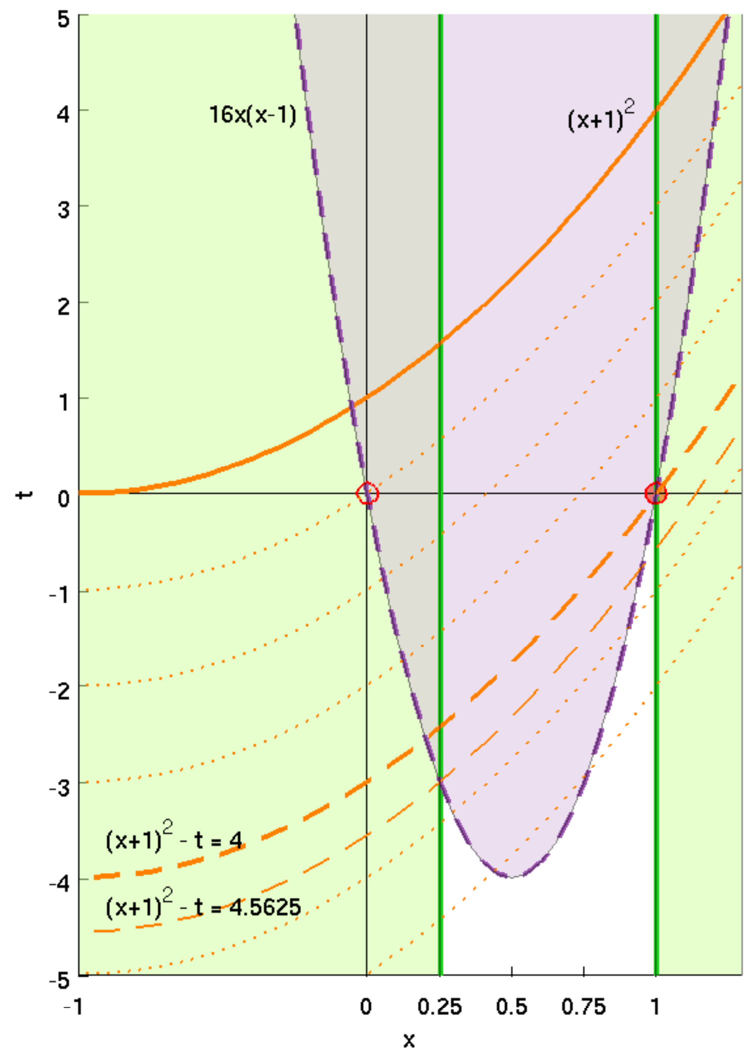
$$\lambda(2\tilde{y}-e)'(y-\tilde{y}) \geq t-\tilde{t} \quad (8)$$

In the computational trials we have used a relaxed cut:

$$\lambda(2\tilde{y}-e)'(y-\tilde{y}) \geq t-\beta\tilde{t} \quad (9)$$

with $\beta < 1$.

$$\min\ (x+1)^2 - t$$
$$\text{s.t.: } \lambda x(x-1)^2 \geq t \quad \frac{1}{4} \leq x \leq 1$$



Reverse convex cut

We consider the reverse convex constraint linearizing it at the current point $\widetilde{x}$, $\widetilde{y}$ and $\widetilde{t}$ (where $\widetilde{t} = \lambda \widetilde{y}'(\widetilde{y} - e)$):

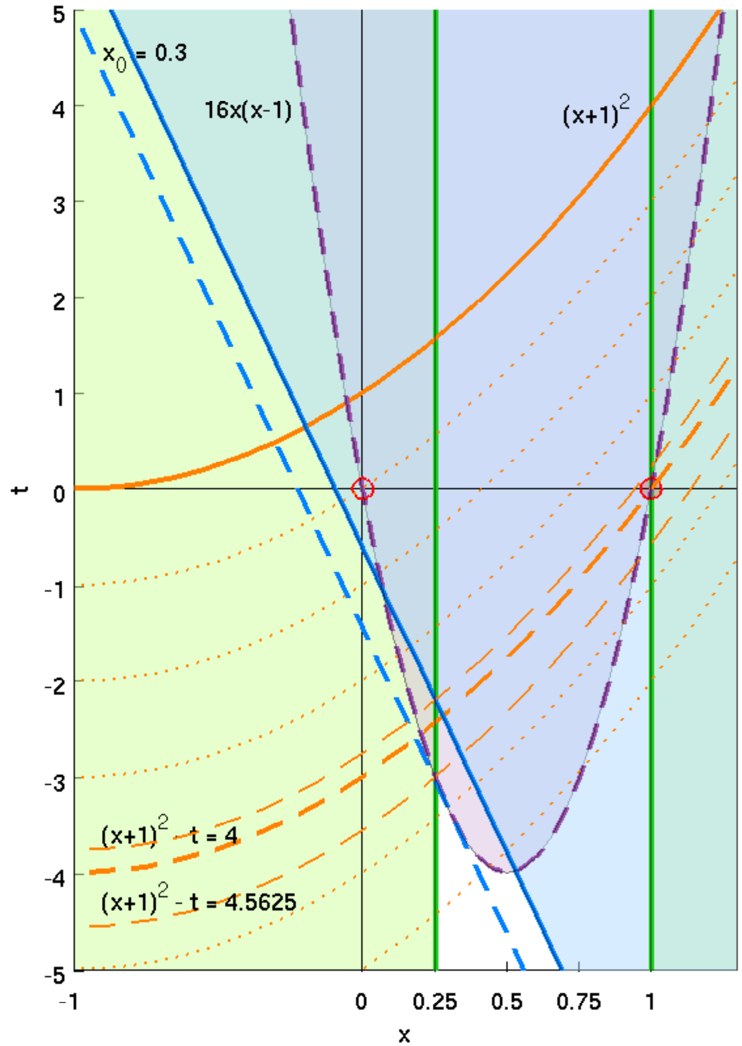
$$\lambda(2\widetilde{y} - e)'(y - \widetilde{y}) \geq t - \widetilde{t} \quad (8)$$

In the computational trials we have used a relaxed cut:

$$\lambda(2\widetilde{y} - e)'(y - \widetilde{y}) \geq t - \beta\widetilde{t} \quad (9)$$

with $\beta < 1$.

$$\min\ (x+1)^2 - t$$
$$\text{s.t.}: \ -6.4x \geq t - 0.6 \quad \tfrac{1}{4} \leq x \leq 1$$



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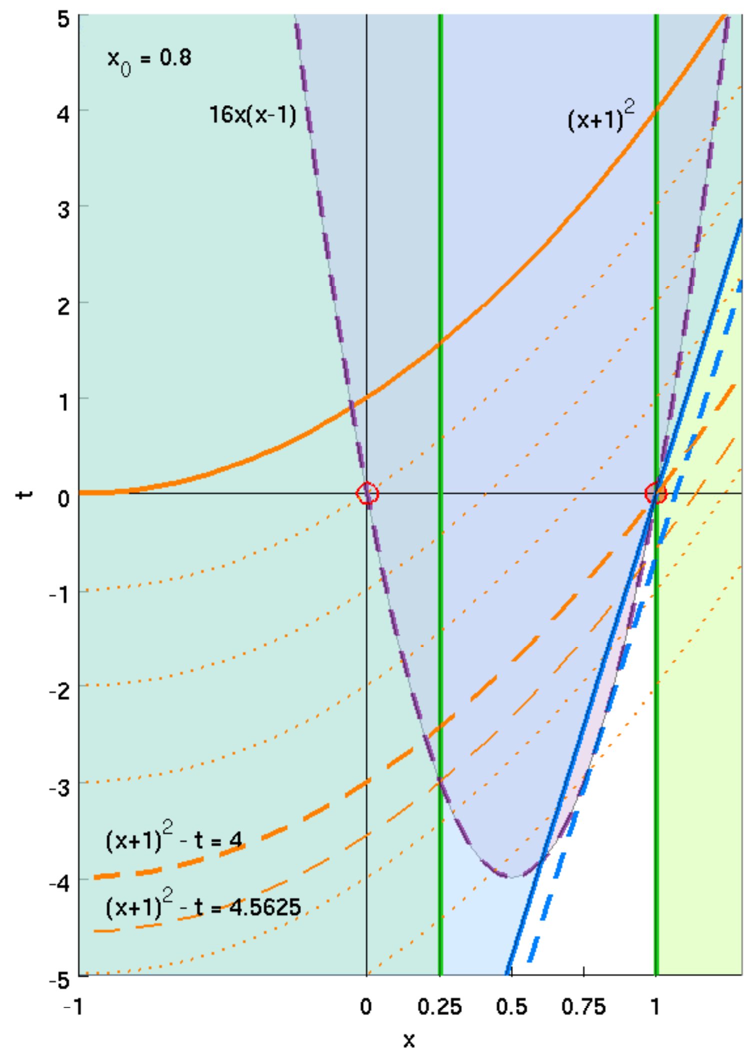
with $\beta < 1$.

$$\begin{aligned} &\min\ (x+1)^2 - t \\ &\text{s.t.}:\ 9.6x \geq t - 9.6 \quad \tfrac{1}{4} \leq x \leq 1 \end{aligned}$$



Transcending constraints

Given that not all the maintenance patterns are feasible we analyze the evolution of the value of the binary variables.

In our application we can distinguish three types of elements in the y vector: s, d and u. Given a specific value for one type of element the rest become fixed. Therefore we focus on the d variable which indicates when the maintenance period will start.

Let us define the following sets:

- Φ_1: Set of units j whose d_j^i values are either nearly 0 or 1 ($d_j^i \leq \varepsilon$ or $d_j^i \geq 1 - \varepsilon$ for all i).
- $\Phi_d := \Phi \backslash \Phi_1$
- $I_{1,j}$: Set of interval numbers where $d_j^i \geq 1 - \varepsilon$ for $j \in \Phi_1$.
- $I_{d,j}$: Set of interval numbers where $d_j^i \geq \varepsilon$ for $j \in \Phi_d$. These are called dominant intervals.

Transcending constraints

When we detect that we are heading to a local solution which is non-feasible we add a new constraint to prevent getting stuck in that point. There are two situations:

- We compute a target point by rounding the current solution. If this is a maintenance pattern but is infeasible (because of the minimum capacity constraints) we add the following constraint:

$$\sum_{j \in \Phi} \sum_{i \in I_{1,j}} d_j^i \leq \sum_{j \in \Phi} \sum_{i \in I_{1,j}} \dot{d}_j^i - 1 \tag{10}$$

  Given that the current point is far away enough of the target point we do not reinitialise the algorithm.

- We keep track of the trend of the binary variables towards 0 and 1. If $I_{d,j}$ has not changed from the last special iteration means that we are in a local solution with some variables far from 0 or 1 and we cannot scape from that point. We add the following transcending constraint:

$$\sum_{j \in \Phi_1} \sum_{i \in I_{1,j}} d_j^i + \sum_{j \in \Phi_d} \sum_{i \in I_{d,j}} d_j^i \leq \sum_{j \in \Phi_1} \sum_{i \in I_{1,j}} \widetilde{d}_j^i + \sum_{j \in \Phi_d} \sum_{i \in I_{d,j}} \widetilde{d}_j^i - 1 \tag{11}$$

  and we restart the variables to an arbitrary (big) value.

Interior point procedure

Interior-point procedure

- i Initialize the variables at a strictly interior point.
- ii Compute the infeasibility of the FOC, ξ_p, ξ_d and ξ_μ, the barrier parameter μ and the gap.

  If the gap is smaller than ε_g then END.
- iii Determine the centering parameter, ρ, and find the Newton direction Δ.
- iv Compute the maximum step length for the primal and dual variables: α_p and α_d.
- v Update the variables.

Interior point procedure

Interior-point procedure

i. Initialize the variables at a strictly interior point.

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   If the gap is smaller than ε_g then END.

iii. Determine the centering parameter, ρ, and find the Newton direction Δ.

iv. Compute the maximum step length for the primal and dual variables: α_p and α_d.

v. Update the variables.

vi. **Check whether a special iteration is required.**

Special iteration

- If $0 \leq y_j^i \leq 1$ and $\xi_p \leq 0.1$ and $\xi_d \leq 0.1$:
  - Compute the target point (round y): $\dot{y}$.
    If $\dot{y}$ is non-feasible for the LTMP problem then add a transcending constraint (10).
  - Update the linearized RC cut (8).
  - Compare the trend of the binary variables.
    If $I_{d,j}$ $\forall j$ is the same as the last special iteration then:
    - Add a transcending constraint (11).
    - Drop the linearized RC cut.
    - Reinitialise all the variables (to a big value).

Application of the method to the solution of LTGMP problems

Data of the test cases

case	n_u	n_i	t^i	= const	≥ const	n_m
ltgmp_1	13	27	14	4	21	6
ltgmp_2	14	27	14	4	20	6
ltgmp_3	12	27	14	4	21	5
ltgmp_4	16	27	14	4	20	8
ltgmp_5	18	27	14	4	13	10
ltgmp_6	34	27	14	4	21	21

Continuous: load balance, max hydro, max nuclear, min special regime, min market share
Extra binary constraints: min system capacity, min company capacity

Realistic cases. Data obtained from public web pages: www.omel.es and www.ree.es

Results

		λ	gap	Obj. Fun	sec	node	ite	RC cuts	Tr
relaxation	14			4979727394	0.05		15		
IP		1.0E+08	1.117E-04	4973540477	21.71		116	5	1
CPLEX			9.995E-05	4973885633	23.64	23300	117079		
relaxation	15			4749325728	0.02		14		
IP		5.0E+09	1.302E-04	4741719276	41.29		268	11	10
CPLEX			9.993E-05	4742611787	12.12	12800	61087		
relaxation	16			6042540366	0.05		16		
IP		4.0E+08	1.063E-04	6029072224	17.47		145	6	1
CPLEX			9.990E-05	6029268414	3.65	2500	11295		
relaxation	17			7071636168	0.02		15		
IP		1.0E+10	1.741E-04	7053603993	23.51		80	4	0
CPLEX			9.993E-05	7055968385	40.37	27600	169642		
relaxation	18			4711181849	0.03		14		
IP		5.0E+08	1.129E-04	4704409665	192.11		369	19	4
CPLEX			8.832E-05	4705130390	2.74	1800	6097		
relaxation	19			7039466093	0.15		14		
IP		5.0E+10	7.757E-04	7005637425	838.28		197	8	0
CPLEX			5.684E-04	7012135215	3648.25	1374700	8103520		

Table 1: Solutions to the LTGMP problem using the interior-point approach and comparison with the CPLEX B&B solution

Summary, conclusions and further research

Summary

- The joint model for the long term power generation and maintenance planning has been presented. It has a quadratic objective function with linear continuous and binary constraints.
- We approached the solution to the LTGMP problem using linearized RC and transcending cuts within an interior point procedure.
- We have tested our algorithm with realistic Spanish test cases.

Conclusions and further research

These results can be improved in the following points:

- We need to find an empiric rule to assign values to the λ parameter.
- The use of appropriate warm start techniques should speed up the solution time.
- The LTGMP problem is an easy problem because it has a large number of feasible solutions. We intend to solve a short term planning model for which CPLEX needs several hours to find one feasible solution.
- The CPLEX branch and bound implementation reaches the best solutions in terms of objective value. Should we develop an efficient implementation of our solution technique, our solutions would be comparable, in terms of time and objective value, to some early feasible solution produced by CPLEX.

Thank you for your attention!