Medium-term generation planning optimization in liberalized electricity markets

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Introduction: Time scales for planning optimization

For power system planning optimization purposes there are several hierarchic time scales, each having a different function:

- *Long-term power planning*. Scope: 2-10 years. Purpose: decide capacity of new generation units and when to start operation in order to match future loads; decide when to retire old units. (Also used for new lines/transformers in the transmission network.)
- *Medium-term power planning*. Scope: 1-2 years. Purpose: decide how much energy is going to generate each unit and when in order to match the predicted loads, so that the budget and the fuels procurement policies can be established; fix target generations for the short-term planning.
- *Short-term power planning*. Scope: 1 day - 1 week. Purpose: decide when to start-up and shut-down units and the amount of power to generate at each hour by each unit while meeting several operational constraints.
- *On-line operation*. Scope: some minutes. Purpose: knowing the state of the transmission network and the flows of power on each line/transformer and satisfying several network, operational and security constraints.

Medium term power planning

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- Medium-term electric power planning is a stochastic optimization problem. Its time scope is one (or two) year(s), which is subdivided into shorter time periods (monthly, bimonthly, ...)
- The variables to be optimized in medium term planning are the expected energies to be generated by each generation unit in each time period.
- It has to be solved for (optimum) new plant planning, (optimum) fuel acquisition for the yearly budget, expected revenue accountancy, and (optimum) medium- and short- term operation.
- It is stochastic because load is uncertain, the generating units may have random outages (whose probability is assumed to be known), and water inflow in reservoirs and renewables' availability are also stochastic.
- Fortunately, we have the convolution method to combine the load duration with unit outages, the Bloom and Gallant formulation for satisfying the load-matching and other constraints, and the stochastic programming techniques (scenarios)

Medium term power planning

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- The most reasonable way of evaluating the impact of renewable energies is through medium term planning.
- Medium term planning can also be used to find the equilibrium solution in electricity markets (through the Nikaido-Isoda algorithm of successive optimizations).
- The increase of risk of profit loss due to the use of renewables can be also evaluated.

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Generation units in medium-term planning

We should first distinguish between the specific generation company (SGC), of which we know its generation units detail, and the rest of participants (RoP) in the market, of which we know their generation units with less detail. The generation units to be considered are:

- all thermal units of the SGC whose production is to participate in the auction process,
- it would be good to consider the reservoir systems of hydro production of the SGC with full detail, but it is usual to model hydrogeneration of the SGC as one or several equivalent simplified single-reservoir systems with or without run-of-the-river,
- the thermal units of the RoP, either as single or as *merged* pseudo-units of similar characteristics (e.g., all available nuclear units of the competitor companies could be merged into a single nuclear pseudo-unit),
- the hydro-systems of the RoP considered as one or more single-reservoir schemes.
- big cascaded reservoirs can be taken into account with a detailed hydro model using extra variables.

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Generation units in medium-term planning

In medium-term planning the relevant parameters of a *thermal* unit are:

⋆ *power capacity*: c_j for the $j^{\underline{\text{th}}}$ unit (MW)

⋆ *outage probability*: q_j for the $j^{\underline{\text{th}}}$

⋆ *linear generation cost*: $\widetilde{f}_j$ for the $j^{\underline{\text{th}}}$ unit (€/MWh)

Let us denote by M the set of units merged into one given pseudo-unit, and let r be the index of one of the composing units. The parameters of the pseudo-unit can be calculated as:

- *maximum power capacity* $c_M = \sum_{r \in M} c_r$
- *linear generation cost* $f_M = \left(\sum_{r \in M} c_r\, f_r\right) / \sum_{r \in M} c_r$
- *outage probability* $q_M = \left(\sum_{r \in M} c_r\, q_r\right) / \sum_{r \in M} c_r$.

Natural water inflows in reservoirs (genuine ones or simplifications) are stochastic in medium-term planning, and scenarios should be employed.

Generation units in medium-term planning

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- Thermal unit generation may also be subject to constraints such as fuel availability and emission limits.
- The wind-power and the photo-voltaic (PV) solar-power generation forecast for a future medium-term period can be split into two parts. A quite reliable one, and a mostly random part, both of them behaving as a pseudo thermal unit with no cost and appropriate capacity and outage parameters.
- Both the reliable part and the random part of wind-power and of PV generation can be deducted from the *load duration curve* of its period in the nodes of an scenario tree using probabilistic methods.

x_j^i will denote the medium-term *expected energy* generated by unit j over the $i^{\underline{\text{th}}}$ period.
There will be n_u thermal units, or pseudo-units considered.

Simplification of reservoir systems

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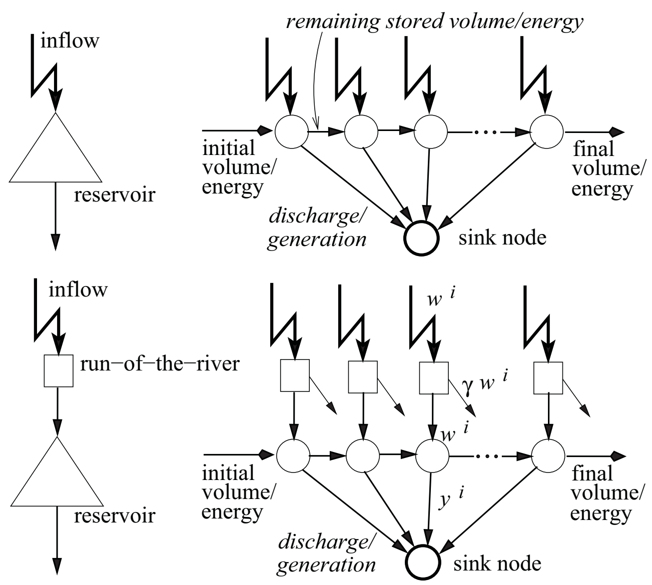
It is common to simplify reservoir systems to a single reservoir with constant head (hydrogeneration proportional to water discharge) in order to avoid using too many variables. In the resulting replicated network, flows are energies instead of water.



Hydrogeneration reservoirs, simplified reservoir systems and run-of-the-river schemes behave as a pseudo-thermal unit with $x_h^i = \gamma w^i + y^i$ subject to additional constraints.

The load duration curve 1

The load duration curve (LDC) is a reasonable way to represent the load of a future period. Its main characteristics are:

- the duration T
- the peak load power $\widehat{p}$
- the base load power $\underline{p}$
- the total energy $\widehat{e}$
- the shape, which is not a single parameter.

The LDC for future periods must be predicted. For a past period, for which the hourly load record is available, the LDC is equivalent to the load over time curve sorted in order of decreasing power.

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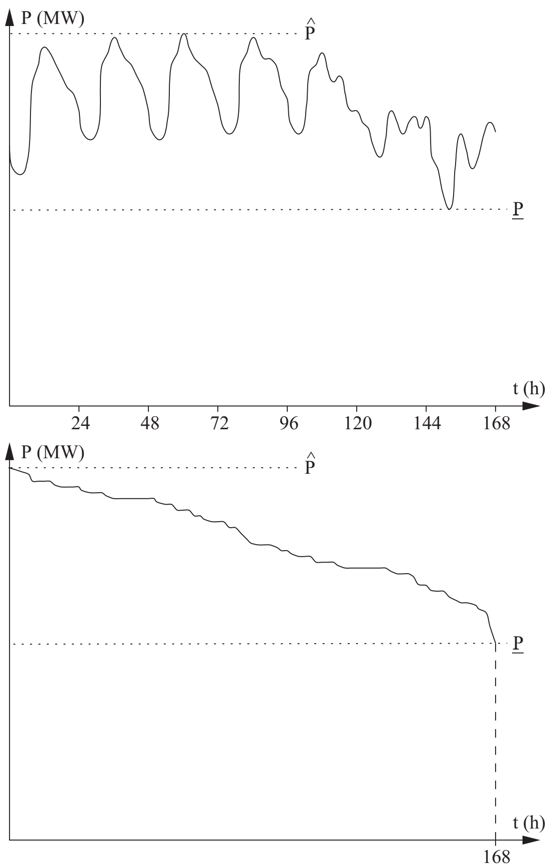
The load duration curve 2

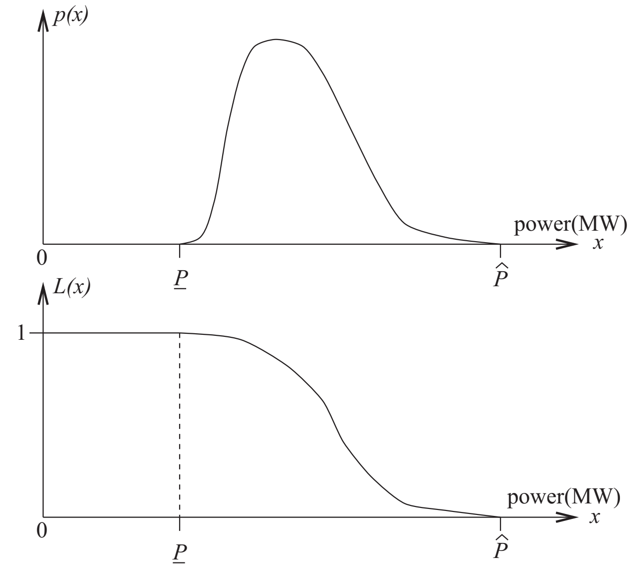


Load over time, above, and load-duration curve (LDC), below . (Data for a week — Monday to Sunday).

The load duration curve 3

Analytically, given the probability density function of load $p(z)$, the load-survival function $S_\emptyset(z)$ is calculated as:

$$S_\emptyset(z) = 1 - \int_0^z p(y)\, dy$$



Probability density function $p(z)$ of load (above), and load-survival function $S_\emptyset(z)$ (below), which, changing axes and probability for time duration scale, is the LDC.

The convolution method to match the load 1

The loading of thermal units to match an LDC was first formulated by Balériaux, Jamoulle and Linard de Guertechin in 1967. Let:

c_j	:	maximum power capacity in MW of unit j
q_j	:	outage probability of unit j
$1 - q_j$	:	in service probability of unit j
U_j	:	set of unit indices $1, 2, \ldots, j$
$S_{U_{j-1}}(z)$	:	load-survival function of unmatched load after loading units $1, 2, ..., j-1$ (z: load in MW)
$S_{U_j}(z)$	:	load-survival function of unmatched load after loading units $1, 2, ..., j-1, j$

the convolution computes $S_{U_j}(z)$ from $S_{U_{j-1}}(z)$ as:

$$S_{U_j}(z) = q_j\, S_{U_{j-1}}(z) + (1 - q_j)\, S_{U_{j-1}}(z + c_j)$$

Recalling that energy$= T \cdot p$, the energy generated by unit j is:

$$x_j = (1 - q_j)\, T \int_0^{c_j} S_{U_{j-1}}(z)\, dz\ .$$

Other associated concepts are:

⋆ *merit order*: units are loaded ordered according to their cost

⋆ *loading order*: units will have load allocated to them in a given order (due to active non-load-matching constraints).

The convolution method to match the load 3

Starting with $S_\emptyset(z)$ and convolving successively the units $1, 2, \ldots$ we will find the distribution of unsupplied load after *loading* these units. Given a set of units whose indices $1, 2, \ldots, n_u$ are the elements of the set of indices Ω, the unsupplied load after loading all the units in Ω will have a load-survival function $S_\Omega(z)$:

$$S_\Omega(z) = S_\emptyset(z) \prod_{m\in\Omega} q_m +$$
$$\sum_{U\subseteq\Omega} (S_\emptyset(z + \sum_{i\in U} c_i)(1-q_i) \prod_{i\in U}(1-q_i) \prod_{i\in U} q_i)$$

We can thus say that $S_\Omega(z)$ (of unsupplied load) is the same no matter in which order the units in Ω have been loaded. The unsupplied energy (external energy to be acquired) $w(\Omega)$ is:

$$w(\Omega) = T \int_0^{\widehat{p}} S_\Omega(z)\, dz$$

The unsupplied load after having loaded the units in subset $U \in \Omega$ is:

$$w(U) = T \int_0^{\widehat{p}} S_U(z)\, dz\,.$$

The convolution method to match the load 4

The contribution of each unit to matching the load will be different depending on the position in loading order, and the lower the order number the bigger the generation will be for the given unit. This is so because $S_{U_{j+1}}(z) \leq S_{U_j}(z)\ \forall z$.
A given unit of index $k \in \Omega$ will have its generation bounded by:

$$\begin{aligned} x_k \geq \quad & \underline{x}_k = (1-q_k)\, T \int_0^{c_k} S_{\Omega\backslash k}(z)\, dz > 0 \\ x_k \leq \quad & \overline{x}_k = (1-q_k)\, T \int_0^{c_k} S_\emptyset(z)\, dz \end{aligned}$$

where $S_{\Omega\backslash k}(z)$ corresponds to the load-survival function after loading all units in Ω but that of index k.
$\underline{x}_k$ and $\overline{x}_k$ correspond respectively to loading unit k the last and the first.
(No non-LMC can force a given unit j in a certain period i to generate beyond $\overline{x}_k$, but it is possible that in some periods non-load-matching constraints (e.g., fuel availability, or emission limits) force a given unit to generate under $\underline{x}_k$, or even at zero.)

The generation duration curve 1

The *generation-duration curve* is the expected production of the thermal units over the time period to which the LDC refers.
The energy generated by each unit is the slice of area under the generation-duration curve which corresponds to the capacity of the thermal unit.
The probability that there are time lapses, within the time period under consideration, where, due to outages, there is not enough generation capacity to cover the current load, is not null. Therefore, *external energy* (from other interconnected utilities), will have to be imported and paid for, at a higher price than the most expensive unit in ownership.
The area under the LDC and the area under the generation-duration curve must coincide.
The peak power of the generation-duration curve is $\sum_{j=1}^{n_u} c_j + \widehat{p}$ and the area above power $\sum_{j=1}^{n_u} c_j$ is the external energy.

The generation duration curve 2

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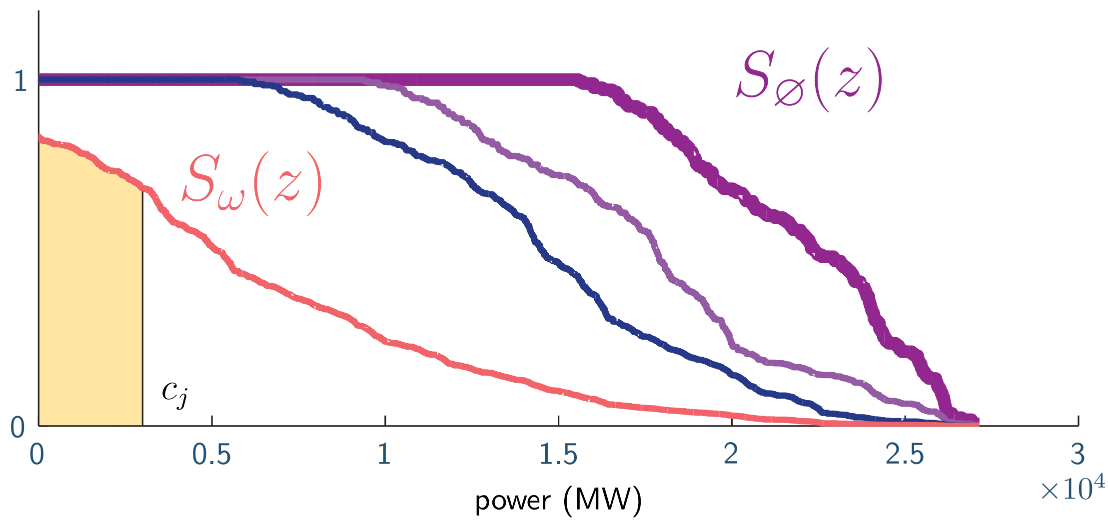

Real load-survival function after loading three units, and expected contribution of the fourth unit.

$S_\emptyset(z)$: Load survival function (original)
$S_\omega(z)$: Load survival function after loading units in set $\omega = \{1, 2, 3\}$
c_j: capacity of unit j loaded after loading units in ω.

The generation duration curve 3

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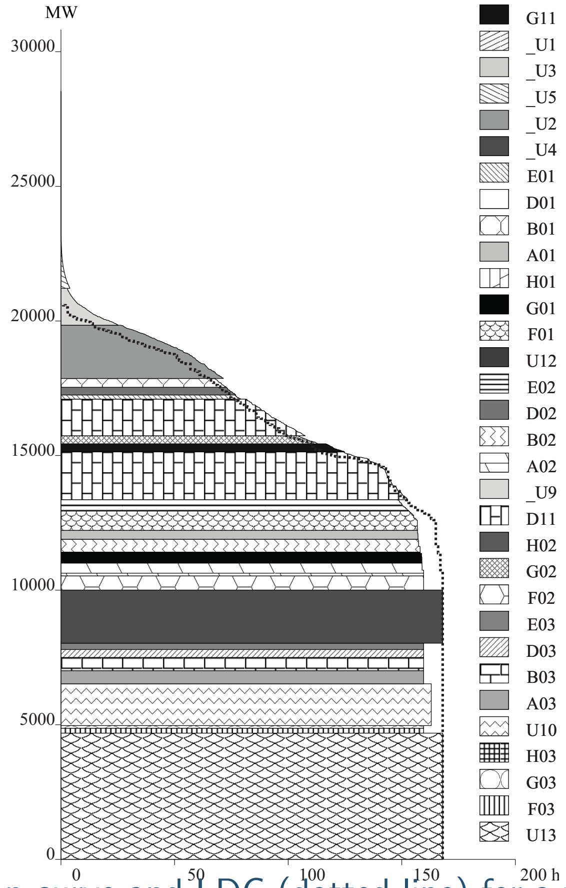

Generation-duration curve and LDC (dotted line) for a weekly period in a 32 unit problem.

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Medium-Term Power Planning Optimization

The Bloom and Gallant formulation 1

Contents

The Bloom & Gallant formulation (1994) of matching the LDC while minimizing a cost function and satisfying other constraints for a single period is:

$$\begin{aligned}
\underset{x_j}{\text{minimize}} \quad & \sum_{j=0}^{n_u} \widetilde{f}_j \, x_j \\
\text{subject to} \quad & \sum_{j\in U} x_j \le \widehat{e} - w(U) \;\forall\, U \subset \Omega = \{1,\dots,n_u\} \qquad (1)\\
& \sum_{j=0}^{n_u} x_j = \widehat{e} \\
& Ax + By \ge r \\
& x_j \ge 0 \qquad j = 0,1,\dots,n_u
\end{aligned}$$

where in (1) there are $2^{n_u} - 1$ *load-matching* constraints (LMCs), which is a large number even for moderate n_u. x is the vector containing the x_j , $\forall\, j$, where x_0 is the external energy, and $\widetilde{f}_0$ its cost. A and B are the matrices of coefficients of x and of the extra variables y, and r are the *rhs* of the non-LMCs. (Equality non-LMCs could also be included.)

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Nested active constraints in the Bloom and Gallant formulation 1

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When no non-LMC of $Ax + By \ge r$ is active at the optimizer, the solution to the problem corresponds to loading the units in *merit order* (order of ascending costs). The active LMCs are then the triangular system:

$$\begin{aligned}
x_1 &= \widehat{e} - w(1) \\
x_1 + x_2 &= \widehat{e} - w(1,2) \\
x_1 + x_2 + x_3 &= \widehat{e} - w(1,2,3) \\
&\cdots \\
x_1 + x_2 + x_3 + \cdots + x_{n_u} &= \widehat{e} - w(1,2,\dots,n_u)
\end{aligned}$$

(Here it is assumed that the cost $\widetilde{f}_j$ of units 1, 2, ..., are in ascending order.)
Should one or several non-LMCs be active, one or many of the above constraints could be nonactive. Loading order may then be different from *merit order* and *splittings* may occur. However, the solution obtained will always have a *nested* set of active LMCs.

Nested active constraints in the Bloom and Gallant formulation 2

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The coefficients in the left-hand side of any LMC is a row vector of ones and zeros, depending on which units there are in the subset the LMC refers to. Regarding these ones and zeros of the *active* LMCs, it can be shown that, in any solution point, the ones must be *nested*.

Let d_ζ represent the row vector of coefficients of the left-hand side of the LMC built with the set of units ζ, which can be any subset of units. It is said that a constraint d_ζ is *nested* into d_θ, where ζ and θ are any sets of units, if $\zeta \subset \theta$.
The following is an example in which d_ζ is nested into d_θ for $n_u = 6$ (assuming that the *merit* order is 1, 2, 3, 4, 5 and 6):

	u_1	u_2	u_3	u_4	u_5	u_6	
d_ζ	.	1	.	1	.	.	$\zeta = \{2, 4\}$
d_θ	1	1	.	1	1	1	$\theta = \{1, 2, 4, 5, 6\}$

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In general, a set of constraints is nested if we can order the constraints in the set in such a way that every constraint is nested in the next. Following the example, a set of *active* nested constraints, corresponding to a solution point, might be:

		u_1	u_2	u_3	u_4	u_5	u_6	
1st	d_η	.	.	.	1	.	.	$\eta = \{4\}$
2nd	d_ζ	.	1	.	1	.	.	$\zeta = \{2, 4\}$
3rd	d_θ	1	1	.	1	1	1	$\theta = \{1, 2, 4, 5, 6\}$
4th	d_Ω	1	1	1	1	1	1	$\Omega = \{1, 2, 3, 4, 5, 6\}$

where d_η is nested in d_ζ, d_θ and d_Ω, d_ζ is nested in d_θ and d_Ω, d_θ is only nested in d_Ω and d_Ω (where the set Ω contains all the units) nests d_η, d_ζ and d_θ.
The phisical meaning of the nested constraints is related to the ordered loading of generation units: once one unit is loaded (appears for the first time in an active LMC), it must also appear in all the ensuing active LMCs, as the unit does not cease to contribute to the satisfaction of the unmatched load.

Nested active constraints in the Bloom and Gallant formulation 4

By reordering the units in *loading* order we could observe that:

		u_4	u_2	u_1	u_5	u_6	u_3	
1st	d_η	1	.	.	.	.	.	$\eta = \{4\}$
2nd	d_ζ	1	1	.	.	.	.	$\zeta = \{4, 2\}$
3rd	d_θ	1	1	1	1	1	.	$\theta = \{4, 2, 1, 5, 6\}$
4th	d_Ω	1	1	1	1	1	1	$\Omega = \{4, 2, 1, 5, 6, 3\}$

has a flight-of-stairs structure (with a landing), and indicates the *loading* order: 4, 2, (1,5,6), 3.

(1,5,6) is a group of units that are loaded at the same time. This means that the loading of one of these unit is initiated before the loading of any other of these units is finished. These three units form a *landing* in the flight-of-stairs structure of the ones in the *active* LMCs (a *splitting* in loading order), and the relative order in which they start loading depends on the values at he solution of x_1, x_5 and x_6.

(*Loading* a unit j means here increasing its generation by augmenting its power output level from zero up to its maximum capacity c_j, during as many hours as the still-unsupplied load permits.)

The multi-period Bloom and Gallant formulation 1

The Bloom and Gallant linear optimization model extended to n_i periods, and with inequality non-LMCs and no extra variables y is:

$$\begin{aligned}
\underset{x_j^i}{\text{minimize}} \quad & \sum_{i=1}^{n_i} \sum_{j=0}^{n_u} \widetilde{f}_j x_j^i \\
\text{subject to:} \quad & \sum_{j \in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i = 1, \dots, n_i \\
& \sum_{j=0}^{n_u} x_j^i = \widehat{e}^i \quad i = 1, \dots, n_i \\
& A^i x^i \ge r^i \quad i = 1, \dots, n_i \\
& \sum_i A^{0i} x^i \ge r^0 \\
& x_j^i \ge \underline{0} \qquad j = 0, 1, \dots, n_u^i \quad i = 1, \dots, n_i
\end{aligned}$$

where supraindex ^{i} means relation with $i^{\underline{\text{th}}}$ period.
Note that $|\, \Omega^i \,| = n_u^i$ (overhauling of units in periods is taken into account), there are single-period and multi-period non-LMCs, and that equality non-LMCs could be also included.

The multi-period Bloom and Gallant formulation 2

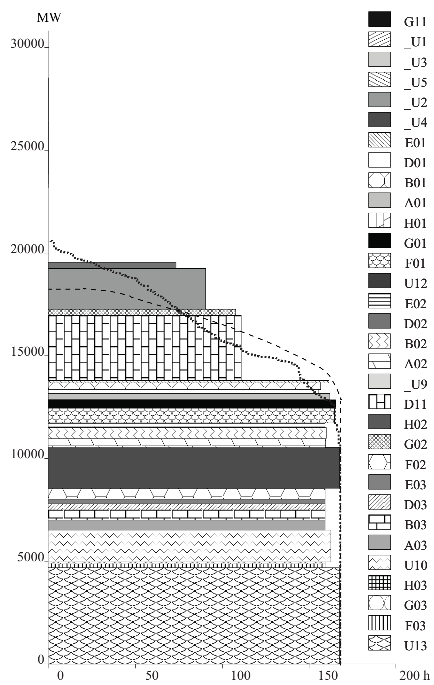
Using the energy-balance equation to extract the external energy x_0^i we could simplify the multi-period problem. Leting f_j be $\widetilde{f}_j - \widetilde{f}_0\,(<0)$, we get:

$$\begin{array}{lll} \underset{x_j^i}{\text{minimize}} & \displaystyle\sum_{i=1}^{n_i}\sum_{j=1}^{n_u} f_j x_j^i & \\ \text{subject to:} & \displaystyle\sum_{j\in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i=1,\dots,n_i \\ & A^i\,x^i \ge r^i \quad i=1,\dots,n_i \\ & \displaystyle\sum_i A^{0i}\,x^i \ge r^0 \\ & x_j^i \ge \underline{0} \qquad j=1,\dots,n_u^i \quad i=1,\dots,n_i \end{array}$$

where in the objective function we have omitted the constant term $\widetilde{f}_0 \sum_{i=1}^{n_i} \widehat{e}^i$.

The Bloom and Gallant formulation with no LMCs

Should we fail to consider the LMCs other than the load-balance and the maximum generation ones for each unit, we would get expected generations that, except for the units that match the part of the LDC up to the base power, are very unlikely to happen or that do not even correspond to a feasible matching of the load.
This is in contrast with the energies obtained considering all LMCs, which are a perfect match of the LDC, and are those more likely to happen.



Generation-duration curve of problem with LMCs omitted.

The GP heuristic to account for the LMCs 1

The GP heuristic (Pagès & Nabona 2005) is a means of avoiding having to create and employ an exponential number of (linear inequality) LMCs in medium-term planning. The GP heuristic requires generating only a reduced subset of LMCs among which there are the *active* (satisfied as an equality) LMCs at the optimizer of the medium-term planning solved.

The GP heuristic is based on the property of the solution points of always having a *nested* set of *active* LMCs (at every period). It can be applied to multi-period problems, but it will be presented here for a single-period problem to simplify the notation.

The experience with the application of the GP heuristic shows that:

- it is highly efficient, allowing the solution of problems with big n_u value in short execution time, and
- it is reliable: its solution corresponds to that found using the full Bloom and Gallant formulation.

The GP heuristic to account for the LMCs 2

Let us consider the single-period long-term power planning problem

$$\begin{array}{ll} \underset{x_j}{\text{minimize}} & \sum_{j=0}^{n_u} \widetilde{f}_j\, x_j \\ \text{subject to} & B_L x \leq r_L \\ & \sum_{j=0}^{n_u} x_j = \widehat{e} \\ & Ax \geq r \\ & 0 \leq x_j \leq \overline{x}_j \qquad j = 1, \ldots, n_u \\ & x_{n_u+1} \geq 0 \end{array} \tag{2}$$

where B_L and r_L are the matrix of zero and one coefficients, and the *rhs* of a subset of the LMCs not including the upper bound of each single unit: $\overline{x}_j = t(1-q_j)\int_0^{c_j} S_\emptyset(z)dz$, which are now explicitly taken into account outside the subset.

The GP heuristic to account for the LMCs 3

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B_L and r_L in problem (2) contain the coefficients and the *rhs* of a list L of LMCs (of which the upper bounds $x_j \le \overline{x}_j$, $\forall\, j$ are excluded). There are three main steps in the GP heuristic:

i **Initialization** where the list L is initialised with the all-one LMC ($L = \{1, 2, \ldots, n_u\} = \{\Omega\}$), and problem (2) is solved.

ii **Self-ordering** where the subset ϕ of units j whose generation x_j in the former solution is at, or close to, its upper bound ($x_j \simeq \overline{x}_j$) is formed, the LMCs made with any subset of units in ϕ is then added to the list L, and (2) is resolved. Let $|\,\phi_0\,|$ indicate the cardinality of set ϕ as it is now.

iii **Step-by-step order** While set ϕ has less elements than Ω, we look in the former solution of (2) for that unit k, not still in ϕ, whose x_k is closer to its upper bound $\overline{x}_k$, set ϕ is updated by adding k to it, the updated set ϕ is then added to the list L, and (2) is solved. Note that in this step we will solve (2) $n_u - |\,\phi_0\,|$ times.

The total number of LMCs generated using the GP heuristic is $2^{|\,\phi_0\,|} + n_u - |\,\phi_0\,|$ (in contrast with $2^{n_u} - 1$ using the complete Bloom and Gallant formulation).

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The GP heuristic to account for the LMCs 4

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i **Initialization**

- $\phi := \emptyset$, $L := \{\Omega\}$ and form B_L and r_L
- solve (2)

ii **Self-ordering**

- $\phi := \{ j \in \Omega \,|\, \rho_j \simeq 1 \}$ where $\rho_j := x_j / \overline{x}_j$
- for $\forall\, \omega \subseteq \phi \,|\, |\omega| > 1\,,\ \sum_{j \in \omega} c_j > \underline{p}$

  $L := L \cup \omega$ and add the newly created LMC to B_L and r_L

  end for
- solve (2)

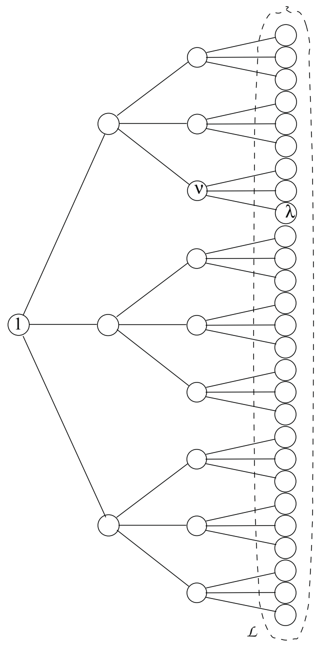
iii While $|\,\Omega \backslash \phi\,| > 1$ **step-by-step order**

- $\phi := \phi \cup \{ j \,|\, \rho_j = \max_{\forall\, k \in \Omega \backslash \phi} \rho_k \}$ where $\rho_k := x_k / \overline{x}_k$
- $L := L \cup \omega$ and add the newly created LMC to B_L and r_L
- solve (2)

end while

Stochastic medium-term power planning 1

Some of the parameters employed (e.g., the inflows in reservoirs, which are a part of vector r of the non-LMCs $Ax \geq r$) vary randomly over time. The formulation employed up to now corresponds to using as parameters their expected value, but we could do better by modeling the medium-term planning over an scenario tree:



A scenario tree is formed by the set of connected tree nodes $\mathcal{N}$. Each node $\nu \in \mathcal{N}$ has a predecessor and an associate period $i(\nu)$, and represents a realization of the uncertainties over this period. 1 is the index of the tree root, which represents the actual value of the uncertainties. Each node ν has an associate probability π_ν such that:

$$\sum_{\forall\, \nu \,|\, i(\nu)=\widetilde{i}} \pi_\nu = 1 \quad \widetilde{i} = 1, 2, \ldots, n_i$$

$\mathcal{H}(\nu) := \{1, \ldots, \nu^-, \nu\}$ path of nodes from root to ν

Stochastic medium-term power planning 2

The formulation of the stochastic medium-term minimum cost planning over an scenario tree would be:

$$\begin{aligned}
\underset{x_j^\nu}{\text{minimize}} \quad & \sum_{\nu \in \mathcal{N}} \pi_\nu \sum_{j=0}^{n_u} \widetilde{f}_j x_j^\nu \\
\text{subject to:} \quad & \sum_{j \in U} x_j^\nu \leq \widehat{e}^{i(\nu)} - w^{i(\nu)}(U) \quad \forall U \subset \Omega^{i(\nu)} \quad \forall \nu \in \mathcal{N} \\
& \sum_{j=0}^{n_u} x_j^\nu = \widehat{e}^{i(\nu)} \quad \forall \nu \in \mathcal{N} \\
& A^{i(\nu)} x^\nu \geq r^{i(\nu)} \quad \forall \nu \in \mathcal{N} \\
& \sum_{\nu \in \mathcal{H}(\lambda)} A^{\lambda, i(\nu)} x^\nu \geq r^\lambda \quad \forall \lambda \in \mathcal{L} \quad \text{(for each leaf !!)} \\
& x_j^{i(\nu)} \geq \underline{0} \qquad j = o, 1, \ldots, n_u \quad \forall \nu \in \mathcal{N}
\end{aligned}$$

where supraindex $^\nu$ means relation with $\nu^{\underline{\text{th}}}$ node, $\mathcal{L} := \{\nu \in \mathcal{N} | \, i(\nu) = n_i\}$ is the set of *leaf* (final period) nodes, and $\mathcal{H}(\lambda)$ the path from the root to node λ. With the notation employed there is no need of *non-anticipativity* constraints.

Liberalized electricity markets

Electricity market types

There are three main types of liberalized electricity market:

- Pure pool markets, where all electricity is exchanged through a *Market Operator*. Generation Companies bid their generation, for each day and hour, to the pool. Distribution Companies bid their demand. The *market operator* clears the market. The price of the last accepted bid for each hour is the *market price* for this hour.
- Pure bilateral markets, where all electricity is traded directly between a given generation company and a given distribution company or consumer for each day and hour for a price agreed among them.
- Mixed system of pool and bilateral contracts, where part of the load is traded as a bilateral contract between generation companies and distributors or (big) consumers, and the rest is bid to the pool where a *market operator* clears the market each hour.

In all three cases a *System Operator* (responsible for the transportation network) checks the feasibility and the security of the proposed exchanges and introduces modifications to the schedule when necessary.

Types of agent in liberalized electricity markets

Regarding the ability of a generation company of provoking an increase of the market price by increasing the price-bids of its generation, we could distinguish two types of company:

- Price taker, when the company is unable to produce a change in market price, and
- Price maker, when the company is able to alter market prices. The increase of market price then comes as a consequence of biding generation at prices higher than marginal production prices while having a non-negligeable share of the generation capacity of the market (above 3%).

A market where there are several price-maker generation companies is an *oligopolistic* market.
Generation companies are no longer interested in generating at the lowest cost but in obtaining the maximum profit (revenue from the market for all accepted bids minus generation cost). There are many models for maximizing the profits of a generation company in an electricity market. They vary depending on market type and behavior, type of company, and the type of risk considered.

Medium-Term Pure-Pool Electricity Markets

Medium-term pure-pool profit maximization

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- In medium-term operation all accepted bids in a time period (a week, or a month) must match the LDC of this period for the whole market
- There is no specific load to be matched by a SGC. The only known loads are the predicted LDC's for the whole market.
- As all generation companies pursue their maximum profit, it is natural to attempt to maximize the profit of all generation companies combined, which is the problem that will be first described. It is called the *generators' surplus* maximization, and means a degree of *collusion* among producers.
- A SGC solving the generators' surplus problem may introduce its own operational constraints (fuel and emission limits, contracts, etc.) and may also introduce a *market-share constraint* for its units in one or several periods.
- The medium-term results will indicate how the SGC should program its units so that its profit be maximized while meeting all constraints.

Medium-term market-price function

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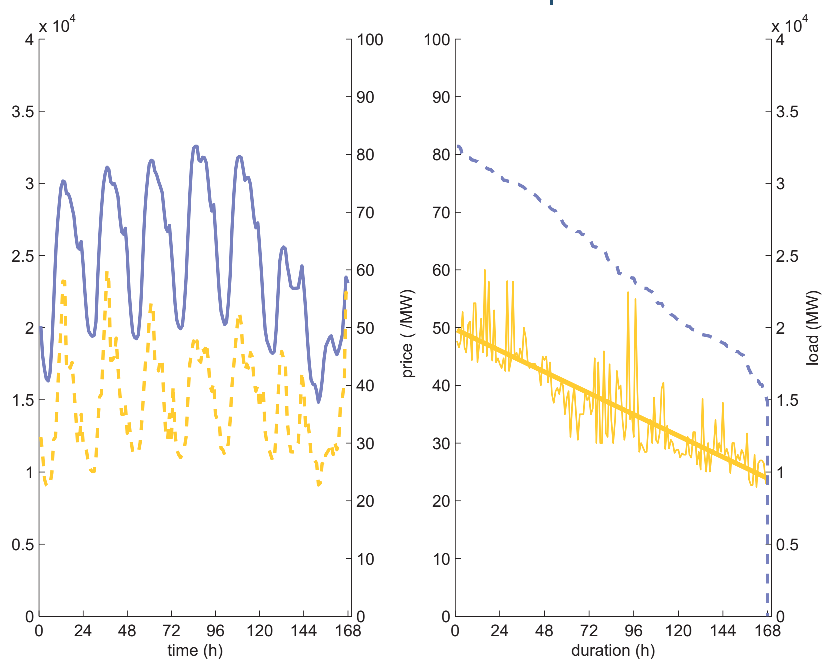
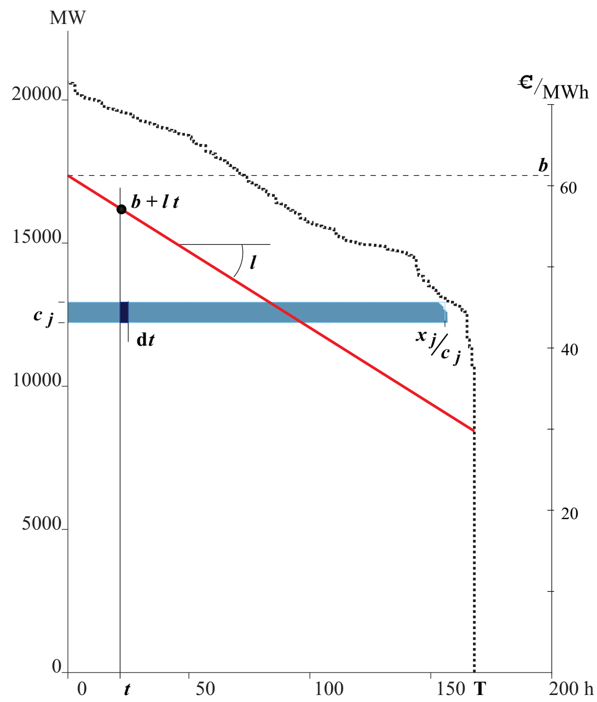
From the records of past market-price and load series it is possible to compute a market-price function for a given period. This function is to be used with expected generations that match the LDC of the period, so market prices should correspond in duration with the duration of loads, from peak to base load in the period.
The purpose of this function is to account for the fact that market price is not constant over the medium-term periods.



Medium-term market-price function 2



Medium-term market-price function w.r.t. the load duration for a time period and contribution of $j^{\underline{\text{th}}}$ unit.

Pure-pool profit maximization function

The profit (revenue minus cost) of unit j in period i will be:

$$\int_0^{x_j^i/c_j} c_j\{b^i + l^i t - \widetilde{f}_j\}\mathrm{d}t = (b^i - \widetilde{f}_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2$$

and adding for all periods and units, and taking into account the external energy, we get the profit function to be maximized:

$$\sum_i^{n_i}\left[\sum_j^{n_u}\left\{(b^i - \widetilde{f}_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2\right\} - \widetilde{f}_0 x_0^i\right]$$

which is quadratic in the generated energies. Using the load balance equation we are led to the equivalent expression:

$$\sum_i^{n_i}\left[\sum_j^{n_u}\left\{(b^i - f_j)x_j^i + \frac{l^i}{2c_j}{x_j^i}^2\right\} - \widetilde{f}_0 \widehat{e}^i\right]$$

with $f_j = \widetilde{f}_j - \widetilde{f}_0$

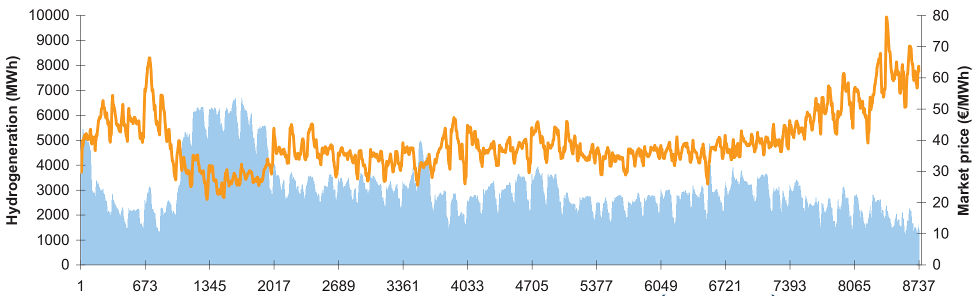
Generators' surplus (Cartel) problem

Given that $f_0\widehat{e}^i$ is a constant, the problem to be solved is:

$$\begin{aligned}
\underset{x_j^i}{\text{minimize}} \quad & \sum_i^{n_i}\sum_j^{n_u}\left\{(f_j - b^i)x_j^i - \frac{l^i}{2c_j}{x_j^i}^2\right\} \\
\text{subject to:} \quad & \sum_{j\in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i = 1,\ldots,n_i \\
& A_{\geq}^i\, x^i \geq R_{\geq}^i \qquad i = 1,\ldots,n_i \\
& \sum_i A_{\geq}^{0i}\, x^i \geq R_{\geq}^0 \\
& A_{=}^i\, x^i = R_{=}^i \qquad i = 1,\ldots,n_i \\
& \sum_i A_{=}^{0i}\, x^i = R_{=}^0 \\
& x_j^i \geq \underline{0} \qquad j = 1,\ldots,n_u, \quad i = 1,\ldots,n_i
\end{aligned}$$

Given that $l^i < 0$, the quadratic of the objective function is positive definite, thus this problem has a unique global minimizer. Moreover, the quadratic of the objective function is diagonal. A multi-scenario version of this problem could be also formulated.

Endogenous modification of the market-price function



Weekly moving average of the market price (orange) and of hydro generation (blue area) during 2007 in the Spanish power pool.

The most obvious endogenous modification of the market-price function is that due to hydro generation. It can be clearly observed from historical records that when the hydro generation level increases, market prices tend to decrease.
Given that both the peak and the base power demand prices appear to be equally affected by the hydro generation level, a linear change in the basic coefficient b^i is introduced:

$$b^i = b_0^i - c_0^i \sum_{k\in H} x_k^i$$

where $H \subset \Omega$ is the set of hydro units and b_0^i and c_0^i are positive coefficients that are estimated from past market-price and hydro generation data.

Endogenous modification of the profit maximization function

Substituting in, integrating and simplyfying the profit maximization function we obtain:

$$\sum_{i}^{n_i}\left[\sum_{j}^{n_u}\{(b_0^i - \widetilde{f}_j + \widetilde{f}_0)x_j^i - c_0^i \sum_{k\in H} x_k^i x_j^i + \frac{l^i}{2c_j} {x_j^i}^2\} - \widetilde{f}_0 \widehat{e}^i\right],$$

which is still quadratic, but its matrix is no longer diagonal and it may be indefinite for values l^i and c_0^i found in practice.

Taking $\widetilde{f}_j - \widetilde{f}_0$ as f_j and removing the constants terms from the objective function we are left with the generators' surplus problem with endogenous influence of hydro.

Pure-pool generators' surplus problem with endogenous influence of hydro

$$\begin{aligned}
\underset{x_j^i}{\text{minimize}} \quad & \sum_{i}^{n_i}\sum_{j}^{n_u}\left\{(f_j - b^i)x_j^i + c_0^i \sum_{k\in H} x_k^i x_j^i - \frac{l^i}{2c_j}{x_j^i}^2\right\} \\
\text{subject to:} \quad & \sum_{j\in U} x_j^i \le \widehat{e}^i - w^i(U) \quad \forall U \subset \Omega^i \quad i = 1,\dots,n_i \\
& A_{\ge}^i x^i \ge R_{\ge}^i \qquad i = 1,\dots,n_i \\
& \sum_i A_{\ge}^{0i} x^i \ge R_{\ge}^0 \\
& A_{=}^i x^i = R_{=}^i \qquad i = 1,\dots,n_i \\
& \sum_i A_{=}^{0i} x^i = R_{=}^0 \\
& x_j^i \ge \underline{0} \qquad j = 1,\dots,n_u, \quad i = 1,\dots,n_i
\end{aligned}$$

in whose solution it can be observed that not all available hydro generation is spent in order to keep market prices, and profits, high.

Given that this situation does not occur in the Spanish pool, a non-LMC constraint is added that forces the reservoir systems to spend all received inflows within each year.

Nash-Cournot equilibrium in electricity markets 1

- A behavioural principle different from the generators' surplus maximization, which is monopolistic on the part of the generation companies, is the oligopolistic Nash equilibrium in a game with Cournot competition type, which means a higher degree of competition than the generators' surplus maximization.
- In a Nash-Cournot equilibrium we can assume either two (the SGC and the RoP), or more players (K generation companies, whose units are $\Omega_k \,|\, \Omega := \{\Omega_1, \Omega_2, \ldots, \Omega_K\}$).
- In the Cournot model of competition we assume that the decision (generation) of one player is conditioned by the decisions (generations) of the rest of the players and that the market price is a function of the overall decisions (total expected generation).
- In a Nash equilibrium no player can increase its revenue by unilaterally changing its decision (generation).
- It is not sure that a given pool behaves more like a Nash-Cournot equilibrium than like a monopolistic generators' surplus maximization.

Nash equilibrium in game theory

Let us assume a game with K players:

- Let $\mathbf{x} = (x_1, x_2, \ldots, x_K) \in \mathcal{X}$ be a decision vector of the set $\mathcal{X}$ of decision vectors.
- Let $\phi_k(\mathbf{x})$ be the (utility) surplus function of player k
- Given the joint decision vectors $\mathbf{x} = (x_1, x_2, \ldots, x_K) \in \mathcal{X}$ and $\widetilde{\mathbf{x}} = (\widetilde{x}_1, \widetilde{x}_2, \ldots, \widetilde{x}_K) \in \mathcal{X}$, we define a new decision vector where all the companies $s \neq k$ play $\widetilde{\mathbf{x}}$, while the agent k plays $\mathbf{x}$:

$$(x_k | \widetilde{\mathbf{x}}) = (\widetilde{x}_1, \ldots, x_k, \ldots, \widetilde{x}_K) \in \mathcal{X}$$

- A point $\mathbf{x}^*$ is a Nash equilibrium point if the following holds:

$$\phi_k(\mathbf{x}^*) = \max_{(x_k | \mathbf{x}^*) \in \mathcal{X}} \phi_k(x_k | \mathbf{x}^*) \quad \forall\, k \in \{1, \ldots, K\}$$

Nash-Cournot equilibrium in electricity markets 2

For the medium-term planning to have *Cournot competition* (and an equilibrium solution) it is necessary to consider that the players mutually condition each other generations. This is so in case we consider the *endogenous model* explained, where the hydro generation of each player influences the market price.

$$\phi_k(x_k|\widetilde{\mathbf{x}}) = \sum_i^{n_i} \sum_{j\in\Omega_k^i} \Big\{ (f_j - b^i)x_j^i + c_0^i \Big[\sum_{l\in H_k} x_l^i x_j^i + \sum_{l\in H_m|m\neq k} \widetilde{x}_l^i x_j^i \Big] - \frac{l^i}{2c_j} x_j^{i\ 2} \Big\}$$

where H_m is the set of hydro generators associated to player m.
The medium-term planning is a *constrained game* because there are constraints that link the generation of several players. Many of the LMCs and the load balance equation, which includes the external energy, link the generations of different players, e.g., the load balance equations:

$$\sum_{k=1}^{K} \sum_{j\in\Omega_k^i} x_k^i + x_0^i = \widehat{e}^i$$

The NIRA algorithm to obtain the Nash-Cournot equilibrium

The Nikaido-Isoda relaxation algorithm (NIRA) (1955) is a succesive-optimization-based procedure to obtain a Nash-Cournot equilibrium point.

- The Nikaido-Isoda function is:

$$\Psi(\widetilde{\mathbf{x}}, \mathbf{x}) := \sum_{k=1}^{K} \big(\phi_k(x_k|\widetilde{\mathbf{x}}) - \phi_k(\widetilde{\mathbf{x}}) \big)$$

- An equivalent formulation of a Nash equilibrium is that $\mathbf{x}^*$ is an equilibrium point if:

$$\max_{\mathbf{x}\in\mathcal{X}} \Psi(\mathbf{x}^*, \mathbf{x}) = 0$$

- We define the *optimal response* function Z as:

$$Z(\widetilde{\mathbf{x}}) := \arg \max_{\mathbf{x}\in\mathcal{X}} \Psi(\widetilde{\mathbf{x}}, \mathbf{x})$$

- The NIRA algorithm updating rule is:

$$\mathbf{x}^{\text{new}} \leftarrow (1-u)\widetilde{\mathbf{x}} + uZ(\widetilde{\mathbf{x}}) \quad u \in \mathbb{R},\ 0 < u < 1$$

- To be shure that the NIRA procedure converges to an equilibrium point the $\Psi(\widetilde{\mathbf{x}}, \mathbf{x})$ should be *weakly convex* w.r.t. $\widetilde{\mathbf{x}}$ and *weakly concave* w.r.t. $\mathbf{x}$, and function $Z(\cdot)$ should be single valued.

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The implementation of the NIRA algorithm to obtain the Nash-Cournot equilibrium

In case of using a scenario tree with nodes ν of the set $\mathcal{N}$, $\max_{\mathbf{x}\in\mathcal{X}} \Psi(\widetilde{\mathbf{x}},\mathbf{x})$ would be equivalent to solving:

$$\begin{aligned}
\underset{x_j^\nu}{\text{minimize}} \quad & \sum_{\nu\in\mathcal{N}} \pi_\nu \sum_{k=1}^{K} \sum_{j\in\Omega_k^{i(\nu)}} \Big\{ (f_j - b^{i(\nu)})x_j^\nu + c_0^{i(\nu)} \Big[\sum_{l\in H_k} x_l^\nu x_j^\nu \\
& + \sum_{l\in H_m | m\neq k} \widetilde{x}_l^\nu x_j^\nu \Big] - \frac{l^{i(\nu)}}{2c_j} x_j^{\nu\,2} \Big\} \\
\text{subject to:} \quad & \sum_{j\in U} x_j^\nu \leq \widehat{e}^{i(\nu)} - w^{i(\nu)}(U) \quad \forall U \subset \Omega^{i(\nu)} \quad \forall \nu\in\mathcal{N} \\
& A_{\geq}^{i(\nu)} x^\nu \geq r_{\geq}^{i(\nu)} \quad \forall \nu\in\mathcal{N} \\
& \sum_{\nu\in\mathcal{H}(\lambda)} A_{\geq}^{\lambda,i(\nu)} x^\nu \geq r_{\geq}^{\lambda} \quad \forall \lambda\in\mathcal{L} \\
& A_{=}^{i(\nu)} x^\nu = r_{=}^{i(\nu)} \quad \forall \nu\in\mathcal{N} \\
& \sum_{\nu\in\mathcal{H}(\lambda)} A_{=}^{\lambda,i(\nu)} x^\nu = r_{=}^{\lambda} \quad \forall \lambda\in\mathcal{L} \\
& x_j^\nu \geq \underline{0} \quad j = 1,\ldots,n_u, \quad \forall \nu\in\mathcal{N}
\end{aligned} \tag{3}$$

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The classic NIRA algorithm to obtain the Nash-Cournot equilibrium

- $\widetilde{\mathbf{x}} \leftarrow \mathbf{x}_0\,, \quad u \leftarrow 0.7$
- **repeat**

  obtain $Z(\widetilde{\mathbf{x}}) = \mathbf{x}^*$ by solving $\max_{\mathbf{x}\in\mathcal{X}} \Psi(\widetilde{\mathbf{x}},\mathbf{x})$ as in (3)

  compute $\Psi^* = \Psi(\widetilde{\mathbf{x}}, Z(\widetilde{\mathbf{x}})) = \sum_{k=1}^{K} \big(\phi_k(x_k^* | \widetilde{\mathbf{x}}) - \phi_k(\widetilde{\mathbf{x}})\big)$

  $\widetilde{\mathbf{x}} \leftarrow uZ(\widetilde{\mathbf{x}}) + (1-u)\widetilde{\mathbf{x}}$
- **until** $\Psi^* \leq \epsilon$

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The representation of wind-power generation

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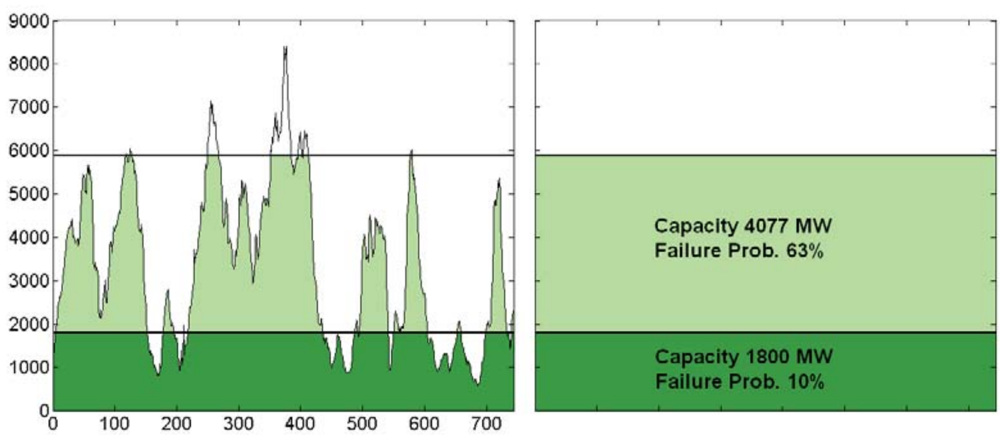
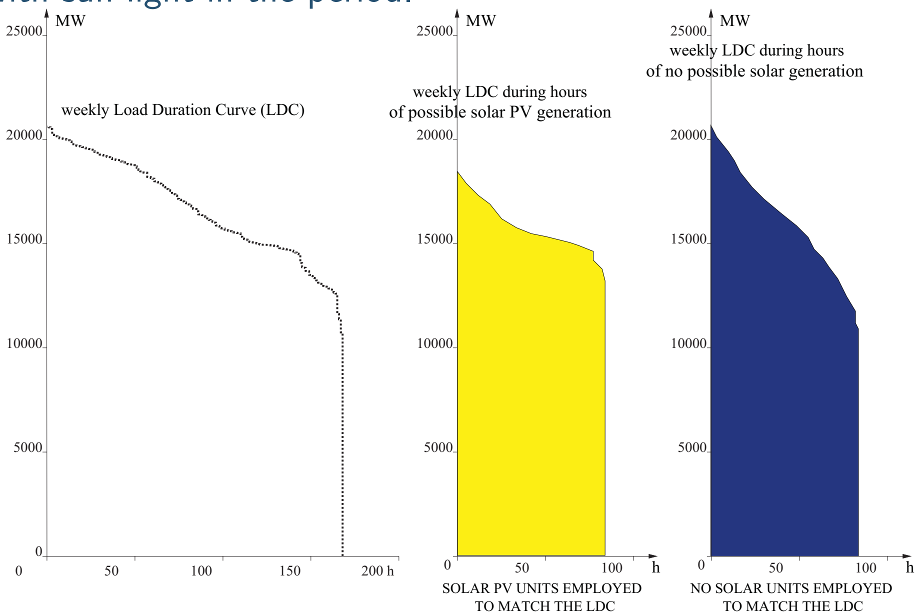
- From the wind-power series corresponding to a given time period we deduce a two-unit model that represents its wind-power generation, with parameters suitable for being employed in the matching of the period LDC.
- Two pseudounits: the base unit and the crest unit. The spikes up to 2% of wind-power energy are neglected.
- In the scenario generation the scenario tree nodes are based on base unit capacity (with fixed failure 10%). Crest units have fixed capacity and fixed failure probability for each period.



The representation of solar Photo-Voltaic (PV) generation

From the PV generation series corresponding to a given time period we deduce a two-unit model that represents its PV generation, using a two-unit model of base PV unit and crest PV unit as for wind-power generation.

An important difference with respect to wind power is that now each time period must be subdivided into two subperiods: one with no PV generation (no sun light hours), and another with it corresponding to hours with sun light in the period.



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Scenario Generation and Scenario Reduction

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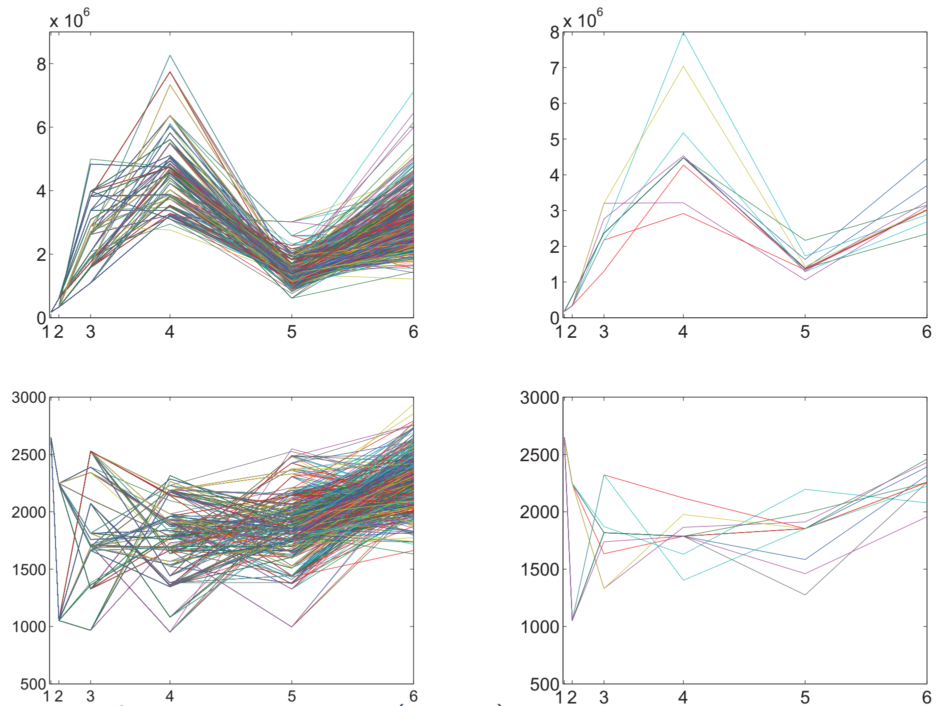
- The scenario tree is created using a mixture of multidimensional vector auto regressive model and Montecarlo methods.
- We reduce the scenario tree to the desired number of scenarios using a backward algorithm



Hydro-inflow scenario tree (above) and wind-power scenario tree

Value of change in stored water in leaves of scenario trees

It is assumed that a generation company (GenCo) will keep or it will spend a larger or lesser part of the water inflows depending on wheather the inflows are above or below its yearly average $\overline{W}_h$. The final water storage in reservoirs v^f_λ is fixed for each inflow scenario path λ.

$$v^f_\lambda = v^0 + 0,4\Big\{\big(\sum_{i\in\mathcal{H}(\lambda)} w^i_h\big) - \overline{W}_h\Big\}$$

(a 40% of inflow excess/shortage is kept/discharged)
The change, positive or negative, in stored water $v^f_\lambda - v^0$ for each path λ is valued at an average market-price value $\overline{\rho}$, so for each path there is an extra term in the objective function that corresponds to the value of the change in the stored water:

$$\overline{\rho} \times 0,4\Big\{\big(\sum_{i\in\mathcal{H}(\lambda)} w^i_h\big) - \overline{W}_h\Big\}$$

Medium-Term Mixed-System Electricity Markets

Definition and principles of bilateral contracts in medium-term power planning

- A bilateral contract (BC) is an agreement between two parties, normally a SGC and a big consumer or distributor, to supply or exchange electric power under a set of specified conditions such as power (MW), energy amount (MWh), time of delivery, duration and price. Bilateral contracts can take the form of futures and forward contracts.
- One of the most extended electricity market types is the mixed market with pool auction and BCs. In it, generation companies may have BCs to supply energy in given amounts and instants, and they bid the remaining available generation capacity to the pool Market Operator to get extra benefits.
- We will not pay attention here to the procedure for reaching BC agreements (auctions, capacity rentals, ...), and we will only care that these BC agreements are honored. No account will be taken of the revenues from the BC agreements in our optimization since their supply produces a previously-known fixed revenue.
- It will be assumed that information of total system load (from the System Operator) and of total market load (from the Market Operator) are available. Subtracting the market load from the system load we get the load supplied through BCs.

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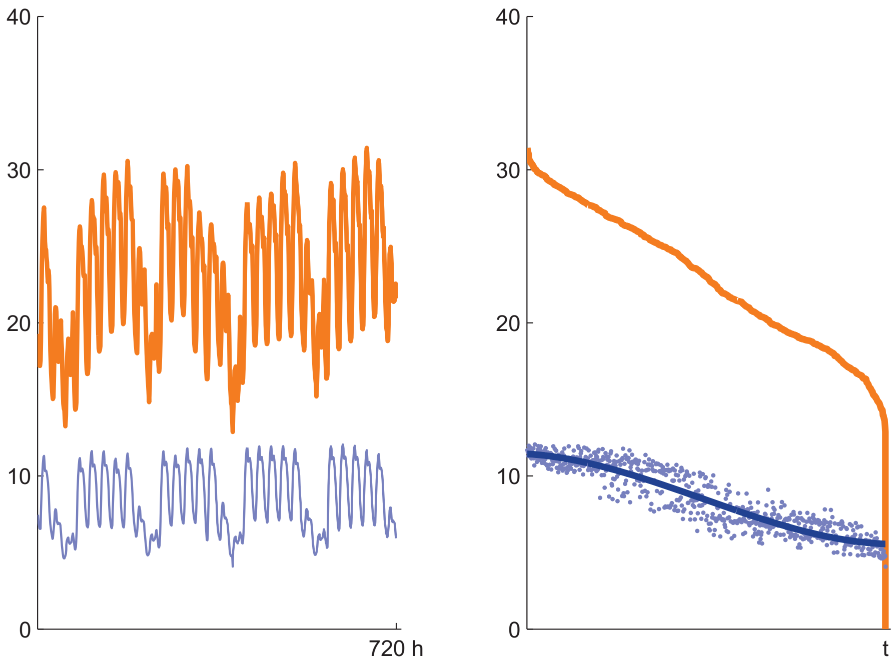
SGC gen. for BCs & Mk.
M.t. pl. with BCs
Hyd.-to-market lim. cntr.
Pr. equil. mix. elec. mkt.
Comput. results & Conc.

- It is here assumed that, from current and past records of system and market load, acceptable predictions of load duration curves (LDCs) of system load and of BC load can be obtained, and that,
- through subtracting its own future BCs, the SGC is able to compute estimated future BC LDCs of the rest of participants (RoP) in the market, and that the SGC knows which are the technologies and capacities of the units of the RoP and has a sufficiently approximate knowledge of their generation cost and other parameters (such as the outage probability). Such information about loads and other generators' units is available at the Spanish Power Pool.
- In such conditions we are able to optimize the revenue from participating in the market while satisfying the BC load, but we must see how can we model that the SGC matches its own BC LDCs in successive periods while also contributing to match the market LDCs, and the RoP match their joint BC LDC while also contributing to match the market LDCs.
- The matching of an LDC will be modelled here through the linear inequality LMCs

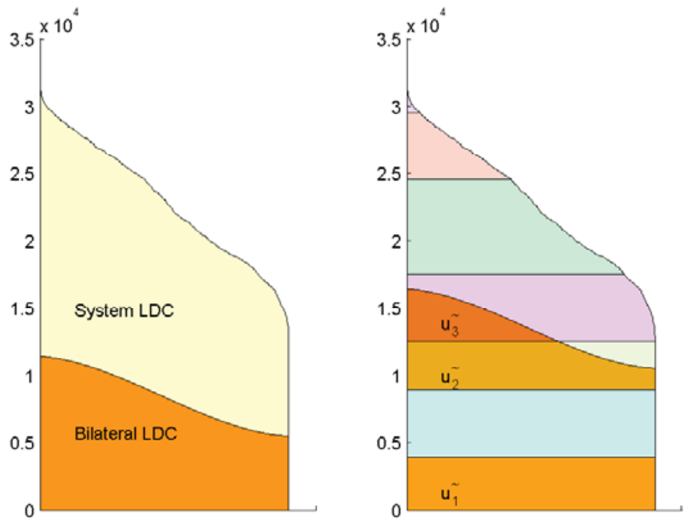
System load and bilateral contracts load in a medium-term period



Series of the system demand and energy traded through bilateral contracts during June 2007 (left). LDC, bilateral data ordered according to the LDC and non-increasing fitted polynomial (right).

The time-share hypothesis in medium-term power planning with BCs

A time-share hypothesis is made to address the problem of each unit having the possibility of matching two different LDCs over a given period.



LDC of the system and part corresponding to the bilateral contracts LDC (shaded part, left), optimal load-matching with production for bilateral contracts (right). Zero outage probabilities assumed.

Generation of SGC units for BCs and for Market

The revenue obtained from the market comes from the energy produced exceeding that devoted to the BCs. Let:
x_j: the total expected energy produced by unit j, and
$\widetilde{x}_j$: the expected energy devoted to match de BCs.
$x_j - \widetilde{x}_j$: the energy going to the market, which is paid at market price.
If we assume that the contribution of a unit has rectangular shape with height equal to its capacity, the market revenue for a unit is:

$$c_j \int_{\frac{\widetilde{x}_j}{c_j}}^{\frac{x_j}{c_j}} (b + lt)\, dt = b(x_j - \widetilde{x}_j) + \frac{1}{2}\frac{l}{c_j}(x_j^2 - \widetilde{x}_j^2)$$

which is a difference of convex functions.

Note that the part of the price function integrated starts after the expected time $\frac{\widetilde{x}_j}{c_j}$ devoted to generate for the BCs, where $\widetilde{x}_j \le x_j$ stands for the energy generated by SGC unit j for the SGC BCs. The same type of revenue function applies to the units of the RoP using their generation $\breve{x}_k \le x_k$ for the RoP BCs.
The costs incurred are:

- the generation costs for the whole generation x_j, and
- the cost of the external generation.

The medium-term power planning in a liberalized market with BCs

$$\begin{array}{lll}
\underset{x,\widetilde{x},\breve{x}}{\text{maximize}} & \displaystyle\sum_{i=1}^{n_i}\Big[\sum_{j\in\widetilde{\Omega}^i}\{b^i(x_j^i-\widetilde{x}_j^i)+\frac{l^i}{2c_j}(x_j^{i\,2}-\widetilde{x}_j^{i\,2})\}+ & \\
& \displaystyle\sum_{j\in\breve{\Omega}^i}\{b^i(x_j^i-\breve{x}_j^i)+\frac{l^i}{2c_j}(x_j^{i\,2}-\breve{x}_j^{i\,2})\}-\sum_{j\in\Omega}f_jx_j^i-f_0x_0^i\Big] & \\
\text{subject to} & \widetilde{x}_j^i \le x_j^i \quad j\in\widetilde{\Omega}^i & \forall i \\
& \breve{x}_j^i \le x_j^i \quad j\in\breve{\Omega}^i & \forall i \\
& \sum_{j\in\widetilde{\phi}^i}\widetilde{x}_j^i \le \widetilde{e}^i - w^i(\widetilde{\phi}^i) & \forall\,\widetilde{\phi}^i\subset\widetilde{\Omega}^i \quad \forall i \\
& \sum_{j\in\breve{\phi}^i}\breve{x}_j^i \le \breve{e}^i - w^i(\breve{\phi}^i) & \forall\,\breve{\phi}^i\subset\breve{\Omega}^i \quad \forall i \\
& \sum_{j\in\phi^i}x_j^i \le e^i - w^i(\phi^i) & \forall\,\phi^i\subseteq\Omega^i \quad \forall i \\
& \sum_{j\in\widetilde{\Omega}^i}\widetilde{x}_j^i = \widetilde{e}^i - w^i(\widetilde{\Omega}^i) & \forall i \\
& \sum_{j\in\breve{\Omega}^i}\breve{x}_j^i = \breve{e}^i - w^i(\breve{\Omega}^i) & \forall i \\
& \sum_{j\in\Omega^i}x_j^i + x_0^i = e^i & \forall i \\
& Cx \ge d & \\
& 0 \le \widetilde{x}_j^i \le \overline{\widetilde{x}}_j^i \quad j\in\widetilde{\Omega}^i & \forall i \\
& 0 \le \breve{x}_j^i \le \overline{\breve{x}}_j^i \quad j\in\breve{\Omega}^i & \forall i \\
& 0 \le x_j^i \le \overline{x}_j^i \quad j\in\Omega^i & \forall i\,,
\end{array}$$

where the o.f. is the difference of two convex functions (DC).

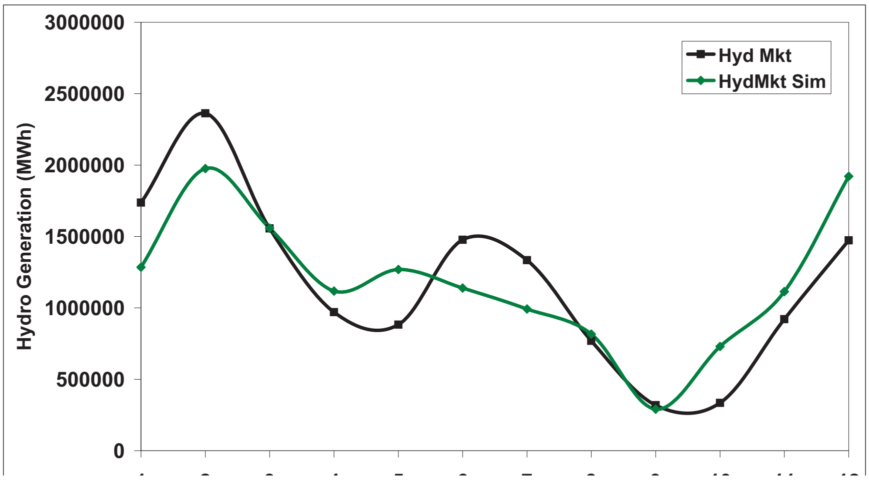
The hydro-to-market limit constraint

Contents

Both with cartel and with equilibrium behaviour GenCos would tend to conceal hydro from market by using it for BCs. This does not happen due to the regulations of the Energy Authorities.

A constraint is incorporated so that the amount of hydro generation bid in the market auction is similar to that observed in practice. The amount of recorded hydro traded in the market in each subperiod has been fit by a linear function of several parameters: the natural inflows w_h^i, the demand $\widehat{e}^i$, the stored hydro reserves $v^i = v^0 + \sum_1^i (w_h^l - x_h^l)$ and tha average market price π^i

$$\sum_{h \in H} (x_h^i - \tilde{x}_h^i) \geq \alpha w_h^i + \beta \hat{e}^i + \gamma v^i + \delta \pi^i \quad \forall i \in 1..n_i$$



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Solution procedure for finding the equilibrium in mixed electricity markets

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The expression of the utility of GenCo k has linear terms

$$b_0^i(x_j^i - \widetilde{x}_j^i) - f_j x_j^i + (x_j^i - \widetilde{x}_j^i) c_0^i \sum_{h \in \{H \setminus H_k\}} ({x'}_h^i - \widetilde{x'}_h^i)$$

(given that ${x'}_h^i$ and $\widetilde{x'}_h^i$ are here fixed), and quadratic nonconvex terms

$$\frac{l^i}{2c_j}({x_j^i}^2 - {\widetilde{x}_j^i}^2) + (x_j^i - \widetilde{x}_j^i) c_0^i \sum_{h \in H_k} (x_h^i - \widetilde{x}_h^i)$$

which, bearing in mind that c_0^i and l^i are negative, it can be decomposed as the difference of two concave (DC) functions:

$$\frac{l^i}{2c_j}{x_j^i}^2 + \frac{c_0^i}{4}\left\{x_j^i - \widetilde{x}_j^i + \sum_{h \in H_k}(x_h^i - \widetilde{x}_h^i)\right\}^2$$

$$-\left[\frac{l^i}{2c_j}{\widetilde{x}_j^i}^2 + \frac{c_0^i}{4}\left\{x_j^i - \widetilde{x}_j^i - \sum_{h \in H_k}(x_h^i - \widetilde{x}_h^i)\right\}^2\right]$$

Solution procedure for finding the equilibrium in mixed electricity markets

An alternative formulation, employed in global optimization for DC nonconvex problems is to maximize the concave part of the objective function, subject to a *reverse convex constraint* (RCC) that contains the convex part of the objective function:

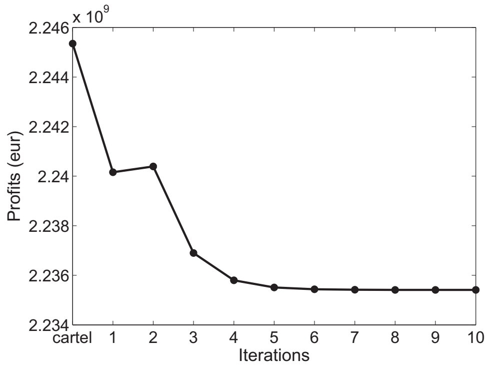
$$\begin{aligned}
\underset{x_j^i, \widetilde{x}_j^i}{\text{maximize}} \quad & \sum_{i=1}^{n_i} \Bigg\{ \sum_{j\in\Omega} \Big[b_0^i (x_j^i - \widetilde{x}_j^i) - f_j x_j^i + (x_j^i - \widetilde{x}_j^i) c_0^i \sum_{h\in\{H\setminus H_k\}} ({x'}_h^i - \widetilde{x'}_h^i) \\
& + \frac{l^i}{2c_j} {x_j^i}^2 + \frac{c_0^i}{4} \Big\{ x_j^i - \widetilde{x}_j^i + \sum_{h\in H_k} (x_h^i - \widetilde{x}_h^i) \Big\}^2 \Big] - f_0 x_0^i \Bigg\} + z \\
\text{subject to} \quad & -\sum_{i=1}^{n_i} \Bigg\{ \sum_{j\in\Omega} \Big[\frac{l^i}{2c_j} {\widetilde{x}_j^i}^2 + \frac{c_0^i}{4} \Big\{ x_j^i - \widetilde{x}_j^i - \sum_{h\in H_k} (x_h^i - \widetilde{x}_h^i) \Big\}^2 \Big] \Bigg\} - z \geq 0 \\
& \text{rest of constraints: LMCs, nonLMCs, and bounds}
\end{aligned}$$

where the explicit constraint is the RCC.
Linearizing the RCC about previously obtained points and resolving the problem could be a strategy for approaching the global optimizer.

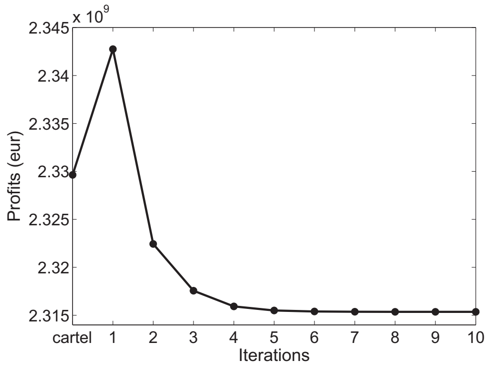
Computational Results and Conclusions

Computational results. Case study characteristics and Evolution of the ob. fn. in NIRA equilibrium solution

- Real data from the Spanish Market
- First case: 10+2 aggregated generation units (2 hydro, 2 coal, 4 fuel/gas, 1 nuclear, 1 special regime, 2 wind-power pseudounits)
- Second case: 14+2 aggregated generation units (4 hydro, 4 coal, 4 fuel/gas, 1 nuclear, 1 special regime, 2 wind-power pseudounits)
- Model implemented and solved with AMPL/IPOpt

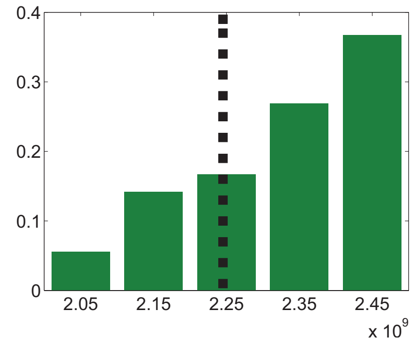


Deterministic case

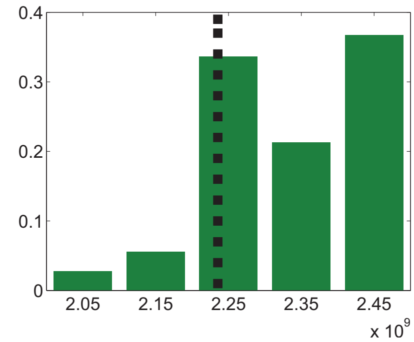


12 scenarios case

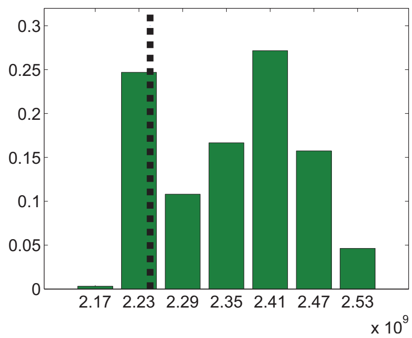
Computational results. Prob. dist of scenario profit value for cartel and equilibrium solution



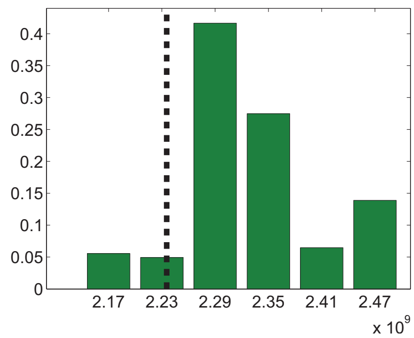
12 scenarios cartel case



12 scenarios case with equilibrium



40 scenarios cartel case



40 scenarios case with equilibrium

Blak dashed line: expected value solution.

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Bigger case studies

Case	Initial day	n_u	n_i	Scenarios	Nodes	Variables
20	17/11/2006	26	11	12	100	2700
				40	271	7317
				100	541	14607
				150	753	20331
21	17/11/2006	51	11	12	90	2430
				40	264	7128
				100	539	14553
				150	767	20709
22	23/09/2006	26	12	12	114	3078
				40	308	8316
				100	621	16767
				150	867	23409

The planning horizon is one year.

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Bigger case study results

	# scenr.	Obj. fn.	GP ite	IPOpt ite	t (sec)	
cas 20	expc. val.	1736101112	27	475	5	
	12	1773735820	46	10257	3558	
	40	1723513764	3	1841	13842	(∼4 hores)
	100	1735132059	3	4124	442240	(∼5 dies)
	150	1732223605	3	3825	584684	(∼7 dies)
cas 21	expc. val.	1620989260	50	518	14	
	12	1670364384	3	8495	6072	
	40	1620489880	3	13187	274234	(∼3 dies)
	100					
	150					
cas 22	expc. val.	2085790926	28	320	4	
	12	2143892645	39	8817	3512	
	40	1879514089	3	11738	64919	(∼18 hores)
	100	1931252082	3	7604	241300	(∼2.8 dies)
	150					

Conclusions

- A new model for a mixed market using a time-share hypothesis has been presented.
- The resulting problem has a non convex objective function.
- A Hydro-to-Market constraint is necessary.
- We found both the solution for the Cartel behaviour and Equilibrium behaviour using the Nikaido Isoda Relaxation Algorithm.
- The Equilibrium solution has profits lower than the Cartel solution, as expected.
- In the model presented, if not for the endogenous function due to hydro generation, we would not get an equilibrium solution.
- A new way to represent the wind-power generation with two pseudounits with given capacity and failure probability in each node of the scenario tree has been presented.
- No procedure that systematically obtains the best optimizer has been found yet for solving the DC mixed market power planning.

Thank you for your attention!