Network Model of Short-Term Optimal Hydrothermal Power Flow with Security Constraints.

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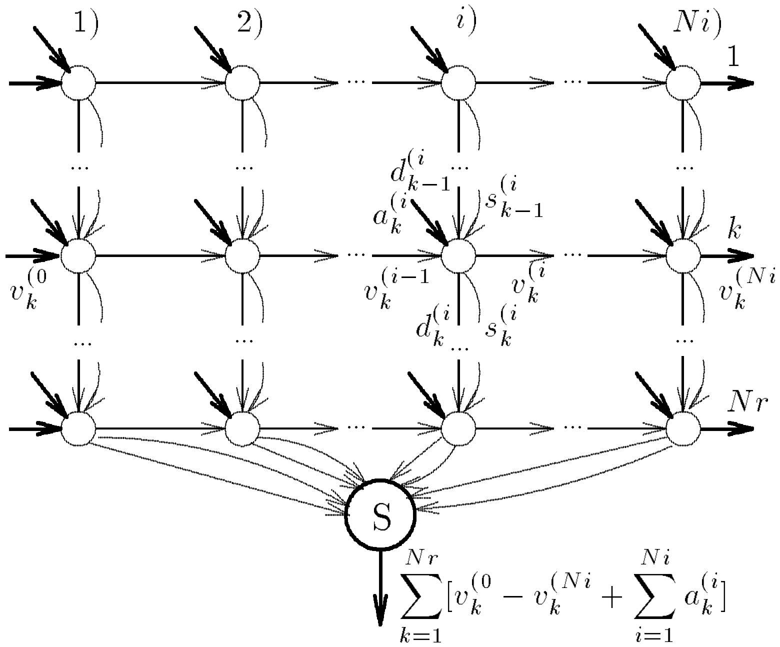
DESCRIPTION OF THE PROBLEM

- **GIVEN :**
  - ∗ A hydro system with Nr reservoirs.
  - ∗ A thermal system with Nu thermal units committed.
  - ∗ A set of Nl load nodes.
  - ∗ A dc transmission network model.
  - ∗ **A list of line outages.**
- **FIND, for each time interval of the period studied:**
  - ∗ The reservoir discharges and storages.
  - ∗ The thermal unit power output.
  - ∗ The power flows on the transmission network.
- **SUCH THAT minimize:**
  - ∗ The thermal generation cost.
- **SATISFYING:**
  - ∗ The forecasted load requirements.
  - ∗ The spinning reserve requirements.
  - ∗ The KCL and KVL in the transmission network.
  - ∗ **Security constraints.**
  - ∗ **Multi interval emission constraints.**

(the unit commitment problem is not solved)

HYDRO NETWORK



- **Balance equation at reservoir k over interval i:**

$$a_k^{(i} + v_k^{(i-1} + d_{k-1}^{(i} + s_{k-1}^{(i} = v_k^{(i} + d_k^{(i} + s_k^{(i} \tag{1}$$

(delays and pumping are considered but not depicted here)

HYDROGENERATION FUNCTION

- **Hydrogeneration at reservoir k over interval i :**

$$H_k^{(i} = \mu {\rho_k}^{(i} {h_k}^{(i} {d_k}^{(i} \tag{2}$$

  * Reservoir head function :

$$h_k = s_{bk} + s_{lk} v_k + s_{qk} {v_k}^2 + s_{ck} {v_k}^3 \tag{3}$$

  * Efficiency :

$$\begin{aligned} \rho_k^{(i} =& \rho_{k0} + \rho_{kh} h_k^{(i} + \rho_{kd} d_k^{(i} + \\ & + \rho_{khd} h_k^{(i} d_k^{(i} + \rho_{khh} (h_k^{(i})^2 + \rho_{kdd} (d_k^{(i})^2 \end{aligned} \tag{4}$$

- **Total hydrogeneration over interval i :**

$$H^{(i} = \sum_{k=1}^{Nr} H_k^{(i} \tag{5}$$

- **Hydrogeneration linearization :**

$$H_k^{(i} \approx H_{Lk}^{(i} = \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \tag{6}$$

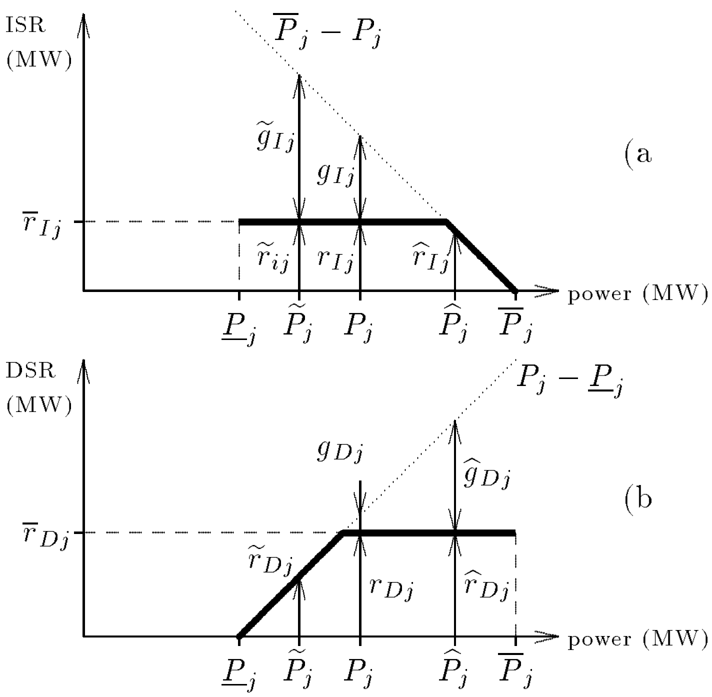
THERMAL SYSTEM MODELIZATION

- **Variables associated to the generation of a single thermal unit:**

$$\underline{P}_j \leq P_j \leq \overline{P}_j \tag{7}$$

$$r_{Ij} = \min\{\overline{r}_{Ij}, \overline{P}_j - P_j\} \tag{8}$$

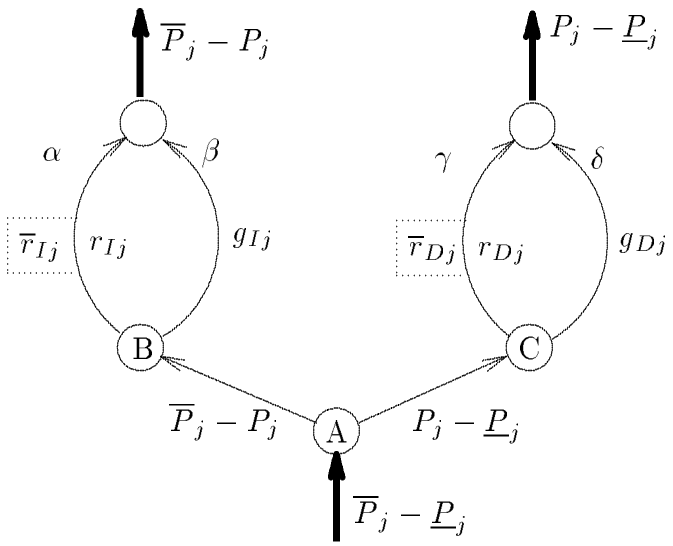
$$r_{Dj} = \min\{\overline{r}_{Dj}, P_j - \underline{P}_j\} \tag{9}$$



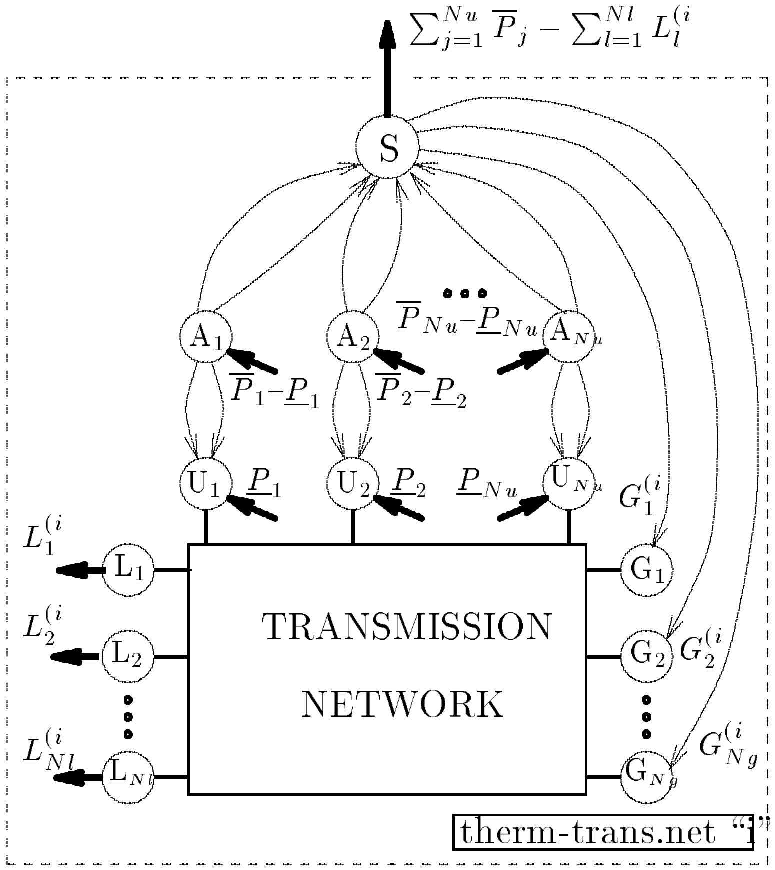
EQUIVALENT THERMAL NETWORK UNIT j

$$r_{Ij} \le \min\{\overline{r}_{Ij}, \overline{P}_j - P_j\} \begin{cases} r_{Ij} \le \overline{r}_{Ij} & (10) \\ r_{Ij} \le \overline{P}_j - P_j \\ \quad\downarrow \\ r_{Ij} + g_{Ij} = \overline{P}_j - P_j & (11) \end{cases}$$

$$r_{Dj} \le \min\{\overline{r}_{Dj}, P_j - \underline{P}_j\} \begin{cases} r_{Dj} \le \overline{r}_{Dj} & (12) \\ r_{Dj} \le P_j - \underline{P}_j \\ \quad\downarrow \\ r_{Dj} + g_{Dj} = P_j - \underline{P}_j & (13) \end{cases}$$



THERMAL + TRANSMISSION NETWORK



- Side constraints :

$$G_j^{(i} = \sum_{k\in I_j} H_k^{(i} \approx \sum_{k\in I_j} \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i}$$

$$j = 1, \ldots, Ng \quad (14)$$

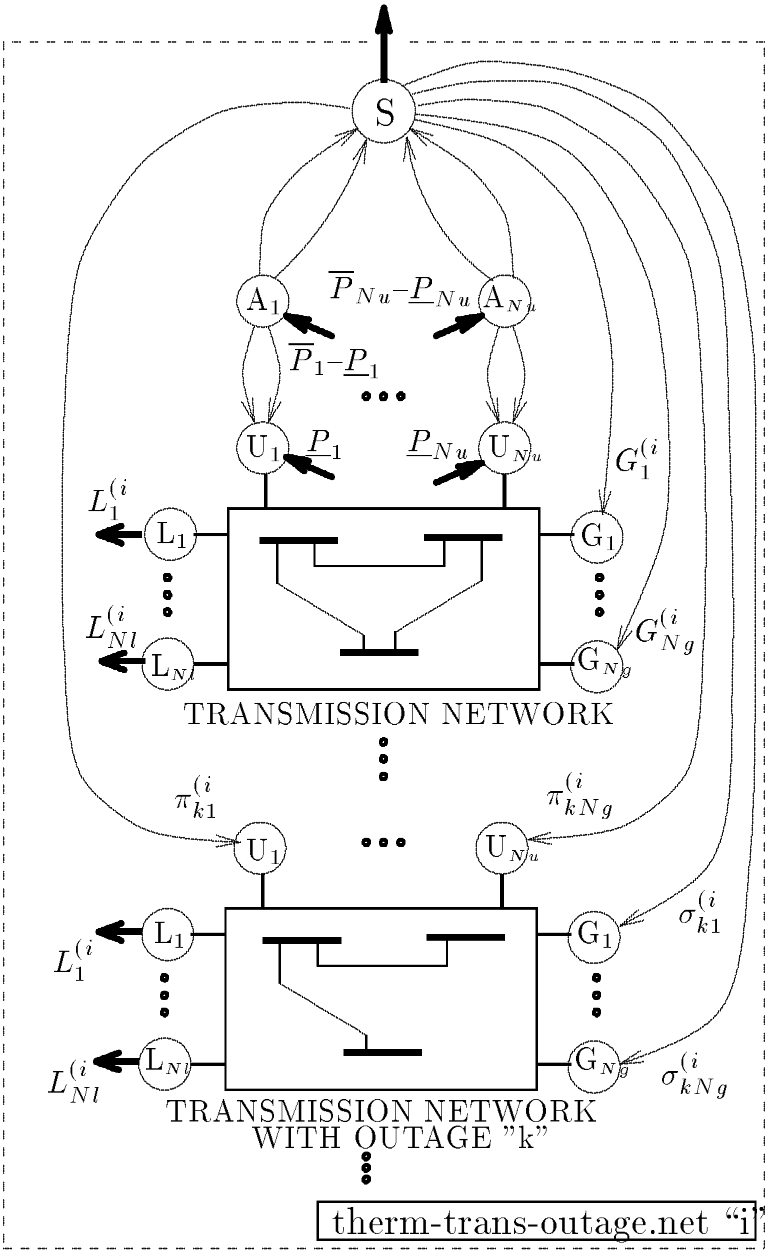
$$\sum_{k=1}^{Nr} \overline{H}_k^{(i} - \sum_{j=1}^{Ng} G_j^{(i} + \sum_{j=1}^{Nu} r_{Ij}^{(i} \geq R_I^{(i} \quad (15)$$

$$\sum_{j=1}^{Ng} G_j^{(i} + \sum_{j=1}^{Nu} r_{Dj}^{(i} \geq R_D^{(i} \quad (16)$$

$$\sum_{m\in \mathcal{C}_j} X_m p_m^{(i} = 0 \quad ; \quad j = 1, \ldots, Nc \quad (17)$$

THERMAL + TRANSMISSION + OUTAGES NETWORK

$\sum_{j=1}^{Nu} \overline{P}_j - (1 + Nk) \times \sum_{l=1}^{Nl} L_j^{(i}$

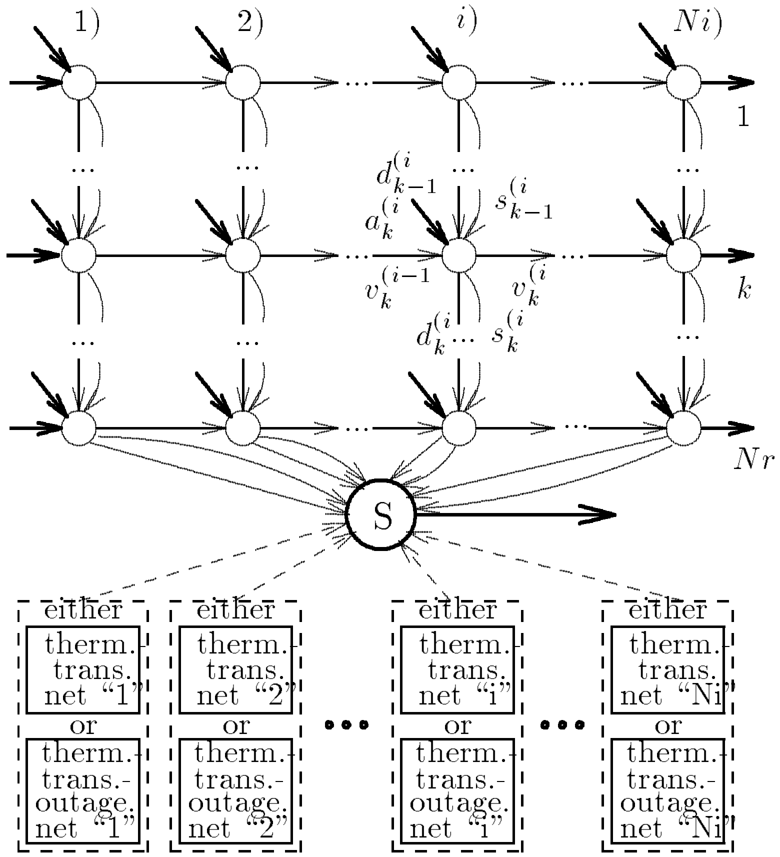


- ... additional side constraints :

$$
\begin{aligned}
G_j^{(i} - \sigma_{kj}^{(i} &= 0 \\
P_j^{(i} - \pi_{kj}^{(i} = r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j - \pi_{kj}^{(i} &= 0
\end{aligned}
\left\{
\begin{array}{l}
k = 1, \ldots, Nk \\
j = 1, \ldots, Nu \\
i = 1, \ldots, Ni
\end{array}
\right. \tag{18}
$$

... plus KVL for the trans. net. with outages

HYDRO-THER.-TRANS.-OUTAGE. EXT. NET. (HTTOEN)



- Multi-interval emission side constraints :

$$\sum_{i=1}^{Ni} \nu_j \sum_{j \in \mathrm{F.T.}} P_j^{(i} \leq \overline{E}_{\mathrm{F.T.}} \tag{19}$$

VARIABLES AND OBJECTIVE FUNCTION

- **Variables :** $i = 1, \ldots, Ni$
  - $*$ Hydro variables : $d_k^{(i}$, $s_k^{(i}$, $v_k^{(i}$, $k = 1, \ldots, Nr$
  - $*$ Thermal variables : $r_{Ij}^{(i}$, $g_{Ij}^{(i}$, $r_{Dj}^{(i}$, $g_{Dj}^{(i}$, $j = 1, \ldots, Nu$
  - $*$ Transmission variables : $p_m^{(i}$, $m = 1, \ldots, Nm$
  - $*$ Hydrogen. variables : $G_j^{(i}$, $j = 1, \ldots, Ng$
  - $*$ Trans. network with outages variables : $\sigma_{kj}^{(i}$, $\pi_{kj}^{(i}$, $k = 1, \ldots, Nk$
- **Objective function : two parts**
  - $*$ Thermal generation costs :

$$\sum_{i=1}^{Ni}\sum_{j=1}^{Nu}[c_{lj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j)^2] \qquad (20)$$

  - $*$ Costs of power losses:

$$\sum_{i=1}^{Ni}\pi^{(i}\sum_{m=1}^{Nm} r_m(p_m^{(i})^2 \qquad (21)$$

$$\min\sum_{i=1}^{Ni}\Big\{\sum_{j=1}^{Nu}[c_{lj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j) + c_{qj}(r_{Dj}^{(i} + g_{Dj}^{(i} + \underline{P}_j)^2]$$

$$+\pi^{(i}\sum_{m=1}^{Nm} r_m(p_m^{(i})^2\Big\} \qquad (22)$$

OPTIMIZATION PROBLEM

- **Exact hydrogeneration function :**

$$
\text{(NNNC)} \begin{cases} \min & f(x) & & (23a) \\ \text{subj. to :} & Ax = r & & (23b) \\ & g(x) + \mathbf{I}_z z = b & & (23c) \\ & 0 \leq x \leq u_x & & (23d) \end{cases}
$$

- **Linearized hydrogeneration function :**

$$
\text{(NNLC)} \begin{cases} \min & f(x) & & (24a) \\ \text{subj. to :} & Ax = r & & (24b) \\ & Tx + \mathbf{I}_z z = b & & (24c) \\ & 0 \leq x \leq u_x & & (24d) \end{cases}
$$

SOLUTION METHODS

- (NNLC) **problem : NOXCB (Heredia & Nabona)**
  - Specialised code for the nonlinear network flow problem with linear side constraints :

$$H_{Lk}^{(i} = \lambda_{0k}^{(i} + \lambda_{v(i-1)k}^{(i} v_k^{(i-1} + \lambda_{v(i)k}^{(i} v_k^{(i} + \lambda_{dk}^{(i} d_k^{(i} \tag{14}$$

  - Successive linearizations.

- (NNNC) **problem : MINOS (Murtagh & Saunders)**
  - General purpose nonlinear optimization package :

$$H_k^{(i} = \mu \rho_k{}^{(i} h_k{}^{(i} d_k{}^{(i}$$

ITERATIVE SOLUTION METHOD

0 **Initializations.**

- **0.1** Definition of the network equations of $\mathbf{(NNLC)}^0$
- **0.2** Selection of the initial solution $[x]^0$; $k := 1$.
- **0.3** Maximum hydrogeneration error : $\epsilon_L \approx 0.02$

1 Major iterations.

- **1.4** Linearization about $[x]^{k-1} \rightarrow \mathbf{(NNLC)}^k$.
- **1.5** Optimization of $\mathbf{(NNLC)}^k$ with NOXCB $\rightarrow [x]^k$
- **1.6** If $\mid [H_L^{(i}]^k - [H^{(i}]^k \mid < \epsilon_L \, L^{(i}$, $\forall i$ then $x^* := [x]^k$; STOP
- **1.7** $k := k + 1$. go to **1.4** .

COMPUTATIONAL RESULTS

Table I : Power system size of case examples

Problem	Power system size					
	Nr	Nu	Nm	Nb	Ni	Nk
A	9	8	21	13	48	-
Ac	9	8	21	13	48	3
B	9	14	30	23	48	-
Bc	9	14	30	23	48	1

Table II : Problem dimensions of case examples

Problem	Optimization problem size			
	arcs	nodes	l.s.c.	n.s.c.
A	4704	1441	529	48
Ac	6816	2298	1453	48
B	7056	2209	480	48
Bc	9792	3314	1104	48

Table III : Computational results

Problem	CPU time (sec.)	iter.	Cost (10^6 Pts.)	# sat. lin. tn	cn
A	6631.7	42361	196.951	1	-
Ac	11698.1	48211	197.015	0	4
B	20809.8	85699	47.775	5	-
Bc	27123.7	85763	47.789	0	21

Line flow values along intervals in a given line

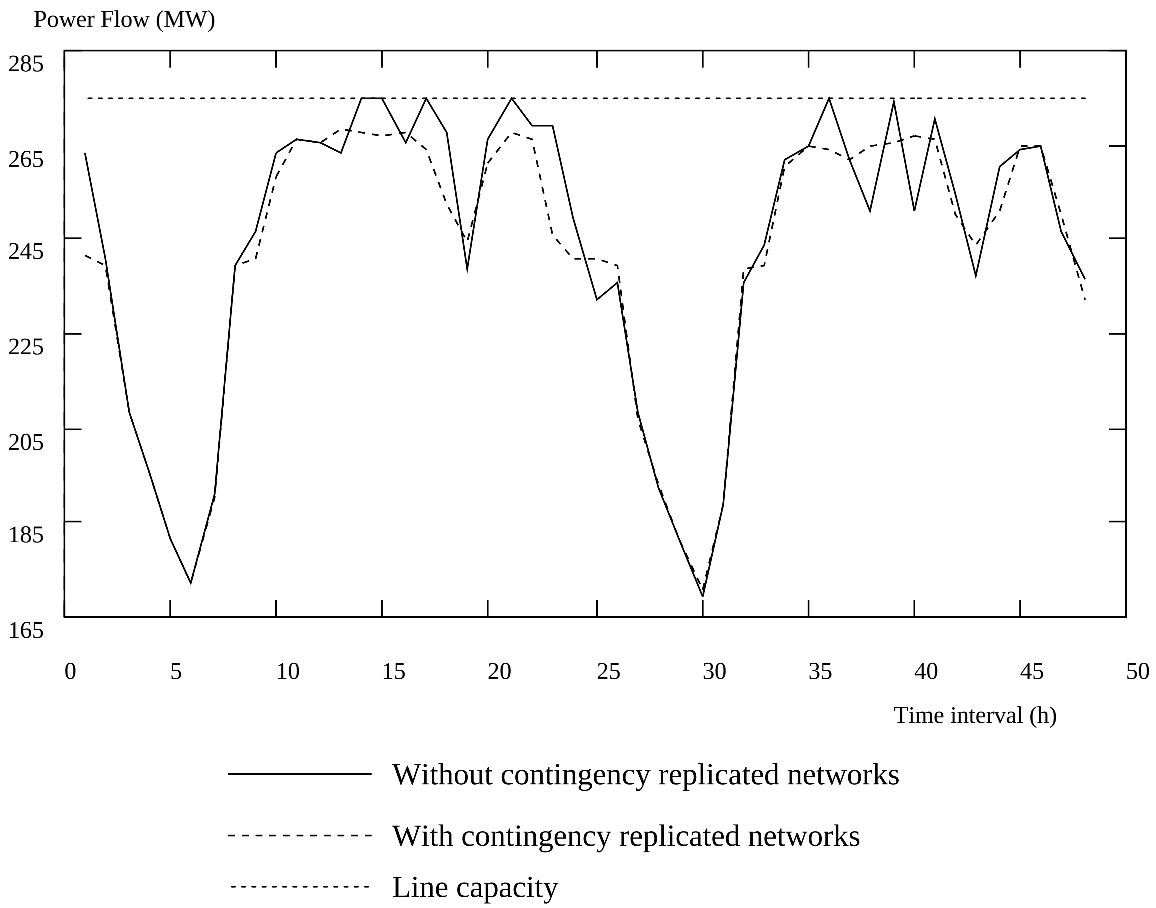


Fig. 5 Line flow values along intervals in a given line with or without contingencies considered.

CONCLUSIONS

- **Thermal equivalent network :**
  - * Network flow model for the thermal generation and spinning reserve.
- **Coupled model :**
  - * Optimization of generation costs and trans. losses.
  - * HTTOEN : Hydro, thermal and trans. with outages network.
  - * Coupling constraints : hydrogeneration arcs and spinning reserve.
  - * Muti–interval hydrothermal dc OPF with security and emission constraints.
- **Successive linearizations :**
  - * $\sim$ one order of magnitude faster.
  - * Deviation from the exact optimum : $< 0.14\%$.
  - * Hydrogeneration linearization error : $< 2\%$.