A DECISION SUPPORT PROCEDURE FOR THE SHORT-TERM SCHEDULING PROBLEM OF A GENERATION COMPANY OPERATING ON DAY-AHEAD AND PHYSICAL DERIVATIVES ELECTRICITY MARKETS

Maria Teresa VESPUCCI(*), Cristina CORCHERO(#), Mario INNORTA(*), F. Javier HEREDIA(#)

(*)*Department of Information Technology and Mathematical Methods*
University of Bergamo
Viale Marconi, 5, 24044 Dalmine (BG) - Italy
E-mail: maria-teresa.vespucci@unibg.it, mario.innorta@fastwebnet.it

(#)*Department of Statistics and Operations Research*
Universitat Politecnica de Catalunya
c. Jordi Girona 1-3, 08034 Barcelona - Spain
E-mail: cristina.corchero@upc.edu, f.javier.heredia@upc.edu

Abstract:

We consider a generation company operating in the liberalized electricity market, whose production system consists of hydro and thermal plants. Production is sold either directly to customers, by means of bilateral contracts, or on the spot market, where the electricity price is unknow until the market clearing process has taken place. Price risk may be hedged by financial tools provided by the Derivative Electricity Market. In this work futures contracts are considered, i.e. agreements to sell electricity in the future for a specified price. A Mixed Integer Linear Programming model is introduced for determining the unit commitment of thermal units and the dispatchment of available thermal units and hydro plants, aiming at maximizing profits. Numerical results on a case study are reported.

Keywords:

Electricity production systems, day-ahead and physical derivatives electricity markets, short-term production scheduling, decision support models, optimization.

1. INTRODUCTION

In the last decade the electric power industry has undergone a fundamental transformation from one dominated by regulated vertically integrated monopolies to an industry where electricity is produced and traded as a commodity through competitive markets, in order to enhance efficiency. In the previous monopolistic context, production resource scheduling aimed at satisfying load demand at minimum production cost, taking into account transmission security constraints; in liberalized markets each power producer aims at maximizing his own profit. Power producers can sell their production directly to consumers, on the basis of bilateral contracts. Moreover, producers and consumers may present sell and buy bids to the Day-Ahead Market (DAM) for each hour of the following day: the Market Operator determines the equilibrium point of supply and demand on the basis of the aggregated supply and demand curves and taking into account the transmission system constraints defined by the Transmission System Operator. Since the electricity price is unknown until the market clearing process has taken place, power producers face price risk. Therefore in many countries the electricity sector deregulation has been followed by the creation of Derivatives Electricity Markets (DM), that provide financial tools for reducing uncertainty and hedging risk. One such tool is the futures contract, an exchange-traded derivative that represents the agreement to sell electricity in the future for a specified price. Futures contracts can have either physical or financial settlement. Financial futures contracts have cash settlement only: they are only an exchange of money that does not affect the producer short-term operation. Physical futures contracts have cash settlement and physical delivery: it entails a quantity of energy that has to be produced mandatorily by the power producer, therefore it changes the daily operation of the units. Futures contracts can be either base or peak load. In base load futures contracts the quantity to be procured is constant in all hours of the delivery period. In peak load futures contracts there is procurement only in peak hours (from 8 am to 24 pm, Monday to Friday). In the most common products the delivery period is a year, a quarter, a month, a week or a day.

The model developed in this work refers to the Spanish DM regulation. As in the DAM, producers and other participants send their bids for futures contracts to the DM Operator who does the clearing process. In order to participate to the DM, a producer must define *virtual units*, i.e. subsets of his own generation units. Two virtual units may bid to the same futures contract only if they are independent, i.e. they do not share any generation unit. The electricity to be delivered on the basis of a futures contract cannot be purchased on the market and must be produced by generation units belonging to the virtual units assigned to it. Three days before the delivery period of a futures contract starts, the DM Operator determines, for the matched virtual units, the amount of energy to be delivered every hour and the future contract settlement price. The information is sent to both the producer and the DAM Operator, as the delivery is done by means of instrumental bids to DAM, i.e. zero-price bids, that are therefore all accepted by DAM Operator.

In this work we assume that the production system consists of hydro and thermal plants. Since electricity cannot be stored, the hourly schedule of the generation resources must take into account the hourly load demand deriving from bilateral and futures contracts. Different time horizons are considered in power production resource scheduling problem. A time horizon of at least one year (medium term) is considered when determining the optimal maintenance plans of hydro and thermal plants and the optimal weekly discharge of seasonal basins. A time horizon of a week or ten days (short term) is considered for determining the unit commitment of thermal plants, i.e. the start-up and shut-down manouvres of the available (not in maintenance) thermal plants, as well as the production levels of the committed thermal plants and of the available hydro plants in each hour. However, the unit commitment of thermal plants introduces various elements of complexity in the short-term scheduling problem (see [3] and [4]): thermal generation costs are nonlinear functions of the production level; binary variables need to be used for modeling the state (*ON*/*OFF*) of thermal plants as well as for determining the scheduling of start-up and shut-down manouvres; a large number of constraints is necessary for describing the technical characteristics of thermal plants. Solution procedures based on dynamic programming [1] have been introduced to deal with the high dimensionality of the solution space in hydro-thermal coordination problems. These procedures, however, may not guarantee an optimal solution and may just provide a suitably defined local optimum. In recent years very powerful solvers for Mixed Integer Linear Programming problems have become available, that can compute the optimal solution of instances of very large dimension. Mathematical programming models and methods have proven to be efficient tools for analysing and solving operation scheduling problems [2]. This has opened the way to the development of resource scheduling models containing very detailed descriptions of both the generation technologies owned by the producer and the market in which the producer operates (see the review in [5]).

The participation in the DAM and in the DM has been studied independently, but the inclusion of physical futures contracts in the electricity markets affects directly the unit commitment and the technical operation of the units, therefore a joint approach is needed. In this paper a Mixed Integer Linear Programming model is developed for determining the profit maximizing short-term hydro-termal scheduling model with delivery commitments from bilateral contracts and futures contracts. In Section 2 the model of the hydro system is introduced and in Section 3 the model for the unit commitment and the despatchment of the thermal units is discussed, as well as the constraints used to linearize the quadratic functions representing the thermal generation cost. The scheduling horizon is discretized in hours: T denotes the number of hours considered and t, $0 \leq t \leq T$, is the hour index, with t=0 denoting the last hour of the scheduling period immediately preceeding the one in consideration. Energy delivery commitments derive both from bilateral contracts with customers and from the futures contracts portfolio resulting from the Derivatives Market clearing process. The futures contracts considered are physical and base load, i.e. agreements to sell some constant quantity of electricity at some price with physical delivery and cash settlement. The constraints for representing the interactions with the DAM and for determining the futures contracts dispatch are discussed in Section 4. In Section 5 the objective funtion and the mathematical model are presented. Finally, in Section 6 numerical results are discussed.

2. MODEL OF THE HYDRO SYSTEM

The hydro system consists of a number of cascades, i.e. sets of hydraulically interconnected hydro plants, pumped-storage hydro plants and basins. It is mathematically represented by a directed multi-graph: water storages (basins) correspond to a set J of nodes, water flows (either power generation, or pumping, or spillage) correspond to a set I of arcs and the interconnections are represented by the arc-node incidence matrix, whose (i, j)-entry is denoted by $A_{i,j}$ ($A_{i,j}$=−1, if arc i leaves node j; $A_{i,j}$=1, if arc i enters node j; $A_{i,j}$=0, otherwise). The power producer has to determine the optimal use of the hydro resources which are available in the planning period: they are given by the initial storage volumes $v_{j,0}$ [10^3 m^3] in all basins $j \in J$ and the natural inflows $B_{j,t}$ [10^3 m^3/h] in all basins $j \in J$ and hours $1 \leq t \leq T$. The decision variables of the hydro scheduling problem are $v_{j,t}$ [10^3 m^3], the storage volume in basin j at the end of hour t, and $q_{i,t}$ [10^3 m^3/h], the water flow on arc i in hour t (turbined volume, if arc i represents generation; pumped volume, if arc i represents pumping; spilled volume, if arc i represents spillage). The mathematical relations that describe the hydro system are the model constraints (2)-(5) in Section 6. Constraints (2) require that the water flow on arc i in hour t is nonnegative and bounded above by the maximum volume $\bar{q}_i$ that can be either turbined, or pumped, or spilled on arc i. Constraints (3) require that the storage volume in basin j at the end of hour t is nonnegative and bounded above by the maximum storage volume $\bar{v}_j$. Constraints (4) impose that the storage volume in basin j at the end of hour t equals the basin storage volume at the end of hour t−1 plus inflows in hour t, taking into account the time ρ_i [h] required by the water flow leaving node i to reach node j, minus outflows in hour t. Basin inflows are natural inflows, turbine discharge from upstream hydro plants, pumped volumes from downstream hydro plants, spilled volumes from upstream basins. Basin outflows are turbine discharge to downstream hydro plants, pumped volumes to upstream hydro plants and spilled volumes to downstream basins. Finally, constraints (5) impose that the storage volume in basin j at the end of hour T is bounded below by the minimum storage volume $\underline{v}_{j,T}$ determined by the medium-term (i.e. one year) resource scheduling, in order to provide the required initial storage volume at the beginning of the next planning period. A positive energy coefficient k_i [$MWh/10^3m^3$] is associated to every arc i representing generation; the product $k_i \cdot q_{i,t}$ expresses the energy produced in hour t. A negative energy coefficient k_i is associated to every arc i representing pumping; the product $k_i \cdot q_{i,t}$ represents the energy used for pumping in hour t. Zero energy coefficients k_i are associated to arcs representing spillage.

3. MODEL OF THE THERMAL SYSTEM

The power producer owns a set K of thermal units for which he has to solve the unit commitment problem, that is to decide the subset of the thermal units which are *ON*, i.e. available for production, in every hour t. The unit commitment decision variables are the binary variables $\alpha_{k,t}$, $\beta_{k,t}$ and $\gamma_{k,t}$, for $k \in K$ and $1 \leq t \leq T$, defined in constraints (6): $\alpha_{k,t}$ and $\beta_{k,t}$ represent respectively the start-up and shut-down manouvres for unit k at hour t, while $\gamma_{k,t}$ represents the status of unit k at hour t:

- $\alpha_{k,t}=1$ [$\beta_{k,t}=1$] if unit k is to be started-up [shut-down] in hour t; $\alpha_{k,t}=0$ [$\beta_{k,t}=0$] otherwise;
- $\gamma_{k,t}=1$ if unit k is to be *ON* in hour t; $\gamma_{k,t}=0$ otherwise.

The values of the binary variables representing states in hours $t-1$ and t and manouvres in hour t must be coherent, i.e. no status change can take place without the corresponding manouvre: these restrictions are imposed by constraints (7). Information about the status of unit k at the beginning of the scheduling period are given by data $\gamma_{k,0}$ [*0/1*] and nh_k [h]: $\gamma_{k,0}=1$ and $nh_k \neq 0$ indicate that unit k is *ON* at the beginning of the scheduling period and was started-up in hour $T-nh_k$ of the previous scheduling period; $\gamma_{k,0}=nh_k=0$ indicate that unit k is *OFF* at the beginning of the scheduling period. The unit commitment must satisfy *minimum up-time constraints* (i.e. after a start-up manouvre a thermal unit must be *ON* for at least ta_k hours) and *minimum down-time constraints* (i.e. after a shut-down manouvre a thermal unit must be *OFF* for at least ts_k hours): these restrictions are imposed by constraints (8)-(11).

The producer has to decide the production level $p_{k,t}$ [MWh] of every thermal unit $k \in K$ in every hour t, $1 \leq t \leq T$, taking into account the following technical restrictions:

- if unit k is *OFF* in hour t, the hourly production $p_{k,t}$ must be zero: this is imposed by constraints (12);
- if unit k is *ON* in hour t, the hourly production $p_{k,t}$ must be neither less than the minimum level $\underline{p}_k$ [MWh] nor greater than the maximum production $\overline{p}_k$ [MWh]: this is imposed by constraints (12)-(14);
- if unit k is started-up in hour t, the hourly production $p_{k,t}$ cannot be greater than vsu_k [MWh], the maximum production at start-up; moreover, if $p_{k,t-1} \leq p_{k,t}$, the production variation is bounded above by δu_k [MWh], the maximum production increase per hour of unit k; these constraints, called *ramp-up constraints*, are imposed by (15);
- if unit k is shut-down in hour t, the hourly production $p_{k,t}$ cannot be greater than vsd_k [MWh], the maximum production of unit k at shut-down; if $p_{k,t-1} \geq p_{k,t}$, the production

variation is bounded above by δv_k [*MWh*], the maximum production decrease per hour of unit *k*; these constraints, called *ramp-down constraints*, are imposed by (16).

The input parameter $p_{k,0}$ [*MWh*] represents the production of unit *k* at the beginning of scheduling period.

Two types of costs are associated to the thermal production: costs of manouvres and generation costs. For every unit *k* csu_k [*Euro*] and csd_k [*Euro*] represent the costs associated to a start-up and to a shut-down manouvre respectively. The thermal generation cost $G_k(p_{k,t})$ of unit *k* in hour *t* is a convex quadratic function of the production level $p_{k,t}$, with coefficients $g_{2,k}$ [*Euro/MWh*^{2}], $g_{1,k}$ [*Euro/MWh*] and $g_{0,k}$ [*Euro*]. Since the mathematical model of the unit commitment problem requires binary decision variables, we linearize the quadratic thermal generation cost functions in order to obtain a Mixed Integer Linear Programming model. For every unit *k*, the interval $[\underline{p}_k, \overline{p}_k]$ is divided in *H* subintervals: let $p_{k,h-1}$ and $p_{k,h}$ denote the extreme points of subinterval *h*, $1 \le h \le H$, let $\overline{pl}_{k,h}$ denote its width and let $cl_{k,h}$ denote the slope of the straight line segment passing through points ($p_{k,h-1}$, $G_k(p_{k,h-1})$) and ($p_{k,h}$, $G_k(p_{k,h})$). The quadratic generation cost of thermal unit *k* in hour *t* is then approximated by the linear expression $cgm_k \cdot \gamma_{k,t} + \sum_{k=1}^{H} cl_{k,h} \cdot pl_{k,t,h}$, where $cgm_k = g_{2,k} \cdot \underline{p}_k^2 + g_{1,k} \cdot \underline{p}_k + g_{0,k}$ and the decision variables $pl_{k,t,h}$, $1 \le t \le T$, are subject to constraints (13) and (14). A more detailed description of the linear approximation of the quadratic thermal generation costs can be found in [6].

4. FUTURES CONTRACTS DISPATCH AND MARKET CONSTRAINTS

Let *F* denote the set of futures contracts assigned to the power producer and let *f* denote the futures contract index. Let I_f and K_f denote the subset of hydro plants and the subset of thermal units, respectively, assigned to futures contract *f*. Let L_f denote the constant quantity of electricity to be delivered every hour and let λ_f denote the futures contract settlement price. Decision variables of the futures contracts dispatch problem are $gh_{i,t,f}$ [*MWh*] and $gt_{k,t,f}$ [*MWh*], subject to the nonnegativity constraints (17)-(18), that denote the amount of energy produced by hydro plant *i* and by thermal unit *k*, respectively, for delivery in hour *t* related to futures contract *f*. The energy quantity L_f must be produced only by the units assigned to futures contract *f*: this restriction is imposed by constraints (19). For all hydro plants *i* that belong to at least one set I_f, the total production used for futures contract delivery in hour *t* must not exceed the total hourly production $k_i \cdot q_{i,t}$ of the hydro plant: these restrictions are imposed by constraints (20), where $F_i \subseteq F$ denotes the subset of futures contracts in which hydro plant *i* participates. For all thermal units *k* that belong to at least one set K_f, the total production used for futures contract delivery in hour *t* must not exceed the total hourly production $p_{k,t}$ of the thermal unit: these restrictions are imposed by constraints (21), where $F_k \subseteq F$ denotes the subset of futures contracts in which thermal unit *k* participates.

Moreover, the producer must satisfy in every hour *t* the commitments deriving from bilateral contracts. The energy car_t [*MWh*] to be delivered on the basis of bilateral contracts may be either produced or bought on the spot market. If the total production in hour *t* exceeds the load from bilateral contracts, the excess quantity $sell_t$ [*MWh*] is sold on the spot market; if his total production is less than the load from bilateral contracts, the producer has to buy on the market the amount of energy buy_t [*MWh*] necessary to meet the bilateral contract load demand. The market constraints are represented by equations (23), with the nonnegativity constraints (22) on decision variables $sell_t$ and buy_t.

5. THE OBJECTIVE FUNCTION AND THE SCHEDULING MODEL

The profit, to be maximized over the set defined by the described constraints, is the sum of

- revenues from selling energy on the spot market at price λ_t, $1 \le t \le T$;
- revenues from the futures contracts: they are settled by differences, i.e., each futures contract has daily cash settlement of the price differences between the spot reference price λ_t and the futures settlement price λ_f, $f \in F$;
- the cost of buying energy at price $\mu_t \ge \lambda_t$;
- thermal generation costs, i.e. cost of manouvres and generation costs.

The short-term hydro-termal scheduling model with delivery commitments from bilateral contracts and futures contracts is as follows.

For $1 \le t \le T$ find values of decision variables $sell_t$, buy_t, $q_{i,t}$, for $i \in I$, $gh_{i,t,f}$, for $i \in I_f$ and $f \in F$, $v_{j,t}$, for $j \in J$, $\alpha_{k,t}$, $\beta_{k,t}$, $\gamma_{k,t}$, $p_{k,t}$, for $k \in K$, and $gt_{k,t,f}$, for $k \in K_f$ and $f \in F$, so as to

$$maximize \quad \left\{ \sum_{t=1}^{T} \left[\lambda_t \cdot sell_t + \sum_{f \in F} (\lambda_f - \lambda_t) \cdot L_f \right] + \right.$$

$$\left. - \sum_{t=1}^{T} \left[\mu_t \cdot buy_t + \sum_{k \in K} \left(csu_k \cdot \alpha_{k,t} + csd_k \cdot \beta_{k,t} + cgm_k \cdot \gamma_{k,t} + \sum_{k=1}^{H} cl_{k,h} \cdot pl_{k,t,h} \right) \right] \right\} \tag{1}$$

subject to

$$0 \le q_{i,t} \le \overline{q}_i \qquad i \in I \qquad 1 \le t \le T \tag{2}$$

$$0 \le v_{j,t} \le \overline{v}_j \qquad j \in J \qquad 1 \le t \le T \tag{3}$$

$$v_{j,t} = v_{j,t-1} + B_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t-\rho_i} \qquad j \in J \qquad 1 \le t \le T \tag{4}$$

$$\underline{v}_{j,T} \le v_{j,T} \qquad j \in J \tag{5}$$

$$\alpha_{k,t} \in \{0,1\} \qquad \beta_{k,t} \in \{0,1\} \qquad \gamma_{k,t} \in \{0,1\} \qquad k \in K \qquad 1 \le t \le T \tag{6}$$

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \qquad k \in K \qquad 1 \le t \le T \tag{7}$$

$$\gamma_{k,t} = 1 \qquad if \quad \gamma_{k,0} = 1 \quad , \qquad k \in K \qquad 1 \le t \le ta_k - nh_k \tag{8}$$

$$\sum_{\tau=t+1}^{min(t+ta_k-1,T)} \gamma_{k,\tau} \ge \alpha_{k,t} \cdot min(ta_k - 1, T - t) \qquad k \in K \quad 1 \le t \le T \tag{9}$$

$$\gamma_{k,t} = 0 \qquad if \quad \gamma_{k,0} = 0 \quad , \qquad k \in K \qquad 1 \le t \le ts_k - nh_k \tag{10}$$

$$\sum_{\tau=t+1}^{min(t+ts_k-1,T)} \gamma_{k,\tau} \le (1 - \beta_{k,t}) \cdot min(ts_k - 1, T - t) \qquad k \in K, \quad 1 \le t \le T \tag{11}$$

$$0 \le p_{k,t} \le \overline{p}_k \cdot \gamma_{k,t} \qquad k \in K \qquad 1 \le t \le T \tag{12}$$

$$0 \le pl_{k,t,h} \le \overline{pl}_{k,h} \qquad k \in K \qquad 1 \le t \le T \qquad 1 \le h \le H \tag{13}$$

$$p_{k,t} = \underline{p}_k \cdot \gamma_{k,t} + \sum_h pl_{k,t,h} \qquad k \in K \qquad 1 \le t \le T \qquad (14)$$

$$p_{k,t} - p_{k,t-1} \le \delta u_k + (vsu_k - \delta u_k) \cdot \alpha_{k,t} \qquad k \in K \qquad 1 \le t \le T \qquad (15)$$

$$p_{k,t} - p_{k,t-1} \ge -\delta d_k + (\delta d_k - vsd_k) \cdot \beta_{k,t} \qquad k \in K \qquad 1 \le t \le T \qquad (16)$$

$$gh_{i,t,f} \ge 0 \qquad i \in I_f \qquad 1 \le t \le T \qquad f \in F \qquad (17)$$

$$gt_{k,t,f} \ge 0 \qquad k \in K_f \qquad 1 \le t \le T \qquad f \in F \qquad (18)$$

$$\sum_{i \in I_f} gh_{i,t,f} + \sum_{k \in K_f} gt_{k,t,f} = L_f \qquad 1 \le t \le T \qquad f \in F \qquad (19)$$

$$\sum_{f \in F_i} gh_{i,t,f} \le k_i \cdot q_{i,t} \qquad i \in \bigcup_{f \in F} I_f \qquad 1 \le t \le T \qquad (20)$$

$$\sum_{f \in F_k} gt_{k,t,f} \le p_{k,t} \qquad k \in \bigcup_{f \in F} K_f \qquad 1 \le t \le T \qquad (21)$$

$$sell_t \ge 0 \qquad buy_t \ge 0 \qquad 1 \le t \le T \qquad (22)$$

$$\sum_{i \in I} k_i \cdot q_{i,t} + \sum_{k \in K} p_{k,t} + buy_t - sell_t = car_t \qquad 1 \le t \le T \qquad (23)$$

6. COMPUTATIONAL RESULTS

A set of computational tests has been performed in order to validate the described model. The model has been implemented in GAMS and solved by CPLEX. Real data of a generation company, with 17 thermal units and 12 hydro plants, and historical data on spot prices were used.

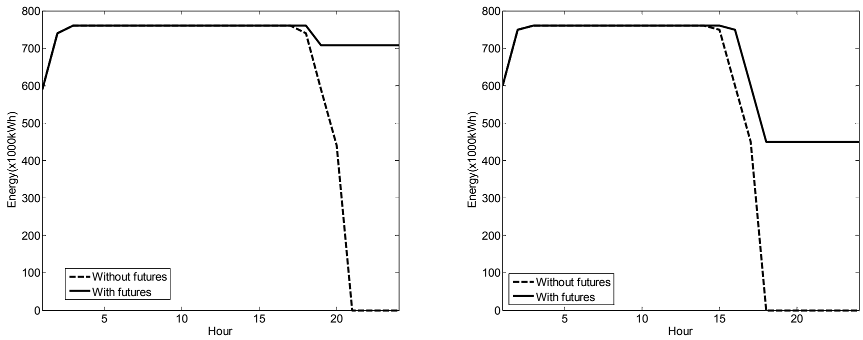

Figure 1: Production of two thermal units with and without futures

In Figure 1 the optimal production schedules of two thermal units over a 24-hour period are compared: the solid line represents the optimal schedule when delivery commitments based on futures contracts are present; the dashed line represents the optimal schedule when there are no commitments deriving from futures contracts. In Figure 2 the production allocated to futures contracts for each unit and each hour is shown: some units commit in all hours full capacity to cover deliveries on futures contracts.

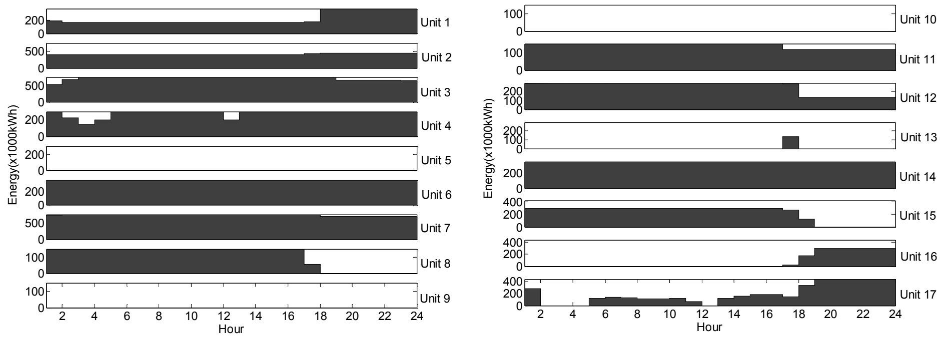

Figure 2: Unit commitment and futures economic dispatch for the thermal units

7. CONCLUSIONS

In this paper a Mixed Integer Linear Programming model has been introduced for the profit maximizing short-term resource scheduling problem of a power producer operating in the liberalized electricity market. Energy delivery commitments derive both from bilateral contracts and from the futures contracts used for hedging spot market price risk. The optimization model allows the producer to determine the optimal resourse schedule, taking into account the operational constraints of hydro and thermal plants. This work can be extended by considering the inclusion of spot price uncertainty into the model: a stochastic version of the short-term hydro-termal scheduling model can be developed, where spot price stochasticity is represented by means of a scenario tree.

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