Solving electric market problems by perspective cuts

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Abstract The electric market regulation in Spain (MIBEL) establishes the rules for bilateral contracts in the day-ahead optimal bid problem. Our model allows a price-taker generation company to decide the unit commitment of the thermal units, the economic dispatch of the bilateral contracts between the thermal units and the optimal sale bids for the thermal units observing the MIBEL regulation. The uncertainty of the spot prices is represented through scenario sets. We solve this model as a deterministic MIQP problem by using perspective cuts to improve the performance of Branch and Cut. Numerical results are reported.

1 Introduction

In liberalized electricity markets, a Generation Company (GenCo) must build an hourly bid that is sent to the market operator, who selects the lowest price among the bidding companies in order to match the pool load. GenCos need to know the prices at which the energy will be paid in order to decide how to bid and how to schedule their resources for maximizing their profit (in this work, for minimizing the costs). But, the market price is a random variable whose realization is only known once the market has been cleared. A forecast procedure based on an auto-regressive integrated moving average model [Nogales et al (2002)] gives us

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the probability distribution of this random variable [Corchero and Heredia (2009)]. The set of scenarios built from the forecasting results are used to feed a stochastic optimization model that finds the optimal day-ahead bid of a price-taker GenCo operating in the MIBEL and holding bilateral and future contracts. The optimization model used in this work extends the model of [Heredia et al (2010)], with the addition of physical future contracts. This model is a Mixed-Integer Quadratic Program (MIQP), which is difficult to solve efficiently, especially for large-scale instances. We approximate the quadratic objective function by means of *perspective cuts* [Frangioni and Gentile (2006)], so that this problem can be solved as a Mixed-Integer Linear Program (MILP) by general-purpose MILP solvers.

2 Model DABFC

2.1 Parameters and variables

Day-Ahead electricity market (DAM) with Bilateral and Futures Contracts *(DABFC)* model is built for a price-taker GenCo owning a set of thermal generation units $\mathscr{I}$ that bid to the $t \in \mathscr{T} = \{1,2,..,24\}$ hourly auctions of the DAM.

The parameters for the i^{th} thermal unit are these:

- generation costs with constant, linear and quadratic coefficients, c_i^b (€), c_i^l (€/MWh) and c_i^q (€/MWh^2).
- $\overline{P}_i$ and $\underline{P}_i$ are the upper and lower bound on the energy generation (MWh).
- start-up, c_i^{on}, and shut-down, c_i^{off}, costs (€).
- minimum operation and minimum idle time, t_i^{on} and t_i^{off}.

Moreover, for each $j \in \mathscr{F}$ the base load futures contract, the parameter L_j^{FC} is the amount of energy (MWh) to be procured each interval of the delivery period by the set U_j of generation units and the parameter λ_j^{FC} is the price of the contract (€/MWh). In addition, for each bilateral contract k, the parameter L_{kt}^{BC} is the amount of energy (MWh) to be procured at interval t of the delivery period by the whole set of generation units and the parameter λ_k^{BC} is the price of the contract (€/MWh).

The 1^{st} stage variables for $t \in \mathscr{T}$ and $i \in \mathscr{I}$ are:

- For the unit commitment: u_i^t (binary) , c_{it}^u, c_{it}^d.
- For the instrumental price offer bid: q_{it}.
- For the scheduled energy for futures contract $j \in \mathscr{F}$: f_{itj}.
- For the scheduled energy for bilaterals contract: b_{it}.

The 2^{nd} stage variables for each scenario $s \in \mathscr{S}$ and $t \in \mathscr{T}$, $i \in \mathscr{I}$ are the total generation, p_{it}^s, and the matched energy in the day-ahead market, $p_{it}^{M,s}$.

2.2 Constraints

2.2.1 Bilateral and future contracts constraints

The future contract constraints for $j \in \mathscr{F}$, $t \in \mathscr{T}$ are:

$$\sum_{i \in U_{jt}} f_{itj} = L_j^{FC} \quad \text{and} \quad f_{itj} \geq 0 \,, \quad i \in \mathscr{I}$$

and bilateral contract constraints for $t \in \mathscr{T}$ are:

$$\sum_{i \in U_t} b_{it} = L_t^{BC} \quad \text{and} \quad 0 \leq b_{it} \leq \overline{P}_i \,, \quad i \in \mathscr{I}$$

2.2.2 Day-ahead market and total generation constraints

The matched energy for $i \in \mathscr{I}$, $t \in \mathscr{T}$, $s \in \mathscr{S}$ are:

$$p_{it}^{M,s} \leq \overline{P}_i u_{it} - b_{it} \quad \text{and} \quad p_{it}^{M,s} \geq q_{it}$$

The instrumental price bid for $i \in \mathscr{I}$, $t \in \mathscr{T}$ holds the following constraints:

$$q_{it} \geq \underline{P}_i u_{it} - b_{it} \,, \qquad q_{it} \geq \sum_{j \,|\, i \in U_j} f_{itj} \,, \quad \text{and} \quad q_{it} \geq 0$$

The total generation constraints for $t \in \mathscr{T}$, $i \in \mathscr{I}$, $s \in \mathscr{S}$ are: $p_{it}^s = b_{it} + p_{it}^{M,s}$

2.2.3 Unit commitment constraints

This formulation follows that proposed by [Carrión and Arroyo (2006)].

For the start-up and shut-down costs for $i \in \mathscr{I}$:

$$\begin{aligned} c_{it}^u &\geq c_i^{on}[u_{it} - u_{i,(t-1)}] \quad &, t \in \mathscr{T} \setminus \{1\} \\ c_{it}^d &\geq c_i^{off}[u_{i,(t-1)} - u_{it}] \quad &, t \in \mathscr{T} \setminus \{1\} \\ c_{it}^u, c_{it}^d &\geq 0 \quad &, t \in \mathscr{T} \end{aligned}$$

Parameters G_i and H_i are the on- and off-initial time periods for $i \in \mathscr{I}$:

$$\sum_{j=1}^{G_i} (1 - u_{ij}) = 0 \quad \text{and} \quad \sum_{j=1}^{H_i} u_{ij} = 0$$

For t^{on} and t^{off}, minimum up and down time for $i \in \mathscr{I}$,

$$\sum_{n=t}^{t+t_i^{on}-1} u_{in} \geq t_i^{on}[u_{it} - u_{i,(t-1)}]\,, \quad t = G_i + 1, \ldots, |\mathscr{T}| - t_i^{on} + 1$$

$$\sum_{n=t}^{t+t_i^{off}-1} (1 - u_{in}) \geq t_i^{off}[u_{i,(t-1)} - u_{it}]\,, t = H_i + 1, \ldots, |\mathscr{T}| - t_i^{off} + 1$$

For the last $t^{on} - 1$ and $t^{off} - 1$ time periods and $i \in \mathscr{I}$

$$\sum_{n=t}^{|\mathscr{T}|}(u_{in} - [u_{it} - u_{i(t-1)}]) \geq 0\,,\, t = |\mathscr{T}| - t_i^{on} + 2, \ldots, |\mathscr{T}|$$

$$\sum_{n=t}^{|\mathscr{T}|}(1 - u_{in} - [u_{i(t-1)} - u_{it}]) \geq 0\,,\, t = |\mathscr{T}| - t_i^{off} + 2, \ldots, |\mathscr{T}|$$

2.3 Objective function

$$\min_{p,q,f,b} \mathrm{E}_{\lambda^D}\left[B(u, c^u, c^d, p^M, p; \lambda^D)\right] =$$

$$\sum_{t\in\mathscr{T}} \sum_{i\in\mathscr{I}} \left(c_{it}^u + c_{it}^d + c_{it}^b u_{it} + \sum_{s\in\mathscr{S}} P^s \left[(c_i^l p_{it}^s + c_i^q (p_{it}^s)^2) - \lambda_t^{Ds} p_{it}^{M,s} \right] \right)$$

Model DABFC is the deterministic equivalent program associated with the two-stage stochastic problem with a set $\mathscr{S}$ of scenarios for the spot price λ_t^D, where $t \in \mathscr{T}$. This deterministic program is a convex MIQP with a well defined global optimal solution.

3 Perspective cuts

In order to solve DABFC by commercial MILP software, the quadratic part of the objective function must be linearized. Since the sum of the probabilities P^s is one, we can include the products $c_{it}^b u_{it}$ in the quadratic parenthesis for each block (i,t,s) in this way:

$$c_i^q (p_{it}^s)^2 + c_i^l p_{it}^s + c_{it}^b u_{it},$$

where the variables u_{it} are binary. For notational simplicity we drop the indices.

The issue is then how to best represent the quadratic function $f(p,u) = c^q p^2 + c^l p + c^b u$ by means of a piecewise-linear one. There is an effective way based on ideas developed in [Frangioni and Gentile (2006)]. The function $f(p,u)$ is only relevant at points (p,u) of its (disconnected) domain $\mathscr{D} = [0,0] \cup \left[[\underline{P}, \overline{P}] \times \{1\} \right]$. However, standard branch-and-cut approaches typically solve the continuous relaxation of the mixed problem, where $u \in [0,1]$ instead of $\{0,1\}$, in order to obtain lower bounds on the optimal value. This makes sense to use the *convex envelope* of $f(p,u)$ over $\mathscr{D}$, that is, the convex function with the smallest (in set-inclusion sense) epigraph containing that of $f(p,u)$.

As is shown in [Frangioni and Gentile (2006)] the *convex envelope* is

$$h(p,u) = \begin{cases} 0, & \text{if } (p,u) = (0,0) \\ \dfrac{c^q p^2}{u} + c^l p + c^b u, & \left\{ \begin{array}{c} \text{if } u\underline{P} \le p \le u\overline{P}, \\ \text{for } u \in (0,1] \end{array} \right\} \\ +\infty, & \text{otherwise.} \end{cases}$$

This function is strongly related with the *perspective-function* $\breve{f}(p,u) = uf(p/u)$ of $f(p) = c^q p^2 + c^l p + c^b$ [Hiriart-Urruty and Lemaréchal (1993)], which is convex if $f(p)$ is convex. In addition, $h(p,u) \ge f(p,u)$ for $0 < u \le 1$, i.e. h is a tighter objective function than f for the continuous relaxation.

As is well-known, every convex function is the point-wise supremum of affine functions. In fact, the epigraph of h is composed of all and only triples (v,p,u) satisfying $u\underline{P} \le p \le u\overline{P}$, $0 \le u \le 1$ and the infinite system of linear inequalities

$$v \ge (2c^q \widehat{p} + c^l)p + (c^b - c^q \widehat{p}^2)u, \qquad \text{taking} \quad \widehat{p} \in [\underline{P}, \overline{P}].$$

For each $\widehat{p}$ we have an inequality so-called a *perspective cut* (PC), which is the unique supporting hyperplane to the function passing by $(0,0)$ and $(\widehat{p},1)$.

PC formulation (PCF) lies in choosing these supporting hyperplanes and using as an objective function the polyhedral function that is the point-wise maximum of the corresponding linear functions. A small set of initial PCs is chosen to solve the problem with the continuous relaxation. When $u^* > 0$, check whether the solution (v^*, p^*, u^*) satisfies the PC for $\widehat{p} = p^*/u^*$; if not, the obtained cut can be added to PCF, which starts with only two pieces, the ones corresponding with $\underline{P}$ and $\overline{P}$. Additional cuts are then dynamically generated when needed as described in the previous paragraph.

4 Numerical tests

In our numerical tests we have used Cplex 12.1, which allows one to directly input the DABFC problem as a Mixed-Integer Linearly Constrained Quadratic Program and solve it as a MIQP. Moreover, for PCF the dynamic generation of PCs can be easily implemented by means of the `cutcallback` procedure. Thus, apart from the basic formulation, the same sophisticated tools (valid inequalities, branching rules, ...) are used for both formulations: MIQP and PCF.

A few differences remain: e.g. the need for invoking the callback functions disables the more efficient dynamic search of Cplex 12.1 for adding cuts, whereas this skill is used when the DABFC problem is solved by Cplex as a MIQP. Apart from these, the very same machinery is used with both formulations, allowing a fair comparison.

The tests have been performed on DELL OPTIPLEX GX620 Intel Pentium with 4 CPU and 3.40 GHz, under Linux (Suse 10.0).

Table 1 Test problems

Prob.	$\lvert\mathscr{F}\rvert$	$\lvert BC\rvert$	$\lvert\mathscr{S}\rvert$	$\lvert\mathscr{I}\rvert$	$\lvert\mathscr{T}\rvert$	# *var*	# var_{PCF}	# *bin*	# *constr*	t_{MIQP}	t_{PCF}	# *PC*
fcbcuc1	2	2	2	4	6	264	312	24	428	0.19	0.14	166
fcbcuc3	2	2	2	4	24	1056	1248	96	1688	1.31	0.27	784
fcbcuc4	2	2	4	6	24	2160	2736	144	3970	26.64	1.5	2271
fcbcuc5	3	2	4	10	24	3840	4800	240	6596	37.27	3.82	2720
fcbcuc6	3	3	5	10	24	4320	5520	240	7796	21.70	5.47	3665
fcbcuc7	3	3	10	10	24	6720	9120	240	13796	169,5	33.87	9687
ismp09	3	3	61	10	24	31200	45840	240	74996	13231.4	1350.89	45361

In Table 1 $|BC|$ means the number of bilateral contracts, # *var* is the number of variables in DABFC, # var_{PCF} is the number of variables in DABFC for the PC formulation, # *bin* is the number of binary variables, # *constr* represents the number of constraints in DABFC, t points out the CPU-times (in seconds) used to solve these problems for each method, and # *PC* is the number of perspective cuts generated by the PC formulation with Cplex. Note that, if we use PCF, the problem increases the number of variables in $m = |\mathscr{T}| \cdot |\mathscr{I}| \cdot |\mathscr{S}|$ and the number of constraints in $2{\cdot}m$.

As can be observed in Table 1, in the solution of these DABFC problems Cplex with perspective cut formulation has been significantly more efficient than without it.

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