Medium-term generation planning in liberalized mixed electricity markets

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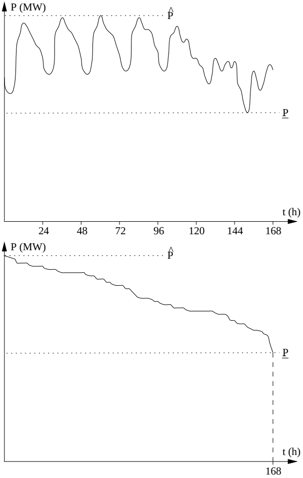
Market types

- Pure pool market: where all electricity is exchanged through a market operator.
- Pure bilateral market: where all electricity is traded directly between a given generation company and a given distribution company.
- Mixed market: where part of the load is traded as a bilateral contract and the rest is bid to the pool.

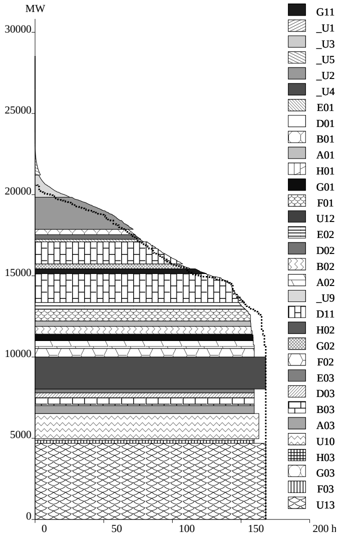
Load Duration Curve (LDC)



LDC is characterized by

- the total energy, $\hat{e}^i$
- the duration, t^i
- the base load power, $\underline{p}^i$
- the peak load power, $\widehat{p}^i$
- the shape, which is not a single parameter
- LDCs would be predicted for future subperiods

Generation Duration Curve (GDC)



The *generation-duration curve* is the expected production of the thermal units over the time interval to which the LDC refers. The energy generated by each unit is the slice of area under the generation-duration curve which corresponds to the capacity of the thermal unit. The area under the LDC and the area under the generation-duration curve must coincide.

The Bloom & Gallant Formulation

$$\underset{x_j}{\text{minimize}} \quad \sum_{j=0}^{n_u} \widetilde{f}_j x_j \tag{1a}$$

$$\text{subject to:} \quad \sum_{j \in \omega} x_j \leq \hat{e} - s(\omega) \qquad \forall\, \omega \subset \Omega \tag{1b}$$

$$Cx \geq d \tag{1c}$$

$$\sum_{j=0}^{n_u} x_j = \hat{e} \tag{1d}$$

$$x_j \geq 0 \qquad j = 0, 1, \ldots, n_u \tag{1e}$$

where x_j is the energy generated by unit j and Ω is the set of generation units of the pool, $\widetilde{f}$ is the linear generation cost and $j = 0$ is the *external energy*. Finally, $s(\omega) = T \int_0^{\widehat{p}} S_\omega(z)dz$

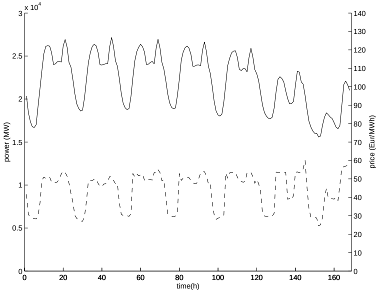
The Non-Load-Matching Constraints

- Maximum hydro generation: water availability is uncertain and the use of stored water is sometimes restricted by demands for irrigation.
- Bonus-scheme coal: some generation units which burn national coal receive a reduction on the coal cost according to a government act.
- Minimum generation time: generation units in the Spanish system are paid for their availability.
- SGC market-share: in medium-term planning it is reasonable to consider the SGC market-share.
- Other constraints

Medium-term profit maximization

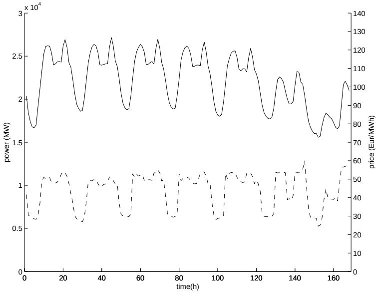
- In medium-term operation all accepted bids in a time interval (a week, or a month) must match the LDC of this interval for the whole market.
- As all generation companies pursue their maximum profit, it is natural to attempt to maximize the profit of all generation companies combined, which is the problem that will be first described. It is called the *generators' surplus (cartel)* maximization.
- The medium-term results will indicate how the Specific Generation Company (SGC) should program its units so that the cartel profits be maximized while meeting all constraints.

Medium-term market price function

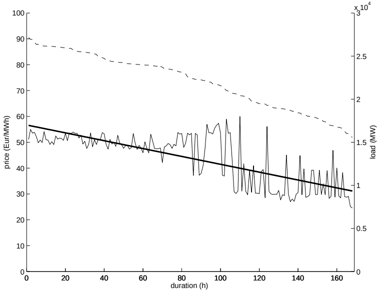


Hourly loads (continuous curve) and market prices (dashed) in a weekly interval

Medium-term market price function

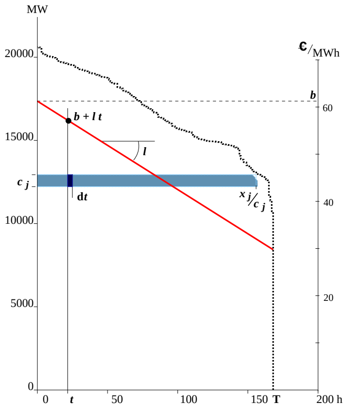


Hourly loads (continuous curve) and market prices (dashed) in a weekly interval



Market prices ordered by decreasing load power (thin continuous curve) in weekly interval,
market price linear function with respect to the load duration (thick line) and LDC (dashed)

Medium-term market price function



Medium-term price function
for a time interval and contribution of unit j

The profit (revenue minus cost) of unit j in interval i will be

$$\int_0^{\frac{x_j^i}{c_j}} c_j\{b^i + l^i t - f_j\}\, dt =$$

$$= (b^i - f_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2$$

Generators' surplus problem (*Cartel* model)

$$\underset{x_j^i}{\text{maximize}} \; \sum_{i=1}^{n_i} \left[\sum_{j\in\Omega} \left\{ \left(b^i - \widetilde{f}_j\right) x_j^i + \frac{l^i}{2c_j} \left(x_j^i\right)^2 \right\} - \widetilde{f}_0 x_0^i \right] \tag{2a}$$

$$\text{subject to:} \; \sum_{j\in\omega} x_j^i \le e^i - s^i(\omega) \quad \forall\, \omega \subseteq \Omega \quad \forall\, i \tag{2b}$$

$$\sum_{j\in\Omega} x_j^i + x_0^i = e^i \quad \forall\, i \tag{2c}$$

$$\sum_{i=1}^{n_i} C^i x^i \ge d \tag{2d}$$

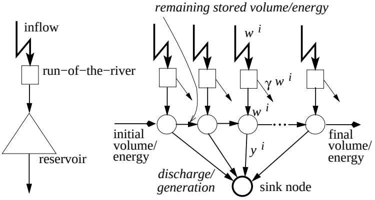
$$x_j^i \ge 0 \quad j \in \Omega \quad \forall\, i \tag{2e}$$

Constraints (2d) makes (2) non separable by periods

The representation of hydro generation

We denote with:

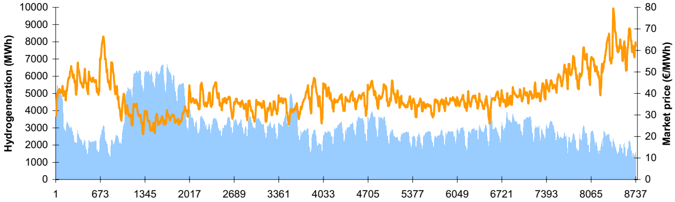
- y_h^i: the amount of water discharged
- w_h^i: the natural water inflows
- $\hat{e}^i$: the total energy
- v^i: the reservoir volume
- π^i: the market price function



Simplification of reservoir systems

$$\left.\begin{array}{l} \sum_{i=1}^{k} y_h^i \leq v_h^0 + \sum_{i=1}^{k} w_h^i \\ \sum_{i=1}^{n_i} y_h^i + v_h^f = v_h^0 + \sum_{i=1}^{n_i} w_h^i \\ x_h^i = y_h^i + \gamma w_h^i \end{array}\right\} \forall\, h \in H$$

Endogenous modification of the market price function



Weekly moving average of the market price (orange) and of hydro generation (blue area) during 2007 in the Spanish power pool

$$b^i = b_0^i - d_0^i \sum_{k \in H} x_k^i$$

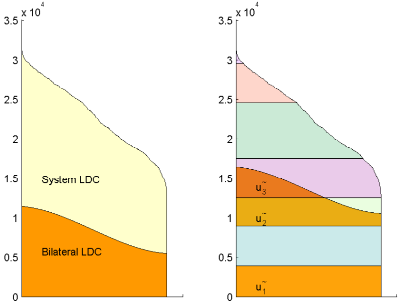
Therefore the new objective function will be

$$\sum_i^{n_i} \Big[\sum_j^{n_u} \{ (b_0^i - \tilde{f}_j) x_j^i - d_0^i \sum_{k \in H} x_k^i x_j^i + \frac{l^i}{2 c_j} {x_j^i}^2 \} - \tilde{f}_0 x_0^i \Big]$$

Bilateral Contracts: Definition and principles

- A Bilateral Contract (BC) is an agreement between two parties, normally a SGC and a big consumer or distributor, to supply or exchange electric power under a set of specified conditions such as power, energy amount, time of delivery, duration and price.
- One of the most extended electricity market types is mixed market with pool auction and BCs. In it, generation companies may have BCs to supply energy in given amounts and instants, and they bid the remaining available generation capacity to the pool Market Operator to get extra benefits.
- It will be assumed that information of total system load and of total market load are available. Subtracting the market load from the system load we get the load supplied through BCs.

The time-share hypothesis in medium term power planning with BCs



LDC of the system and part corresponding to the bilateral contracts LDC (shaded part, left), optimal load-matching with production for bilateral contracts (right)

We assume that

- units are generating at full capacity
- the contribution of a unit has rectangular shape with height equal to its capacity

x is the total expected generation and $\widetilde{x}$ is the energy produced for bilateral contracts.

Profit maximization with the endogenous function and bilateral contracts

The endogenous price function for the hydro generation traded in the market is

$$\pi^i(t, g_h) = b_0^i + l^i t + d \sum_{h \in H} (x_h^i - \widetilde{x}_h^i)\,.$$

Subtracting the generation cost from the revenue we obtain the generation unit profit:

$$\begin{aligned} r_j^i(x_j^i, \widetilde{x}_j^i) = &\ c_j \int_{\frac{\widetilde{x}_j^i}{c_j}}^{\frac{x_j^i}{c_j}} (\pi^i(t, g_h) - f_j)\, dt = \\ = &\ b_0^i(x_j^i - \widetilde{x}_j^i) + d \sum_{h \in H} (x_h^i - \widetilde{x}_h^i)(x_j^i - \widetilde{x}_j^i) \\ &+ \frac{1}{2} \frac{l^i}{c_j} (x_j^{i\,2} - \widetilde{x}_j^{i\,2}) - f_j x_j^i \end{aligned}$$

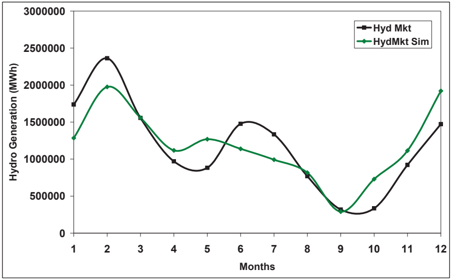
This function r_j^i is indefinite (and it could be decomposed as a difference of two concave functions.)

Hydro-to-Market Constraint

A Hydro-to-Market constraint must be employed, otherwise hydro generation is used for BCs

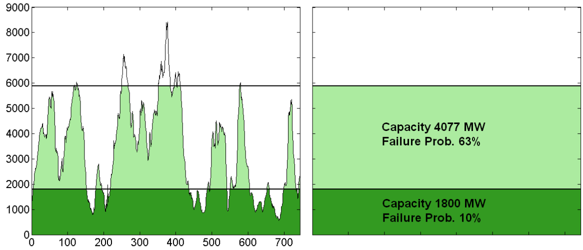
$$\sum_{h \in H} (x_h^i - \tilde{x}_h^i) \geq \alpha w_h^i + \beta \hat{e}^i + \gamma v^i + \delta \overline{\pi}^i \quad \forall\, i \in 1..n$$

where w_h^i is the water inflows for the subperiod i, $\hat{e}^i$ is the total energy, v^i is the reservoir volume and $\overline{\pi}^i$ is the average market price.



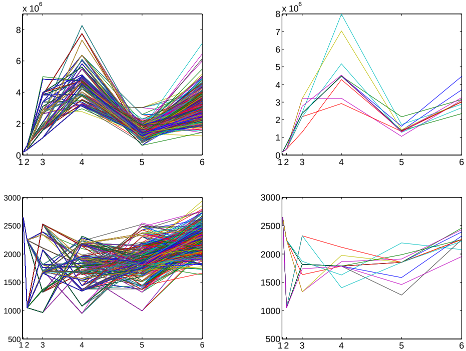
The representation of wind-power generation

- From the wind-power series corresponding to a given time period we deduce the two-unit model that represents its wind-power generation.
- Two pseudounits: the base unit and the crest unit.
- In the scenario generation the scenario tree nodes are based on base unit capacity (fixed failure 10%).



Scenario Generation and Scenario Reduction

- The scenario tree is created using a mixture of multidimensional vector auto regressive model and Montecarlo methods.
- We reduce the scenario tree to the desired number of scenarios using a backward algorithm



The medium-term power planning in a liberalized market with BCs and stochasticity

$$\underset{x_j^\nu, \widetilde{x}_j^\nu}{\text{maximize}} \sum_{\nu \in \mathcal{N}} \pi_\nu \left\{ \sum_{j \in \widetilde{\Omega}} r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu) + \sum_{j \in \Omega \setminus \widetilde{\Omega}} r_j^\nu(x_j^\nu, 0) - f_0 x_0^\nu \right\} \quad (3a)$$

$$\text{subject to: } \widetilde{x}_j^\nu \le x_j^\nu \quad \forall j \in \Omega \quad \forall \nu \in \mathcal{N} \quad (3b)$$

$$\sum_{j \in \widetilde{\omega}} \widetilde{x}_j^\nu \le \widetilde{e}^\nu - s^\nu(\widetilde{\omega}) \quad \forall \widetilde{\omega} \subseteq \widetilde{\Omega} \quad \forall \nu \in \mathcal{N} \quad (3c)$$

$$\sum_{j \in \omega} x_j^\nu \le e^\nu - s^\nu(\omega) \quad \forall \omega \subseteq \Omega \quad \forall \nu \in \mathcal{N} \quad (3d)$$

$$\sum_{j \in \widetilde{\Omega}} \widetilde{x}_j^\nu = \widetilde{e}^\nu - s^\nu(\Omega) \quad \forall \nu \in \mathcal{N} \quad (3e)$$

$$\sum_{j \in \Omega} x_j^\nu + x_0^\nu = e^\nu \quad \forall \nu \in \mathcal{N} \quad (3f)$$

$$\sum_{\nu \in \mathcal{H}(\lambda)} C^{\lambda, i(\nu)} x^\nu \ge d^\lambda \quad \forall \lambda \in \mathcal{L} \quad (3g)$$

$$0 \le \widetilde{x}_j^\nu \quad j \in \widetilde{\Omega} \quad \forall \nu \in \mathcal{N} \quad (3h)$$

$$0 \le x_j^\nu \quad j \in \Omega \quad \forall \nu \in \mathcal{N} \quad (3i)$$

where ν are the nodes, $\mathcal{N}$ is the set of nodes, λ are the leafs of the scenario tree, $\mathcal{L}$ is the set of leafs and $\mathcal{H}(\lambda)$ is the path leading to the leaf λ.

Nash-Cournot equilibrium in electricity markets

- In a Nash-Cournot equilibrium we can assume either two (the SGC and the RoP), or more players (K generation companies, whose units are $\Omega_k | \Omega := \{\Omega_1, \Omega_2, ..., \Omega_K\}$).
- In the Cournot model of competition we assume that the decision (generation) of one player is conditioned by the decisions (generations) of the rest of the players and that the market price is a function of the overall decisions (total expected generation).
- In a Nash equilibrium no player can increase its revenue by unilaterally changing its decision (generation).
- We can find an equilibrium point because the endogenous revenue function relates profits to the hydro generation of each player.

Nash-Cournot equilibrium in electricity markets

For the medium-term planning to have *Cournot competition* (and a equilibrium solution) it is necessary to consider that the players mutually condition each other generations. This is so in case we consider the *endogenous model* explained, where the hydro generation of each player influences the market price.

$$\phi_k(x_k|\hat{\mathbf{x}}) = \sum_i \sum_{j\in\Omega_k^i} \left\{ (f_j - b^i)x_j^i - \frac{l^i}{2c_j}(x_j^i)^2 + \right.$$

$$\left. + d \left[\sum_{l\in H_k} x_l^i x_j^i + \sum_{l\in H_{m|m\neq k}} \hat{x}_l^i x_j^i \right] \right\}$$

The NIRA algorithm to obtain the Nash-Cournot equilibrium

The Nikaido-Isoda relaxation algorithm (NIRA) is an optimization-based procedure to obtain a Nash-Cournot equilibrium point.

- The Nikaido-Isoda function is

$$\Psi(\hat{\mathbf{x}}, \mathbf{x}) := \sum_{k=1}^{K} (\phi_k(x_k|\hat{\mathbf{x}}) - \phi_k(\hat{\mathbf{x}}))$$

- An equivalent formulation of a Nash equilibrium point is that $\mathbf{x}^*$ is an equilibrium point if

$$\max_{x \in \mathcal{X}} \Psi(\hat{\mathbf{x}}, \mathbf{x}) = 0$$

- We define the optimal response function Z as

$$Z(\hat{\mathbf{x}}) := arg \max_{x \in \mathcal{X}} \Psi(\hat{\mathbf{x}}, \mathbf{x})$$

- The NIRA algorithm updating rule is

$$x^{new} \leftarrow (1-u)\hat{\mathbf{x}} + uZ(\hat{\mathbf{x}}) \; u \in \mathbb{R}, \, 0 < u < 1$$

The implementation of the NIRA algorithm to obtain the Nash-Cournot equilibrium

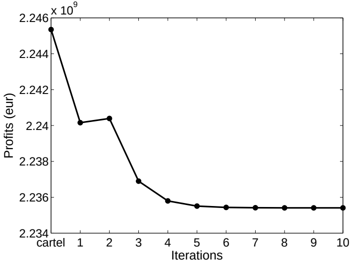
Note that, given $\hat{\mathbf{x}}$, $\phi_k(\hat{\mathbf{x}})$ is a constant, and that $max_{\mathbf{x}\in\mathcal{X}}\Psi(\hat{\mathbf{x}},\mathbf{x})$ is equivalent to solving

$$\begin{aligned}
\underset{x_j^i}{\text{maximize}} \quad & \sum_{i=1}^{n_i}\sum_{k=1}^{K}\sum_{j\in\Omega_k^i}\Bigg\{(b^i - f_j)x_j^i + \frac{l^i}{2c_j}(x_j^i)^2 + \\
& - d\Big[\sum_{l\in H_k} x_l^i x_j^i - \sum_{l\in H_{m|\, m\neq k}} \hat{x}_l^i x_j^i\Big]\Bigg\} \\
\text{subject to:} \quad & \sum_{j\in\omega} x_j^i \le e^i - s^i(\omega) \quad \forall\,\omega\subseteq\Omega \quad \forall\, i \\
& \sum_{j\in\Omega} x_j^i + x_0^i = e^i \quad \forall\, i \\
& \sum_{i=1}^{n_i} C^i x^i \ge d \\
& x_j^i \ge 0 \quad j\in\Omega \quad \forall\, i
\end{aligned}$$

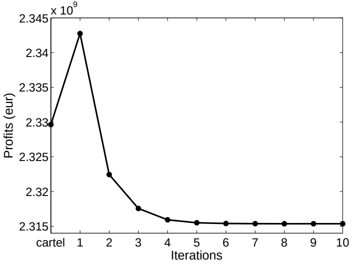
Case study characteristics

- Real data from the Spanish Market
- First case: 9+2 aggregated generation units (2 hydro, 2 coal, 4 fuel/gas, 1 nuclear, 2 wind-power pseudounits)
- Second case: 13+2 aggregated generation units (4 hydro, 4 coal, 4 fuel/gas, 1 nuclear, 2 wind-power pseudounits)
- Model implemented and solved with AMPL/IPOPT

Evolution of the Objective Function

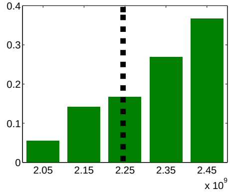


Deterministic case

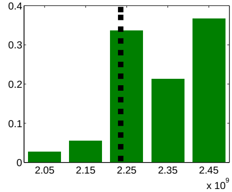


12 scenarios case

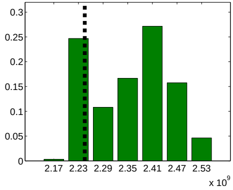
Probability distributions of scenario profit value for cartel and equilibrium solution



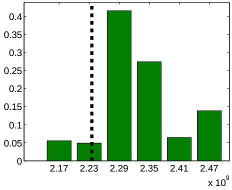
12 scenarios cartel case



12 scenarios case with equilibrium



40 scenarios cartel case



40 scenarios case with equilibrium

Conclusions

- A new model for a mixed market using a time-share hypothesis has been presented.
- The resulting problem has a non convex objective function.
- A Hydro-to-Market constraint is necessary.
- We found both the solution for the Cartel behaviour and Equilibrium behaviour using the Nikaido Isoda Relaxation Algorithm.
- The Equilibrium solution has profits lower than the Cartel solution, as expected.
- In the model presented, if not for the endogenous function due to hydro generation, we would not get an equilibrium solution.
- A new way to represent the wind-power generation with two pseudounits with given capacity and failure probability in each node of the scenario tree has been presented.

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