Stochastic programming models for optimal bid strategies in the Iberian Electricity Market

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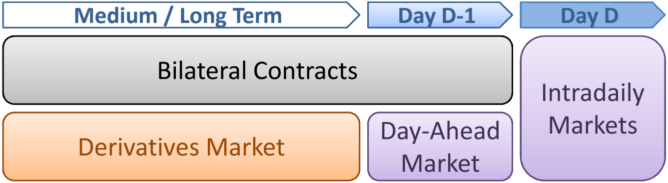
Iberian Electricity Market: MIBEL



Derivatives Market

Physical Futures Contracts
Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery.
Financial Futures Contracts
OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

Bilateral Contracts

Organized markets
- Virtual Power Plants auctions (**EPE**) - Distribution auctions (**SD**) - International Capacity Interconnection auctions - International Capacity Interconnection nomination
Non organized markets
- **National** BC **before** the spot market - **International** BC **before** the spot market - **National** BC **after** the spot market

Day-Ahead Market

Day-Ahead Market
Hourly action. The matching procedure takes place 24h before the delivery period. Physical futures contracts are settled through a zero price bid.

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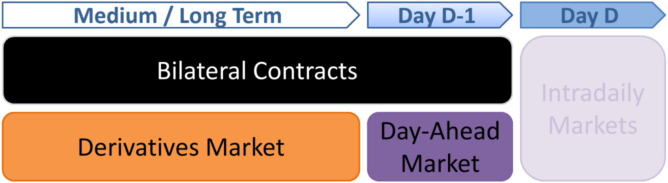

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Characteristics of Physical Futures and Bilateral Contracts

Base Load Futures Contract

- *Base Load Futures Contract j* consists in a pair $(L_j^{FC}, \lambda_j^{FC})$
  - L_j^{FC}: amount of energy (MWh) to be procured each interval of the delivery period by the set U_j of generation units.
  - λ_j^{FC}: price of the contract (c€/MWh).

Bilateral Contracts

- *Bilateral Contract k* consists in a pair $(L_{kt}^{BC}, \lambda_k^{BC})$ $t \in T$
  - L_{kt}^{BC}: amount of energy (MWh) to be procured at interval t of the delivery period by the whole set of generation units.
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Integration of the futures and bilateral contracts in the day-ahead bid

The energies L_j^{FC} and L_{kt}^{BC} should be integrated in the MIBEL's day-ahead bid respecting the two following rules:

1. If generator i contributes with f_{itj} MWh at period t to the coverage of the FC j, then the energy f_{itj} must be offered to the pool for free (instrumental price bid).
2. If generator i contributes with b_{it} MWh at period t to the coverage of the BCs, then the energy b_{it} must be excluded from the bid to the day-ahead market. Unit i can offer its remaining production capacity $\overline{P}_i - b_{it}$ to the pool.

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Problem definition

The objective of the study is to decide:

- the **optimal economic dispatch of the physical futures and bilateral contract** among the thermal units
- the **optimal bidding at Day-Ahead Market** abiding by the MIBEL rules
- the **optimal unit commitment** of the thermal units

maximizing the expected Day-Ahead Market profits taking into account futures and bilateral contracts.

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Model characteristics

- Stochastic mixed integer quadratic programming model
- *Price-taker* generation company
- Set of thermal generation units, I, with quadratic generation costs.
- Optimization horizon of 24h, T
- Set of physical futures contracts, F, of energy L_j^{FC} $j \in F$.
- A pool of bilateral contracts of energy $L_t^{BC} = \sum_k L_{kt}^{BC}$, $t \in T$.
- Set of day-ahead spot price scenarios, $\lambda^s \in \Re^{|T|}$, $s \in \mathcal{S}$

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Variables

First stage variables: $t \in T, \ i \in I$

- Instrumental price offer bid : q_{it}
- Scheduled energy for futures contract j: f_{itj} $j \in F$
- Scheduled energy for bilaterals contract: b_{it}

Second stage variables $t \in T, \ i \in I, \ s \in S$

- Matched energy: $p_{it}^{M,s}$
- Total generation: p_{it}^{s}

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Physical Future and Bilateral Contracts model

Physical future contract coverage:

$$\sum_{i \in U_j} f_{itj} = L_j^{FC} \, , \, j \in F \, , \, t \in T$$

$$f_{itj} \geq 0 \, , \, j \in F \, , \, i \in I \, , \, t \in T$$

Bilateral contract coverage:

$$\sum_{i \in I} b_{it} = L_t^{BC} \, , \, t \in T$$

$$0 \leq b_{it} \leq \overline{P}_i \, , \, i \in I \, , \, t \in T$$

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Objective function

Maximization of the E.V. of the profit from the day-ahead market

$$\max_{p,q,f,b} \sum_{t \in T} \sum_{i \in I} \sum_{s \in S} P^s \left[\lambda_t^s p_{it}^{M,s} - \left(c_i^l p_{it}^s + c_i^l (p_{it}^s)^2 \right) \right]$$

Incomes from Futures and Bilateral contracts (constant):

- **Futures contracts:** $\sum_{t \in T} \sum_{j \in J} \left(\lambda_j^{FC} - \lambda_t \right) L_j^{FC}$
- **Bilateral contracts:** $\sum_{t \in T} \sum_{k \in K} \lambda_k^{BC} L_{kt}^{BC}$

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Summary of the model

Problem OBIFUC

(**O**ptimal bid with **BI**lateral and **FU**tures **C**ontracts)

Max E[Profit from the Day-ahead market]

s.t:

Physical future contract coverage

Bilateral contract coverage

Matched energy

Instrumental price bid

Total energy generation

Problem (OBIFUC) is **concave**.

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Optimal Matched Energy

Lemma

Let $x^{*\prime} = [p^*, p^{M,*}, q^*, f^*, b^*]'$ be an optimal solution of problem (OBIFUC). Then for any thermal unit i the optimal value of the matched energy $p_{it}^{M,s*}$ can be expressed as:

$$p_{it}^{M,s*} = \max\{q_{it}^*, \rho_{it}^s(b_{it}^*)\} \quad (1)$$

where $\rho_{it}^s(x)$ is a known function

(*Proof: KKT conditions of problem (OBIFUC)*)

Function $\rho_{it}^{s}(x)$

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$$\rho_{it}^{s}(x) = \begin{cases} [\underline{P}_i - x]^{+} & \theta_{it}^{s} - x < [\underline{P}_i - x]^{+} \\ \theta_{it}^{s} - x & [\underline{P}_i - x]^{+} \leq \theta_{it}^{s} - x \leq \overline{P}_i - x \\ \overline{P}_i - x & \theta_{it}^{s} > \overline{P}_i \end{cases} \qquad (2)$$

with $\theta_{it}^{s} = (\lambda_t^{s} - c_i^{l})/2c_i^{q}$

Bid function's optimality conditions

Definition (Bid function's optimality conditions)

Let $x^{*\prime} = [p^*, p^{M,*}, q^*, f^*, b^*]'$ be the optimal solution of the (OBIFUC) problem. The bid function λ_{it}^{b*} of a thermal unit i committed on period t is said to be optimal if the value of the matched energy function associated to any scenario's clearing price λ_t^s, $p_{it}^M(\lambda_t^s)$, coincides with the optimal matched energy $p_{it}^{M,s*}$, that is:

$$p_{it}^M(\lambda_t^s) = p_{it}^{M,s*} = \max\{q_{it}^*, \rho_{it}^s(b_{it}^*)\}$$

OBIFUC's optimal bid function

Lemma (Optimal bid function)

Let $x^{*\prime} = [p^{M,*}, p^*, q^*, f^*, b^*]'$ be an optimal solution of the (OBIFUC) problem and i any thermal unit committed on period t at the optimal solution. Then the bid function:

$$\lambda^*_{it}(p_{it}; b^*_{it}, q^*_{it}) = \begin{cases} 0 & \text{if } p_{it} \leq q^*_{it} \\ 2c^q_i\,(p_{it} + b^*_{it}) + c^l_i & \text{if } q^*_{it} < p_{it} \leq (\overline{P}_i - b^*_{it}) \end{cases} \tag{3}$$

is optimal w.r.t. the (OBIFUC) problem and the optimum x^*.

This result is also valid when the unit commitment is included in the (OBIFUC) model.

OBIFUC's optimal bid function

Lemma (Optimal bid function)

Let $x^{*\prime} = [p^{M,*}, p^*, q^*, f^*, b^*]'$ *be an optimal solution of the (OBIFUC) problem and* i *any thermal unit committed on period* t *at the optimal solution. Then the bid function:*

$$\lambda^*_{it}(p_{it}; b^*_{it}, q^*_{it}) = \begin{cases} 0 & \text{if } p_{it} \leq q^*_{it} \\ 2c^q_i\,(p_{it} + b^*_{it}) + c^l_i & \text{if } q^*_{it} < p_{it} \leq (\overline{P}_i - b^*_{it}) \end{cases} \tag{3}$$

is optimal w.r.t. the (OBIFUC) problem and the optimum x^**.*

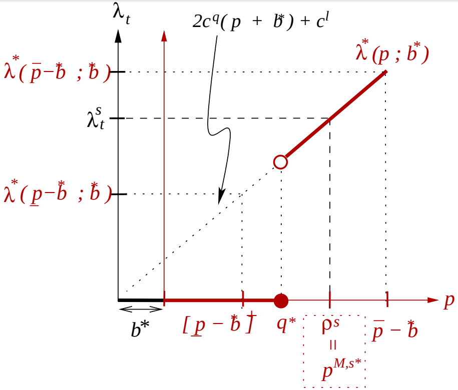
This result is also valid when the unit commitment is included in the (OBIFUC) model.

OBIFUC's optimal bid function

Matched energy at scenario s:

$$p_{it}^{M}(\lambda_t^s) = p_{it}^{M,s*} = \max\{q_{it}^*, \rho_{it}^s(b_{it}^*)\} \quad (4)$$

$$q_{it}^* \le \rho_{it}^s(b_{it}^*) \implies$$
$$\implies p_{it}^{M,s*} = \rho_{it}^s(b_{it}^*)$$



OBIFUC's optimal bid function

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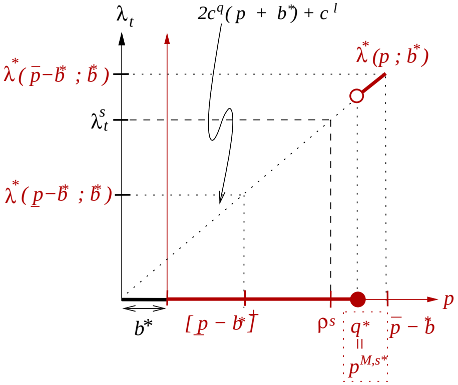
$$q_{it}^* \geq \rho_{it}^s(b_{it}^*) \implies p_{it}^{M,s*} = q_{it}^*$$



Case Study characteristics

- Real data from the Spanish Market about the generation company and the market prices (from January 1^{rst} to December 31^{rst}, 2008).
- 9 thermal generation units (6 coal, 3 fuel) from a Spanish generation company with daily bidding in the MIBEL

$[\overline{P} - \underline{P}]$ (MW)	160-243	250-550	80-260	160-340	30-70
$min_{on/off}$ (h)	3	3	3	4	4

$[\overline{P} - \underline{P}]$ (MW)	60-140	160-340	110-157	110-157
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- 61 scenarios simulated from a multivariate times series + factor model.
- Model implemented and solved with AMPL/CPLEX 11.0.

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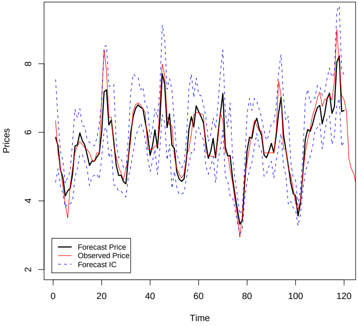
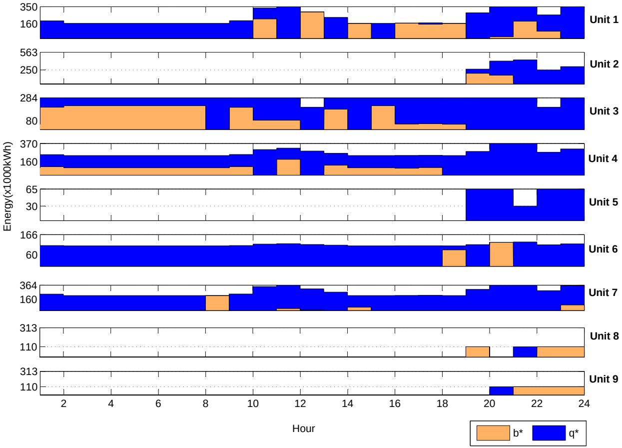
Uncertainty characterization

- Uncertainty source: DAM Price, λ^s, characterized as a time series. The prices for the day in study must be forecasted.
- Price scenario forecasting method:



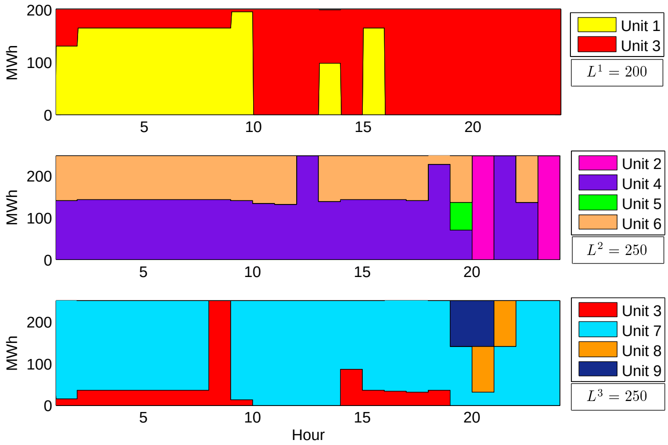
1. 24 time series are considered
2. Estimation of the factor model
3. The forecasting model is specified as a linear multiple regression model with the factors as predictors
4. Simulation of the price scenarios

Results: bilateral and futures contracts settlement



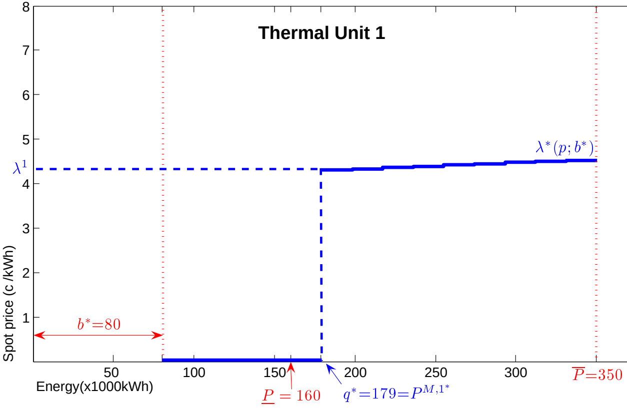
Settlement of the three futures contracts (L^{FC}=700MW) and the portfolio of bilateral contracts (L^{BC}=300MW)

Results: economic dispatch of futures contracts



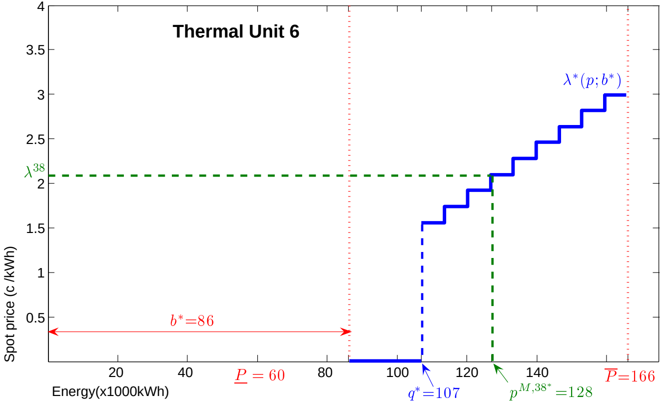
Economic dispatch of each futures contract among the corresponding set of units

Results: optimal bidding curve



Optimal bidding curve for thermal unit 1 at interval 23

Results: optimal bidding curve



Optimal bidding curve for thermal unit 6 at interval 18

Conclusions

- It has been built an Optimal Bidding Model for a price-taker generation company operating both Futures and Bilateral Contracts.
- The stochasticity of the spot market price has been taken into account and it has been represented by a scenario set.
- The model developed gives the producer:
  - The optimal bid for the spot market.
  - Unit commitment
  - Optimal allocation of the physical futures and bilateral contracts among the thermal units

  following in detail the MIBEL rules.

Stochastic programming models for optimal bid strategies in the Iberian Electricity Market

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