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Stochastic Programming Model for the Day-Ahead Bidding and Bilateral Contracts

Settlement Problem

F. Javier Heredia Marcos J. Rider Cristina Corchero

Group on Numerical Optimization and Modeling
Departament d'Estadística i Investigació Operativa
Universitat Politècnica de Catalunya
International Workshop on Operational Research 08
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Work partially supported by the Ministerio de Educación y Ciencia of Spain under Project
DPI2005-09117-C02-01

# Iberic Electrical Energy Market (MIBEL)

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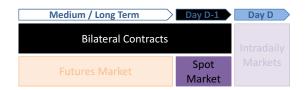
The Problem

VPP auctions
VPP and GPU

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Conclusions



### **Derivatives Market**

# Physical Futures Contracts Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery. Financial Futures Contracts

OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

### **Bilateral Contracts**

Organized markets

- Virtual Power Plants auctions (BPE)
- Distribution auctions (SD)
- International Capacity Interconnection auctions
- International Capacity Interconnection nomination

Non organized markets
- National BC before the spot market
- International BC before the spot market

National BC after the spot market

### Day-Ahead Market

Day-Ahead Market Hourly action. The matching procedure takes place 24h before the delivery period. Physical futures contracts are settled through a zero price bid.

# Iberic Electrical Energy Market (MIBEL)

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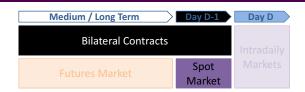
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### Derivatives Market

# Physical Futures Contracts Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery. Financial Futures Contracts OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

### Bilateral Contracts



### Day-Ahead Market

Day-Ahead Market
Hourly action. The matching procedure takes place
24h before the delivery period.
Physical futures contracts are settled through a zero

Physical futures contracts are settled through a ze price bid.

- This work is focused on:
  - Day-ahead market.
  - Virtual Power Plant Auctions (EPE)
  - National BC before and after the day-ahead market.

# Virtual Power Plant auctions (VPP)

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Conclusion

• The Royal Decree 1634/2006 imposes to Endesa and Iberdrola to hold a series of five **Virtual Power Plant** (VPP) auctions (EPE, starting July 2007) offering virtual power capacity at price  $\lambda^{V}$  to any party who is a member of the MIBEL.

# Virtual Power Plant auctions (VPP)

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The Problem

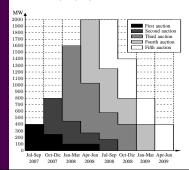
VPP auctions
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# Virtual Power Plant auctions (VPP)

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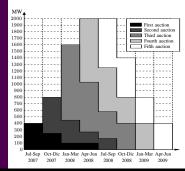
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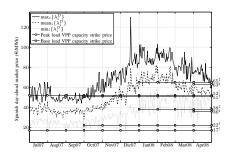
The Model

The Solution

Conclusions

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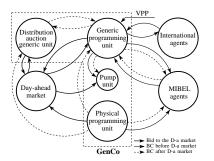
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Conclusion



 The VPP capacity means that the buyer have up to \(\overline{p}^V\) MWh at his disposal.

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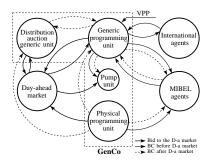
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- The VPP capacity means that the buyer have up to \(\overline{p}^V\) MWh at his disposal.
- The buyer can exercise the right to use energy p̄<sup>ν</sup> against an exercise price λ<sup>ν</sup> ∈/MWh.

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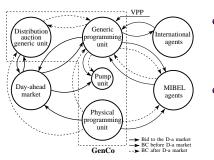
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Conclusion



- The VPP capacity means that the buyer have up to \(\overline{p}^{\nu}\) MWh at his disposal.
- The buyer can exercise the right to use energy p̄<sup>V</sup> against an exercise price λ<sup>V</sup> €/MWh.
- Energy p̄<sup>V</sup> of the VPP is incorporated to the market through the Generic Programming Unit (GPU) and can be used to cover the national and international bilateral contracts duties and/or to sell it to the day-ahead market.

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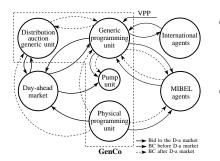
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Conclusion



- The VPP capacity means that the buyer have up to p̄<sup>V</sup>
   MWh at his disposal.
- The buyer can exercise the right to use energy p̄<sup>ν</sup> against an exercise price λ<sup>ν</sup> €/MWh.
- Energy \(\overline{\rho}^V\) of the VPP is incorporated to the market through the **Generic Programming Unit** (GPU) and can be used to cover the national and international bilateral contracts duties and/or to sell it to the day-ahead market.
- GPU can buy/purchase energy from the pool and B.C.

# Objectives of the study

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Conclusions

- The ojective of this study was to develop an stochastic programming model that allows a price-taker producer to decide
  - The economic dispatch of the bilateral contracts among the thermal and generic programming units.
  - The optimal bidding for both thermal and generic programming units, observing the MIBEL regulation.
  - The unit commitment of its thermal units;

# Objectives of the study

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  - The economic dispatch of the bilateral contracts among the thermal and generic programming units.
  - The optimal bidding for both thermal and generic programming units, observing the MIBEL regulation.
  - The unit commitment of its thermal units;

that maximizes the expected profit from its involvement in the spot market, bilateral contracts and virtual power plant capacity.

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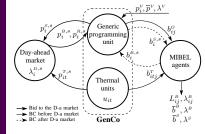
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Price-taker GenCo.



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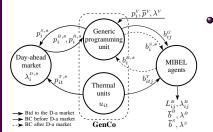
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- Price-taker GenCo.
- T thermal units: convex gen. costs; start-up/shut-downs costs; min up/down time.

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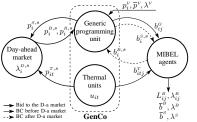
System constraints

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- Price-taker GenCo.
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- GPU associated to a VPP:
   p̄<sup>ν</sup> MWh, λ<sup>ν</sup> €/MWh.

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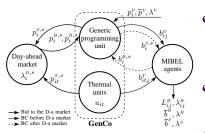
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- Price-taker GenCo.
- T thermal units: convex gen. costs; start-up/shut-downs costs; min up/down time.
- GPU associated to a VPP:  $\overline{p}^{v}$  MWh,  $\lambda^{v} \in /MWh$ .
- Both thermal units and GPU bid to the the day-ahead market.

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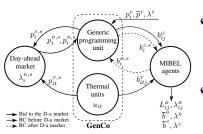
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Conclusions



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- T thermal units: convex gen. costs; start-up/shut-downs costs; min up/down time.
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- Both thermal units and GPU bid to the the day-ahead market.
- $\mathcal{B}$  bilateral contracts before the day-ahead market:  $L^{\mathcal{B}}_{ii}$  MWh,  $\lambda^{\mathcal{B}}_{ii} \in /$ MWh  $\forall j \in \mathcal{B}$

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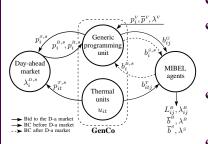
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- Price-taker GenCo.
- T thermal units: convex gen. costs; start-up/shut-downs costs; min up/down time.
- GPU associated to a VPP:
   p̄<sup>ν</sup> MWh, λ<sup>ν</sup> €/MWh.
- Both thermal units and GPU bid to the the day-ahead market.
- $\mathcal{B}$  bilateral contracts before the day-ahead market:  $L^{\mathcal{B}}_{ii}$  MWh,  $\lambda^{\mathcal{B}}_{ii} \in /$ MWh  $\forall j \in \mathcal{B}$
- Purchase/sell bilateral contracts after the day-ahead market:  $(\overline{b}^{B} \text{ MWh}, \lambda^{B} \in /\text{MWh}), (\overline{b}^{S} \text{ MWh}, \lambda^{S} \in /\text{MWh})$

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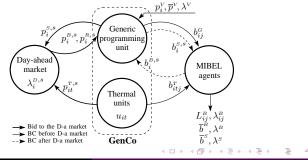
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Conclusions

## First stage variables: $\forall t \in \mathcal{T}, \ \forall i \in \mathcal{I}$

•  $u_{it}$ : unit commitment.



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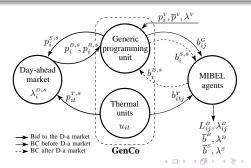
The Solution

Conclusion

### First stage variables: $\forall t \in \mathcal{T}, \ \forall i \in \mathcal{I}$

- $u_{it}$ : unit commitment.
- $\bullet$   $b_{itj}^{\mathsf{T}}$ ,  $b_{ij}^{\mathsf{G}}$ : energy allocated to

B.C.  $j \in \mathcal{B}$ 



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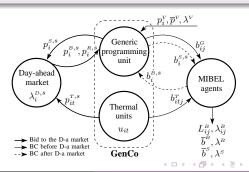
The Solution

Conclusion

### First stage variables: $\forall t \in \mathcal{T}, \ \forall i \in \mathcal{I}$

•  $u_{it}$ : unit commitment.

- $p_i^V$ : exercised VPP energy.
- $b_{itj}^{\mathsf{T}},\ b_{ij}^{\mathsf{G}}$ : energy allocated to B.C.  $j \in \mathcal{B}$



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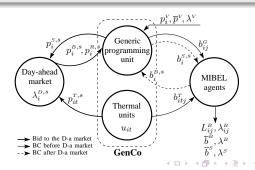
The Solution

Conclusion

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- $b_{itj}^{\mathsf{T}},\ b_{ij}^{\mathsf{G}}$ : energy allocated to B.C.  $j \in \mathcal{B}$
- Selling/purchase bidding.



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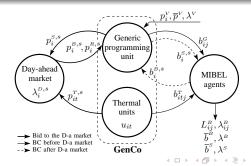
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### First stage variables: $\forall t \in \mathcal{T}, \ \forall i \in \mathcal{I}$

- $u_{it}$ : unit commitment.
- $b_{itj}^{\mathsf{T}}$ ,  $b_{ij}^{\mathsf{G}}$ : energy allocated to B.C.  $i \in \mathcal{B}$
- $p_i^V$ : exercised VPP energy.
- Selling/purchase bidding.
- Many other aux. variables.



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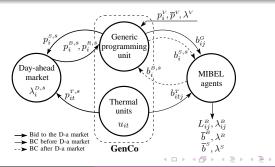
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Conclusions

### Second stage variables: $\forall t \in \mathcal{T}, \ \forall i \in \mathcal{I}, \ \forall s \in \mathcal{S}$

•  $p_{it}^{T,s}$ : Th.U.'s matched energy.



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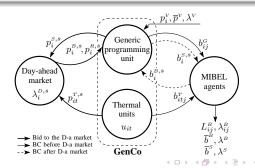
Final mode

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Conclusions

### Second stage variables: $\forall t \in \mathcal{T}, \ \forall i \in \mathcal{I}, \ \forall s \in \mathcal{S}$

- $p_{it}^{T,s}$ : Th.U.'s matched energy.
- $p_i^{\mathcal{B},s}$ ,  $p_i^{\mathcal{S},s}$ ,  $p_i^{\mathcal{R},s}$ : GPU's bought/sold matched energy.



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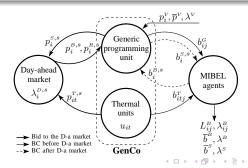
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The Solution

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- $p_{it}^{T,s}$ : Th.U.'s matched energy.
- $p_i^{B,s}$ ,  $p_i^{S,s}$ ,  $p_i^{R,s}$ : GPU's

- $b_i^{S,s}$ ,  $b_i^{B,s}$ : selling/buying B C after the D-A-M
- bought/sold matched energy.



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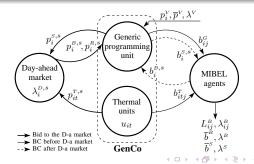
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Conclusions

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- $p_i^{B,s}$ ,  $p_i^{S,s}$ ,  $p_i^{R,s}$ : GPU's bought/sold matched energy.
- $b_i^{s,s}$ ,  $b_i^{g,s}$ : selling/buying B.C. after the D-A-M.
- Many other aux. variables.



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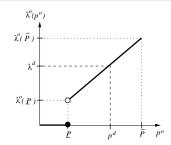
The Solution

Conclusion

### Matched energy at scenario s:

$$p_{it}^{T,s}(b_{it}^{T}) = \begin{cases} p_{it}^{D,s} - b_{it}^{T} & \text{if } b_{it}^{T} \leq p_{it}^{D,s} & \forall i \in \mathcal{I} \\ 0 & \text{if } b_{it}^{T} > p_{it}^{D,s} & \forall s \in \mathcal{S} \end{cases}$$
(1)

Case a:  $b_{it}^T = 0$ 



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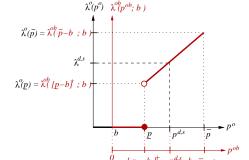
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(1)



Case b:  $0 < b_{i+}^{T} \le p_{i+}^{D,s}$ 

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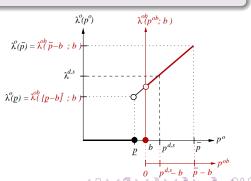
Conclusion

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$$p_{it}^{T,s}(b_{it}^{T}) = \begin{cases} p_{it}^{D,s} - b_{it}^{T} & \text{if } b_{it}^{T} \leq p_{it}^{D,s} & \forall i \in \mathcal{I} \\ 0 & \text{if } b_{it}^{T} > p_{it}^{D,s} & \forall s \in \mathcal{S} \end{cases}$$
(1)

Case b:

$$0 < b_{it}^{\mathsf{T}} \leq p_{it}^{\mathsf{D},s}$$



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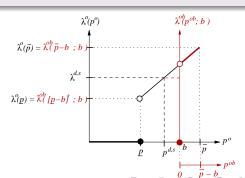
Conclusio

### Matched energy at scenario s:

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(1)

Case c:

$$p_{it}^{\scriptscriptstyle D,s} < b_{it}^{\scriptscriptstyle T}$$



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### Matched energy at scenario s:

$$p_{it}^{T,s}(b_{it}^{T}) = \begin{cases} p_{it}^{D,s} - b_{it}^{T} & \text{if } b_{it}^{T} \leq p_{it}^{D,s} & \forall i \in \mathcal{I} \\ 0 & \text{if } b_{it}^{T} > p_{it}^{D,s} & \forall s \in \mathcal{S} \end{cases}$$
(1)

• The non-differentiable expression (1) can be formulated as a set of linear constraints, with auxiliary variables  $v_{it}^s \geq 0$  and  $z_{it}^s \in \{0,1\}$ .

$$\left. \begin{array}{l} p_{it}^{s} = p_{it}^{\tau,s} + b_{it}^{\tau} \\ p_{it}^{\tau,s}, b_{it}^{\tau} \in \Omega_{it}^{\tau,s}(u_{it}, v_{it}^{s}, z_{it}^{s}) \end{array} \right\} \forall t \in \mathcal{T}, \ \forall s \in \mathcal{S} \ \ (2)$$

• Unit commitment constraints:

$$u_{it} \in \Omega_{it}^{U}(a_{it}, e_{it}) \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}$$

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 Under the price-taker assumption and the MIBEL rules, the optimal selling (OSB) and buying (OBB) biddings for the GPU are:

$$\begin{aligned}
\mathsf{OSB}_{i}(b_{i}^{\mathsf{G}}, p_{i}^{\mathsf{V}}) &= ([p_{i}^{\mathsf{V}} - b_{i}^{\mathsf{G}}]^{+}, \lambda^{\mathsf{S}}) \\
\mathsf{OBB}_{i}(b_{i}^{\mathsf{G}}, p_{i}^{\mathsf{V}}) &= ([b_{i}^{\mathsf{G}} - \overline{p}^{\mathsf{V}}]^{+} + \min\{b_{i}^{\mathsf{G}}, \overline{p}^{\mathsf{V}} - p_{i}^{\mathsf{V}}\}, \lambda^{\mathsf{B}})
\end{aligned}$$

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### GPU model

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$$\begin{aligned}
\mathsf{OSB}_{i}(b_{i}^{\mathsf{G}}, p_{i}^{\mathsf{V}}) &= ([p_{i}^{\mathsf{V}} - b_{i}^{\mathsf{G}}]^{+}, \lambda^{\mathsf{S}}) \\
\mathsf{OBB}_{i}(b_{i}^{\mathsf{G}}, p_{i}^{\mathsf{V}}) &= ([b_{i}^{\mathsf{G}} - \overline{p}^{\mathsf{V}}]^{+} + \min\{b_{i}^{\mathsf{G}}, \overline{p}^{\mathsf{V}} - p_{i}^{\mathsf{V}}\}, \lambda^{\mathsf{B}})
\end{aligned}$$

### with

- $\lambda^s$  and  $\lambda^B$  the price of the selling and buying B.C. after the d-a-m.
- $p_i^V$  the exercised energy of the VPP
- $b_i^c$ : the contribution to the B.C. before the d-a-m.

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$$\begin{array}{l} \bullet \ \ \text{The GPU's } \textit{matched sold } (p_i^{\mathcal{S},s}) \text{ and } \textit{matched bought} \\ (p_i^{\mathcal{B},s} + p_i^{\mathcal{R},s}) \text{ energy at each scenario } s \in \mathcal{S} \text{ are:} \\ p_i^{\mathcal{S},s} (b_i^{\mathcal{G}}, p_i^{\mathcal{V}}) = \left\{ \begin{array}{ll} [p_i^{\mathcal{V}} - b_i^{\mathcal{G}}]^+ & \text{if } s \in \mathcal{M}_i^{\mathcal{S}} \\ 0 & \text{if } s \not \in \mathcal{M}_i^{\mathcal{S}} \\ p_i^{\mathcal{B},s} (b_i^{\mathcal{G}}, p_i^{\mathcal{V}}) = \left\{ \begin{array}{ll} \min\{b_i^{\mathcal{G}}, \overline{p}^{\mathcal{V}} - p_i^{\mathcal{V}}\} & \text{if } s \in \mathcal{M}_i^{\mathcal{B}} \\ 0 & \text{if } s \not \in \mathcal{M}_i^{\mathcal{B}} \\ p_i^{\mathcal{R},s} (b_i^{\mathcal{G}}) = \left\{ \begin{array}{ll} [b_i^{\mathcal{G}} - \overline{p}^{\mathcal{V}}]^+ & \text{if } s \in \mathcal{M}_i^{\mathcal{B}} \\ 0 & \text{if } s \not \in \mathcal{M}_i^{\mathcal{B}} \\ \end{array} \right. \end{array} \right. \ \, \forall s \in \mathcal{S}$$

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• The GPU's matched sold  $(p_i^{s,s})$  and matched bought  $(p_i^{g,s} + p_i^{g,s})$  energy at each scenario  $s \in \mathcal{S}$  are:

$$\begin{aligned} p_i^{S,s}(b_i^G, p_i^V) &= \begin{cases} [p_i^V - b_i^G]^+ & \text{if } s \in \mathcal{M}_i^S \\ 0 & \text{if } s \notin \mathcal{M}_i^S \end{cases} \\ p_i^{B,s}(b_i^G, p_i^V) &= \begin{cases} \min\{b_i^G, \overline{p}^V - p_i^V\} & \text{if } s \in \mathcal{M}_i^B \\ 0 & \text{if } s \notin \mathcal{M}_i^B \end{cases} & \forall i \in \mathcal{I} \\ p_i^{B,s}(b_i^G) &= \begin{cases} [b_i^G - \overline{p}^V]^+ & \text{if } s \in \mathcal{M}_i^B \\ 0 & \text{if } s \notin \mathcal{M}_i^B \end{cases} & \forall s \in \mathcal{S} \end{aligned}$$

• These non-differentiable expressions can be formulated through an equivalent set of linear constraints with auxiliary variables  $w_i^{G,s} \ge 0$  and  $y_i^{G,s} \in \{0,1\}$ .

$$p_i^{s,s}, p_i^{s,s}, p_i^{R,s} \in \Omega_i^{c,s}(w_i^{c,s}, y_i^{c,s}) \quad \forall i \in \mathcal{I} , \ \forall s \in \mathcal{S}$$
 (4)

## Modellization: system constraints

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• The energy  $L_{ij}^{B}$  of the *j*-th B.C. before the d-a-m can be provided both by the thermal units  $\mathcal{T}$  and the GPU:

$$\sum_{t \in \mathcal{T}} b_{itj}^{\mathsf{T}} + b_{ij}^{\mathsf{G}} = L_{ij}^{\mathsf{B}} \quad \forall j \in \mathcal{B} , \forall i \in \mathcal{I} 
b_{it}^{\mathsf{T}} = \sum_{j \in \mathcal{B}} b_{itj}^{\mathsf{T}} \quad \forall t \in \mathcal{T} 
b_{i}^{\mathsf{G}} = \sum_{j \in \mathcal{B}} b_{ij}^{\mathsf{G}}$$

$$\begin{cases}
\forall i \in \mathcal{I} \\
\forall i \in \mathcal{I}
\end{cases} (5)$$

## Modellization: system constraints

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Conclusions

• The energy  $L_{ij}^{B}$  of the j-th B.C. before the d-a-m can be provided both by the thermal units  $\mathcal{T}$  and the GPU:

$$\sum_{t \in \mathcal{T}} b_{itj}^{\mathcal{T}} + b_{ij}^{\mathcal{G}} = L_{ij}^{\mathcal{B}} \quad \forall j \in \mathcal{B}, \ \forall i \in \mathcal{I} \\
b_{it}^{\mathcal{T}} = \sum_{j \in \mathcal{B}} b_{itj}^{\mathcal{T}} \quad \forall t \in \mathcal{T} \\
b_{i}^{\mathcal{G}} = \sum_{j \in \mathcal{B}} b_{ij}^{\mathcal{G}}$$

$$\begin{cases}
\forall i \in \mathcal{I} \\
\forall i \in \mathcal{I}
\end{cases} (5)$$

• At each hour  $i \in \mathcal{I}$  the net energy balance of the GPU must be zero (PDBF=0 constraint).

$$p_i^V + p_i^{B,s} + p_i^{R,s} + b_i^{B,s} = p_i^{S,s} + b_i^{S,s} + b_i^G$$

$$\forall s \in \mathcal{S} , \forall i \in \mathcal{I}$$
 (6)

## The spot price

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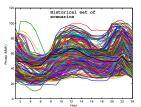
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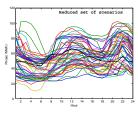
System constraints Stochasticity Final model

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Conclusio

- Spot market price,  $\lambda_i$  is a random variable that has to be represented through a set of scenarios
- Price scenario construction:
  - Set of 261 historical daily scenarios, from the start-up of the MIBEL (July 1, 2007) to the day in study (May 8, 2008).
  - Reduction of the number of scenarios preserving at maximum the characteristics of the observed data <sup>1</sup>





Gröwe-Kuska et al. Scenario Reduction and Scenario Tree Construction for Power Management Problems

#### Modellization: final model

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• The final model is a two-stage mixed quadratic stochastic programming problem:

máx 
$$B_{\lambda^D}(u, a, e, p^T, p^V, p^S, p^B, p^R, b^S, b^R)$$
  
s.t.:

- Eq. (2) Thermal's matched energy  $p_{it}^{T,s}$  Eq. (3) Unit commitment const.
- Eq. (4) GPU's matched energy  $p_i^{s,s}$ ,  $p_i^{g,s}$ ,  $p_i^{g,s}$
- Eq. (5) Bilateral contracts  $\mathcal{B}$  covering
- Eq. (6) GPU's net energy balance const.

#### Modellization: final model

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 The final model is a two-stage mixed quadratic stochastic programming problem:

$$\left(\begin{array}{l} \text{máx } B_{\lambda^D}(u, a, e, p^T, p^V, p^S, p^B, p^R, b^S, b^R) \\ \text{s.t.} : \end{array}\right)$$

- Eq. (2) Thermal's matched energy  $p_{it}^{\tau,s}$
- Eq. (3) Unit commitment const.
- Eq. (4) GPU's matched energy  $p_i^{S,s}$ ,  $p_i^{B,s}$ ,  $p_i^{R,s}$
- Eq. (5) Bilateral contracts  ${\cal B}$  covering
- Eq. (6) GPU's net energy balance const.

where  $B_{\lambda^D}$  represents the expected profit w.r.t the spot prices from the GenCo's involvement in the spot market, bilateral contracts and virtual power plan capacity.

## Case study

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onclusion

- The model was solved with real data of a Spanish generation company and market prices.
  - 50 Day-ahead market price scenarios;
  - 24 hours of study;
  - 10 thermal units;
  - 2 bilateral contracts;

## Case study

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Case study Results

Results Analysis

- The model was solved with real data of a Spanish generation company and market prices.
  - 50 Day-ahead market price scenarios;
  - 24 hours of study;
  - 10 thermal units;
  - 2 bilateral contracts;
- The model was tested for three different cases:
  - (a) A GenCo without GPU;
  - (b) A GenCo with GPU but without VPP capacity; and
  - (c) A GenCo with GPU and VPP capacity.

### Case study: results

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• The mathematical characteristics of the model.

Case	Constraints	Real variables	Binary variables	CPU time <sup>(1)</sup>
(a)	79921	31417	12720	142s
(b)	86026	35086	12792	108s
(c)	89758	37525	12816	1500s

(1): AMPL/CPLEX11 (default options)

2\*CPU AMD Opteron 2222 (3 GHz) dual core 32GB RAM

#### Case study: results

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• The mathematical characteristics of the model.

	Case	Constraints	Real variables	Binary variables	CPU time <sup>(1)</sup>
	(a)	79921	31417	12720	142s
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	(c)	89758	37525	12816	1500s
-					

(1): AMPL/CPLEX11 (default options) 2\*CPU AMD Opteron 2222 (3 GHz) dual core 32GB RAM

• The expected profit values for all study cases

Case (a)	Case (b)	Case (c)
609.150,08€	664.349,62€	898.642,41€
	(GPU)	(GPU+VPP)

## Case study: analysis of the solution

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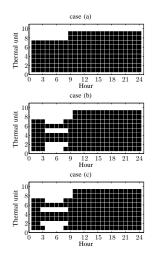
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Unit commitment of the thermal units for all study cases.



## Case study

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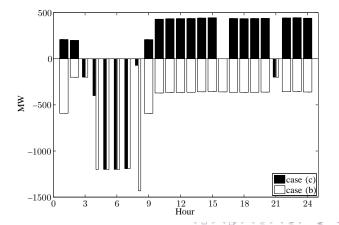
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 Optimal selling and buying biddings (OSB<sub>i</sub>, OBB<sub>i</sub>) of the GPU for the study cases (b) (GPU) and (c) (GPU+VPP).



## Case study: analysis of the solution

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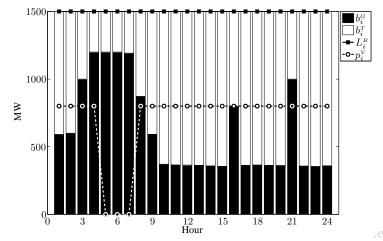
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• Operation planning for study case (c) (GPU+VPP).



## Case study: analysis of the solution

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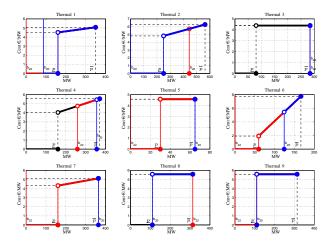
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 Optimal thermal bidding curves for study case (c) (GPU+VPP).



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Conclusions

• A new two-stage stochastic programming model has been presented and tested with the following characteristics:

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- A new two-stage stochastic programming model has been presented and tested with the following characteristics:
  - A new model for the optimal thermal bidding function and matched energy who takes into account the presence of bilateral contracts.

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- A new two-stage stochastic programming model has been presented and tested with the following characteristics:
  - A new model for the optimal thermal bidding function and matched energy who takes into account the presence of bilateral contracts.
  - The mathematical modellization of the generic programming units and the Virtual Power Plants.

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The Solution

- A new two-stage stochastic programming model has been presented and tested with the following characteristics:
  - A new model for the optimal thermal bidding function and matched energy who takes into account the presence of bilateral contracts.
  - The mathematical modellization of the generic programming units and the Virtual Power Plants.
  - The modellization of the optimal bidding functions and matched energy of the GPU.

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Conclusions

 A new two-stage stochastic programming model has been presented and tested with the following characteristics:

- A new model for the optimal thermal bidding function and matched energy who takes into account the presence of bilateral contracts.
- The mathematical modellization of the generic programming units and the Virtual Power Plants.
- The modellization of the optimal bidding functions and matched energy of the GPU.
- The inclusion in the optimization model of the bilateral contracts after the day-ahead market.

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 A new two-stage stochastic programming model has been presented and tested with the following characteristics:

- A new model for the optimal thermal bidding function and matched energy who takes into account the presence of bilateral contracts.
- The mathematical modellization of the generic programming units and the Virtual Power Plants.
- The modellization of the optimal bidding functions and matched energy of the GPU.
- The inclusion in the optimization model of the bilateral contracts after the day-ahead market.
- The consideration of the most recent regulations of the MIBEL energy market.



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# Stochastic Programming Model for the Day-Ahead Bidding and Bilateral Contracts Settlement Problem

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Universitat Politècnica de Catalunya
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