A Three-stage Short-term Electric Power Planning Procedure for a Generation Company in a Liberalized Market ⋆

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Abstract

In liberalized electricity markets, generation companies bid their hourly generation in order to maximize their profit. The optimization of the generation bids over a short-term weekly period must take into account the action of the competing generation companies and the market-price formation rules and must be coordinated with long-term planning results. This paper presents a three stage optimization process with a data analysis and parameter calculation, a linearized unit commitment, and a nonlinear generation scheduling refinement. Although the procedure has been developed from the experience with the Spanish power market, with minor adaptations it is also applicable to any generation company participating in a competitive market system.

Key words: Mixed-Integer Programming, Nonlinear Programming, Short-Term Power Planning, Long-Term Power Planning, Liberalized Electricity Markets, Market-Price Forecasting, Electricity Generation.
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1 Introduction and Motivation

Generation companies in liberalized electricity markets do not have a load of their own to satisfy, but they must bid their hourly generation to the *market*

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operator, who selects the lowest-price among bidding companies to match the pool load. A specific generation company (SGC) expects to have most of its bids accepted, i.e., to have them priced below the *market price* determined hourly by matching the lowest price bids with the pool load.

The less efficient thermal units will not have their bids accepted at hours with lower demand (and lower market price), and this poses the problem of making feasible bids with these units, which satisfy the operating conditions of minimum service and down times. Since feasibility is a condition for bid acceptance, it is clear that a unit commitment should be within the procedure employed to prepare the generation bids. Many of the methods described in the literature for optimizing the generation bids of an SGC consider only the units of the SGC and solve a unit commitment.

The procedure proposed in [17] first finds probability distribution functions of market clearing prices for power and for reserve. From these, estimates of pairs of energy and reserve prices, their probability, price variances, and transition probabilities between pairs at adjacent hours are calculated. Then a unit commitment for the units of the SGC is formulated as a stochastic maximization of profit minus a risk term, also subject to self-scheduling requirement constraints. From this commitment, energies and reserve levels are associated with each price pair to form the energy and reserve offer curves. In [21] a unit commitment schedule is obtained by optimizing the self-commitment of each unit separately using prices obtained by a stochastic model that takes into account demand, unit reliabilities and temperature.

For an SGC owing a portfolio of units capable of altering market-clearing prices, a procedure has been proposed [6] for which it is necessary to know the price quota curve of the SGC for each hour. With this procedure, a maximum-profit linear unit-commitment problem for the SGC units can be set up and solved over a planning horizon. From its solution market prices for each hour are estimated and generation bids can be established. Should the SGC be a price taker (unable to alter the market price) for the proposal in [3] it is necessary to know the market-price probability distribution function for each hour. From this, a linear expression of the expected revenue and the expected profit can be formulated, which is maximized in a unit commitment solution for each separate unit of the price taker, and a bidding strategy is derived from the results. An extension of this procedure to account for the loss-of-profit risk, can be found in [4].

Another method considering the units of the SGC plus a unit commitment is that in [2], which relies on the prediction of a residual demand function (also called price-quota curve) for each hour, expressing the change in market price due to the generation of the SGC. From these, a nonlinear and discontinuous mixed-integer optimization problem is formulated to maximize the SGC profit.

The solution technique is a coordinate descent algorithm having mixed-integer linear optimizations as subproblems.

A different type of approach is that in [12], which formulates the profit maximization problem in order to find the best bidding strategy for a SGC in one hourly interval (no unit commitment is made). It is a two-level optimization procedure: on the first level the SGC determines its bid vector (for all generators from the utility), satisfying the constraints, and on the second level an optimal power flow determines the value of the supply vector that minimizes the total system cost. It is assumed that competitors' strategies may be well estimated through the study of past data. Load and its price depends on several nodes and load is forecasted with a multi-normal distribution. The model is improved by taking into account risk aversion using a utility function. Monte Carlo simulations and Genetic Algorithms are used to solve the problem.

The scope of the problem addressed here is no longer that of the generation units of a SGC but that of all units of all companies participating in the same competitive market, enforcing the matching of the load of the whole system. This idea was developed in [13], where a bidding strategy for multi-round auctions based on a two-level optimization procedure was proposed. At the top level a centralized economic dispatch is employed to determine the market price, the production and demand levels of all generation companies and consumers, and at the lower level a self-unit commitment is solved by each generation company to determine a profitable bid. It is assumed that each supplier has enough information about competitors so that it can run a centralized economic dispatch in designing its bidding strategy.

The new procedure presented here is for a single-round auction system. It is also based on a unit commitment solution followed by an economic dispatch, but it takes into account all participants in the generation bidding process both in the unit commitment stage and at the economic dispatch, obtaining estimates of the hourly market clearing prices and generation levels for a weekly period. Two types of prediction of the supply-bid function for every hour are employed: the first linear and static, and the second nonlinear, with the linear term being a linear function of unit generations. Long-term results [16,18], obtained previously for the week to be planned, are enforced through constraint in the short-term planning. An implicit assumption also is that a SGC, besides its own generation portfolio, has sufficient knowledge about the generation units of the rest of the market agents. Part of this information is publicly available in the bulletins released by the System Operator. The rest of it can be approximately inferred from other public sources or by comparison with similar units. (This is so in the Spanish power pool.)

The procedure developed requires data-base queries and curve fittings for the parameter calculation and then two successive optimization processes: a unit commitment, in which the zero-priced bids of the units are determined, and a nonlinear continuous scheduling, in which the generations are refined. These processes have been coded using the modeling language *AMPL* [8] and have been solved with the mixed-integer programming package *Cplex 9.0* [5] and the nonlinear optimizer *Minos* [14].

The remainder of the paper is organized as follows. Section 2 describes the features of generation units and the coordination with long-term results. Section 3 presents the models of supply-bid function employed. In Section 4 the three-stage procedure proposed is described. Each stage is then fully detailed in a separate section: Section 5 for the data analysis and parameter preparation, Section 6 for the unit commitment stage, and Section 7 for the multi-interval economic dispatch. Section 8 follows with computational results. Finally, Section 9 presents some relevant conclusions.

2 Generation units and psudo-units in short-term planning

The generation units to be considered are:

- all thermal units of the SGC whose hourly production is to participate in the auction process during the weekly period,
- the reservoir systems of hydro production of the SGC with full detail of storage and discharge,
- the thermal units of the rest of generating companies that participate in the market auction either as single or as amalgamated pseudo-units of similar characteristics (e.g., all available nuclear units of the competitor companies could be merged into a single nuclear pseudo-unit),
- the hydro-systems of the competitor generation companies considered as one or more pseudo-thermal units.

There will be n_u thermal units, or pseudo-units considered. The eolic generation for each hour of the short-term weekly period should be predicted and deducted from the forecasted load.

2.1 Thermal units and pseudo-units

The relevant parameters of the $j^{\underline{\text{th}}}$ thermal unit are:

⋆ *maximum and minimum power capacity* $\overline{c}_j$ and $\underline{c}_j$
⋆ *linear generation cost* f_j
and should the unit be susceptible of being started up and shut down:
⋆ *minimum operation time and minimum idle time* $\underline{\text{Non}}_j$ and $\underline{\text{Noff}}_j$

⋆ *start-up cost* q_j

Let us denote by M the set of units merged into a given pseudo-unit, and let m be the index of one of the composing units. The parameters of the pseudo-unit that have been employed are:

- *maximum power capacity* $\overline{c}_M = \sum_{m\in M} \overline{c}_m$
- *minimum power capacity* $\underline{c}_M = \min\{\underline{c}_m\,,\ \forall\, m \in M\}$
- *linear generation cost* $f_M = \left(\sum_{m\in M} \overline{c}_m\, f_m\right) / \sum_{m\in M} \overline{c}_m$
  and should the pseudo-unit be susceptible of being started up and shut down:
- *minimum operation time* $\underline{\mathrm{Non}}_M = \left(\sum_{m\in M} \overline{c}_m\, \underline{\mathrm{Non}}_m\right) / \sum_{m\in M} \overline{c}_m$
- *minimum idle time* $\underline{\mathrm{Noff}}_M = \min\{\underline{\mathrm{Noff}}_m\,,\ \forall\, m \in M\}$
- *start-up cost* $q_M = \left(\sum_{m\in M} \overline{c}_m\, q_m\right) / \sum_{m\in M} \overline{c}_m$.

Variables and parameters corresponding to a certain hourly interval in the short-term period will be denoted with a supra-index ^{i} indicating the interval number. Let g_j^i be the power generation of unit j over the $i^{\underline{\text{th}}}$ interval, and $g^i = \sum_{j=1}^{n_u} g_j^i$ the total thermal generation in interval i. Let n_i be the total number of intervals considered. We will normally consider a weekly period subdivided into $n_i = 168$ hourly intervals, but shorter periods could also be considered.

2.2 The hydro-generation

The average generated hydropower h_k^i over the $i^{\underline{\text{th}}}$ interval of duration T^i (one hour) in reservoir k will be:

$$h_k^i = \frac{\rho_k \mathrm{g}}{T^i} d_k^i \widetilde{s}_k^i$$

where $\rho_k < 1$ is the efficiency of the turbine-alternator system, g is the acceleration of gravity, d_k^i is the volume of water discharged over the $i^{\underline{\text{th}}}$ interval, and $\widetilde{s}_k^i$ the equivalent water head:

$$\widetilde{s}_k^i = \mathrm{s}_{\mathrm{b}k} + \frac{\mathrm{s}_{\mathrm{l}k}}{2}(v_k^{i-1} + v_k^i) + \frac{\mathrm{s}_{\mathrm{q}k}}{3}(v_k^i - v_k^{i-1})^2 + \mathrm{s}_{\mathrm{q}k} v_k^{i-1} v_k^i + \frac{\mathrm{s}_{\mathrm{c}k}}{4}({v_k^{i-1}}^2 + {v_k^i}^2)(v_k^{i-1} + v_k^i) \tag{1}$$

where v_k^i is the volume of water in reservoir k at the end of the $i^{\underline{\text{th}}}$ interval. $\mathrm{s}_{\mathrm{b}k}$, $\mathrm{s}_{\mathrm{l}k}$, $\mathrm{s}_{\mathrm{q}k}$, and $\mathrm{s}_{\mathrm{c}k}$ are the basic, linear, quadratic and cubic coefficients respectively of the head to volume function, which are data to the problem.

The water balance in reservoir k over the $i^{\underline{\text{th}}}$ interval in a cascaded reservoir basin would be:

$$v_k^{i-1} + \mathrm{w}_k^i + \sum_{j \in G_k} (d_j^i + p_j^i) = v_k^i + d_k^i + p_k^i \tag{2}$$

where w_k^i is the natural inflow, G_k the set of reservoirs directly upstream of reservoir k, and p_k^i the spillage. Inflows w_k^i are forecasted.

The total hydrogeneration is

$$\begin{aligned} h^i = \sum_{k=1}^{n_k} h_k^i = \frac{\mathrm{g}}{T^i} \sum_{k=1}^{n_k} \rho_k \Big\{ \mathrm{s}_{\mathrm{b}k} + \tfrac{\mathrm{s}_{\mathrm{l}k}}{2}(v_k^{i-1} + v_k^i) + \tfrac{\mathrm{s}_{\mathrm{q}k}}{3}(v_k^i - v_k^{i-1})^2 \\ + \mathrm{s}_{\mathrm{q}k} v_k^{i-1} v_k^i + \tfrac{\mathrm{s}_{\mathrm{c}k}}{4}({v_k^{i-1}}^2 + {v_k^i}^2)(v_k^{i-1} + v_k^i) \Big\} d_k^i \end{aligned} \tag{3}$$

which is nonlinear.

The temporary evolution of water in reservoirs, over n_i successive intervals, can be modeled through a replicated hydro network [20]. The initial volume in reservoir k during the $i^{\underline{\mathrm{th}}}$ interval is the final volume of this reservoir in interval $i-1$. Discharges d_k^i and spillages p_k^i are also flows on arcs of the replicated network, and its node balances are expressed in equations such as (2). Initial and final water volumes in reservoirs in the first and last intervals respectively: v_k^0 and $\mathrm{v}_k^{n_i}$, are to be considered data for the problem. (Initial volumes are usually the current ones, and final volumes are the target values obtainable from a long-term generation planning.)

2.3 Linearization of hydro-generation

Equation (3) is a fourth-order polynomial of hydro-variables, and may make the generation optimization problem hard to solve. A simplification of the problem could be to linearize these equations in the following way: the term in braces of (3), which is the $k^{\underline{\mathrm{th}}}$ reservoir head over the $i^{\underline{\mathrm{th}}}$ interval, will be considered to be a constant $\widehat{s}_k^i$, as if the volumes $\widehat{v}_k^i$ it had been calculated with were known,

$$\widehat{s}_k^i = \mathrm{s}_{\mathrm{b}k} + \frac{\mathrm{s}_{\mathrm{l}k}}{2}(\widehat{v}_k^{i-1} + \widehat{v}_k^i) + \frac{\mathrm{s}_{\mathrm{q}k}}{3}(\widehat{v}_k^i - \widehat{v}_k^{i-1})^2 + \mathrm{s}_{\mathrm{q}k}\widehat{v}_k^{i-1}\widehat{v}_k^i + \frac{\mathrm{s}_{\mathrm{c}k}}{4}(\widehat{v}_k^{i-1\,2} + \widehat{v}_k^{i\,2})(\widehat{v}_k^{i-1} + \widehat{v}_k^i)$$

and the succession of volumes employed $\widehat{v}_k^i$, i=1, …, n_i could be a former solution, or a uniform variation from the initial v_k^0 to the final volume $v_k^{n_i}$. Thus, equations (3) become:

$$\begin{cases} h_k^i = \dfrac{\rho_k \mathrm{g} \widehat{s}_k^i}{T^i} d_k^i & \forall\, k\ ,\ \forall\, i \\ h^i = \sum_{k=1}^{n_k} h_k^i = \frac{\mathrm{g}}{T^i} \sum_{k=1}^{n_k} \rho_k \widehat{s}_k^i d_k^i & \forall\, i \end{cases} \tag{4}$$

Equations (4), linear in the discharges d_k^i, together with the balance equations (2) are the linearized hydro-generation model.

2.4 Coordination with the long-term planning results

This coordination between the long and the short term decision levels is important in order to guarantee that certain aspects of the operation that arise in the long-term level are explicitly taken into account in the short term, so that in no case the short-term planning overrides the long-term results.

Several works have addressed this problem. In [9] there is a discussion concerning the use of *primal* and *dual* coordination between annual resource allocations and short-term operation. Reneses *et al.* [19] extend the range of possibilities with the *marginal resource-valuation functions* and compare this three approaches on a case study.

The *primal-information* approach imposes the production level of each resource obtained in the long-term model to the short-term model as constraints. The *dual-information* approach makes use of the dual prices of the constraints that limit the resources. The dual prices give marginal valuations for the resources. They are incorporated into the short-term objective function, penalizing or encouraging the use of the resource. The *marginal resource-valuation functions* is a continuous valuation of a resource for a range of operating points that the company could face. The main advantages of the primal approach are that is easy to implement and that ensures the feasibility of the long-term planning. However, it does not allow deviations from the forecasted parameters. The dual approach is more flexible than the primal one, as the dual prices do not limit the generation but rather give guidelines on the type of resources to use. The main disadvantage of the dual coordination is the lack of robustness: small changes on the long-term parameters may lead to important changes on the dual prices.

Our choice is to use a primal-information type approach with a tolerance. The units and pseudo-units used in the short-term planning must also appear in the long-term method employed. The short-term planning period (usually one week) has been systematically included in the long-term planning as the first long-term interval. Then the results for the first long-term interval are passed to the short-term as constraints for the whole generation of each unit in the short-term period. Let us assume that E_j is the energy generated by the $j^{\underline{\text{th}}}$ unit over the first long-term period, as obtained by a long-term planning procedure. It is then necessary to impose that, in the short term:

$$(1-\delta)E_j \leq \sum_{i=1}^{n_i} T^i g_j^i \leq (1+\delta)E_j \qquad j = 1,\dots,n_u \tag{5}$$

where δ is a small positive tolerance that must be employed because, in long-term studies, energies obtained take into account outage probabilities, which are not considered in the short term.

Regarding hydrogeneration, although in the long term a model can be employed where a whole hydrogeneration basin is considered as a thermal unit plus a total generation constraint, a more detailed long-term hydrogeneration model where each single reservoir is accounted for is also possible [10]. The stored volumes of water of each reservoir at the end of the first long-term interval are passed as final values for the short-term problem. The initial (current) stored volumes in each reservoir are the same for long- and short-term planning.

Prior to the short-term planning, it is thus necessary to solve a long-term planning problem using the same generation units as in the short term, and using the short-term weekly period as its first interval. Long-term planning for a given utility maximizes profit (revenues at market price minus generation costs) over a yearly, or longer, long-term period [16]. Changes in operation conditions, such as the availability of certain units, or changes in fuel prices, or in demand forecasts, call for a new long-term planning solution and the subsequent short-term planning.

3 The supply-bid function

Generation bids corresponding to a certain past hour ordered by increasing price have a characteristic shape. The function giving the generation price (in €/MWh) for each MWh bid will be referred to as the *supply-bid function*. Its features do not change substantially in the different hours, the most important being that:

- Generation companies bid a considerable part of the capacity of many of their generators at zero price.
- The sum of the hourly zero-priced generation bids of all companies falls below the hourly load but amounts to a large part of the load.
- The shape of the supply-bid function in the part corresponding to positive prices has an irregular shape, which can be reasonably approximated by a polynomial of degree four or higher.

The intersection of the supply-bid function of a given hour with the demand function (in terms of price) or with the forecasted demand of this hour gives the forecasted *market price* m^i. Fig. 1 shows the supply-bid function for an hourly interval in the Spanish pool. Although there is a supply-bid function for demand for each interval, predicting it is of equivalent complexity to predicting

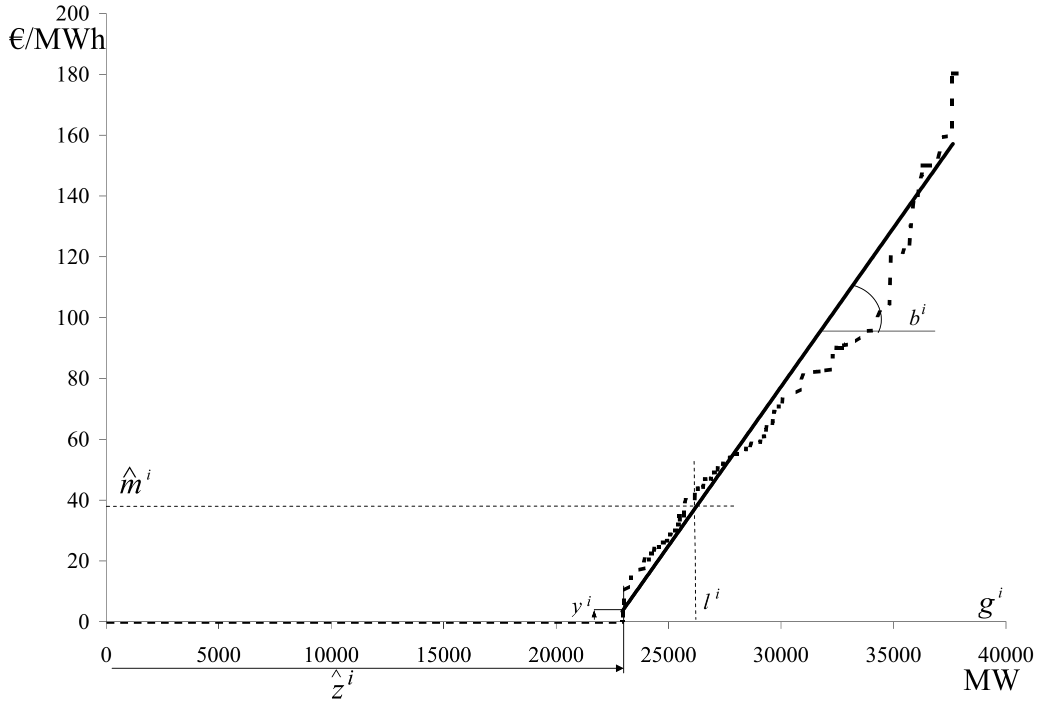

Fig. 1. Supply-bid function of the $i^{\underline{\text{th}}}$ interval corresponding to 10 to 11 a.m. on a Monday in March 2004 in the Spanish power pool (dashed curve) and linearized supply-bid function (continuous broken line).

the matched load l^i in each interval. Using the predicted matched load l^i has been preferred because it makes the model simpler.

An important variable, necessary to determine the supply-bid function, is the amount of zero-priced energy z^i in the interval. Obtaining a good estimate of z^i for each hourly interval i is one of our goals.

Generation bids and zero-priced energies can be expressed in MW, as the load l^i is, because the duration of all intervals considered is one hour.

Fig. 1 also shows the linearized supply-bid function for the $i^{\underline{\text{th}}}$ interval. It is determined by the estimates $\widehat{z}^{\,i}$ of the zero-priced energy in the interval, and by the basic and linear coefficients y^i and b^i giving the approximate linearized supply-bid function of the $i^{\underline{\text{th}}}$ interval.

The linearized supply-bid function is then:

$$\begin{cases} m^i = 0 & g^i + h^i < \widehat{z}^{\,i} \\ m^i = y^i + b^i(g^i + h^i - \widehat{z}^{\,i}) & g^i + h^i \geq \widehat{z}^{\,i} \end{cases}$$

The market-price estimate is then, for $g^i + h^i = l^i$,

$$\widehat{m}^i = y^i + b^i(l^i - \widehat{z}^{\,i}) \qquad \text{for } \; l^i \geq \widehat{z}^{\,i} \quad \text{and} \quad \widehat{z}^{\,i} = \sum_{j=1}^{n_u} \widehat{z}_j^{\;i} + \sum_{k=1}^{n_k} \widehat{z}_k^{\;i} \,. \tag{6}$$

where $\widehat{z}_j^i$ for thermal units and $\widehat{z}_k^{\;i}$ for reservoirs are to be determined. This supply-bid function will be employed in a unit commitment problem to be solved.

3.1 The nonlinear approximation to the supply-bid function

The nonlinear supply-bid function is:

$$\begin{cases} m^i = 0 & g^i + h^i < \widehat{z}^{\,i} \\ m^i = \widetilde{b}^i(g^i + h^i - \widehat{z}^{\,i}) + \gamma_q^i(g^i + h^i - \widehat{z}^{\,i})^2 + & \\ \qquad \gamma_c^i(g^i + h^i - \widehat{z}^{\,i})^3 + \gamma_t^i(g^i + h^i - \widehat{z}^{\,i})^4 & g^i + h^i \geq \widehat{z}^{\,i} \\ \text{with} & \\ \widetilde{b}^i = \beta_0^i + \sum_{j=1}^{n_u} \beta_j^i(g_j^i - \widehat{z}_j^i) & \end{cases}$$

The market-price estimate, for $g^i + h^i = l^i$, is then

$$\begin{cases} \widetilde{m}^i = \widetilde{b}^i(l^i - \widehat{z}^{\,i}) + \gamma_q^i(l^i - \widehat{z}^{\,i})^2 + \gamma_c^i(l^i - \widehat{z}^{\,i})^3 + \gamma_t^i(l^i - \widehat{z}^{\,i})^4 \\ \quad = \widetilde{b}^i(l^i - \widehat{z}^{\,i}) + \Gamma^i = \left\{\beta_0^i + \sum_{j=1}^{n_u} \beta_j^i(g_j^i - \widehat{z}_j^i)\right\}(l^i - \widehat{z}^{\,i}) + \Gamma^i \end{cases} \tag{7}$$

where the g_j^i are to be determined. This function, linear in g_j^i, is used in the refinement stage, where $\widehat{z}^{\,i}$ and $\widehat{z}_j^i$ are data. The computation of the β coefficients is explained in section 5.

Fig. 2 shows the polynomial fit and how it is changed when, through changes in g_j^i, $\widetilde{b}^i$ is increased, or decreased, by 10%.

Several comments are in order:

- $\widehat{z}^{\,i}$ in (7) is here a constant, obtained from the solution to a linearized unit commitment problem using the linearized model;
- the coefficients γ_q^i, γ_c^i and γ_t^i having been previously calculated, and l^i (the load) being a forecasted value, the only variable component of $\widetilde{m}^i$ is $\widetilde{b}^i$;
- $\widetilde{b}^i$ has a constant part β_0^i and a variable part $\widetilde{b}^i$ that changes with g_j^i through the linear expression (7); hydrogeneration is considered not to influence the slope $\widetilde{b}^i$;
- The nonlinear hydromodel (3) would be employed to compute h_k^i.

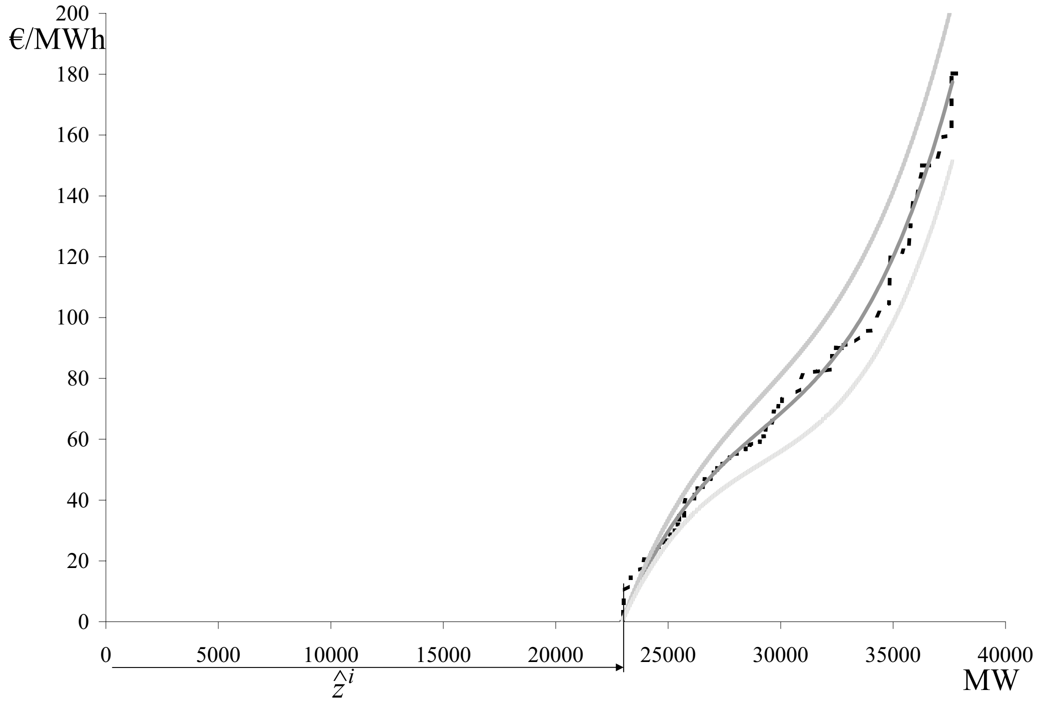

Fig. 2. Nonlinear supply-bid function for the $i^{\underline{\text{th}}}$ interval. (The original supply-bid function is the dashed curve.)

4 The three-stage procedure

The proposed short-term power planning procedure addresses the drawing up of the bidding strategy for a weekly, or shorter than weekly, period of a generation utility participating in a competitive market.

The successive stages in the solution procedure are:

⋆ the *data analysis and preparation* using records of energy bids and loads of preceeding weeks and in similar weekly periods of former years including:
  · analysis of the type of contribution of each unit considered to the bids at zero price (either as a given proportion of its power capacity, or as a given proportion of the pool hourly load),
  · fitting a polynomial to the forecast of each hourly supply-bid function,
  · determining the relative influence of the amount of power bid by each unit in the slope of the hourly supply-bid function,
  · the hourly load forecast

⋆ the *linearized unit commitment* solution using:
  · a linearized market-price estimate as in (6), where y^i and b^i are estimated in the data analysis stage, but $\widehat{z}^i$ is optimized and calculated from the unit commitment results.

  · a linearized hydro-generation function (4) in terms of the reservoir discharges for the reservoirs of the SGC,

  and finding

  · the unit commitment of all units considered,
  · the hourly zero-priced energy bids of all units and their summation $\widehat{z}^{\,i}$,
  · an initial estimate of hourly market prices $\widehat{m}^i$ and of hourly generations.

⋆ the *nonlinear scheduling refinement* using:

  · the unit commitment and the hourly zero-priced bids of the committed units or pseudo-units $\widehat{z}^{\,i}$,
  · a polynomial supply-bid function for each hour with a linear coefficient having a limited range of variation in terms of the generations of the committed units,
  · the nonlinear hydro-generation functions for the reservoirs (3) of the SGC,

  and finding

  · the hourly generation of each unit and the hourly power bids,
  · the estimates of the hourly market prices $\widetilde{m}^i$ and of the expected profit.

Ideally, the second and third stages could be a single one, in which a mixed-integer nonlinearly-constrained nonlinear problem would be solved. To date, efficient practical methods for solving such a problem are not available, so the problem is split into two separate stages, the first solving a mixed-integer linear version of the problem, and the second optimizing a continuous nonlinear scheduling refinement in which the mixed-integer solution results are employed as data.

5 The data analysis and parameter preparation stage

The experience with the auction system in the Spanish electricity market, which has been a pure pool system up to July 2006 [11], shows that there are some regular bidding patterns which, together with the smooth evolution of the demand, bring about a certain regularity in the evolution of the market price.

5.1 Required data and data sources

For each hour in the short-term period considered we must have:

- For the same hour in an equivalent period of some precceding weeks and of several former years
  · Aggregated supply-bid curves of the daily market for each unit.
  · Market clearance price.

- Forecasted load demand l^i: the interval load, to be predicted from historical records of loads in preceeding weeks and in similar weeks of former years. Time-series analysis techniques can be applied to forecast interval loads.
- For each generation unit j in the pool, the data specified in Section 2 should be known for the units of the SGC, and known or estimated for the rest of the units.

One of the criteria employed to determine the units that are merged into a pseudo-unit is type and the uniformity in the coefficients of contribution to the zero-priced energy.

In the Spanish Electricity Pool such data can be obtained from the Network Operator web site `www.ree.es`, or from the Market Operator's `www.omel.es`.

5.2 Parameters for the second stage

The 2$^{\text{nd}}$ stage is a unit commitment leading to the determination of the estimates $\widehat{z}^{\,i}$ and $\widehat{z}^i_j$, and the state u^i_j of committed units.

No previous market-price estimation is employed. Instead, a linearized supply-bid function estimation will be used. The parameters y^i and b^i: basic and linear term of linear approximation to supply-bid function, are to be previously computed by linear regression for a number of market-price curves of the same hour of similar former days and in similar weeks of former years.

The experience with the Spanish power pool shows that the proportion of zero-priced energy bids $\widehat{z}_i$ against load to be supplied l^i has generally been over 65%. Furthermore, the amount of zero-priced bid has similar patterns for several units and hours. Committed thermal units contribute to $\widehat{z}_i$ with $\chi_j * \overline{c}_j$, where χ_j is the proportion of zero-priced energy offered with respect to maximum capacity, and can be calculated from past records. Fig. 3 shows the patterns for the second week of March of two coal pseudo-units.

For hydro units the amount of zero-priced bid is proportional to the forecasted demand: $z^i_j = \lambda_j l^i$, as shown in Fig. 4.

The proportions χ_j and λ_j presented are not the same all day long. (In some hours, they are systematically higher than in others. See figures 3 and 4.) However, by splitting the hours into two subsets: *peak* and *base* we can observe a good deal of uniformity, as can be seen in the figures mentioned. Peak hours go from 9 to 23h on working days and the rest of the week are the base hours.

To summarize, the zero-priced part of the supply-bid function, $\widehat{z}^i$, is the summation of:

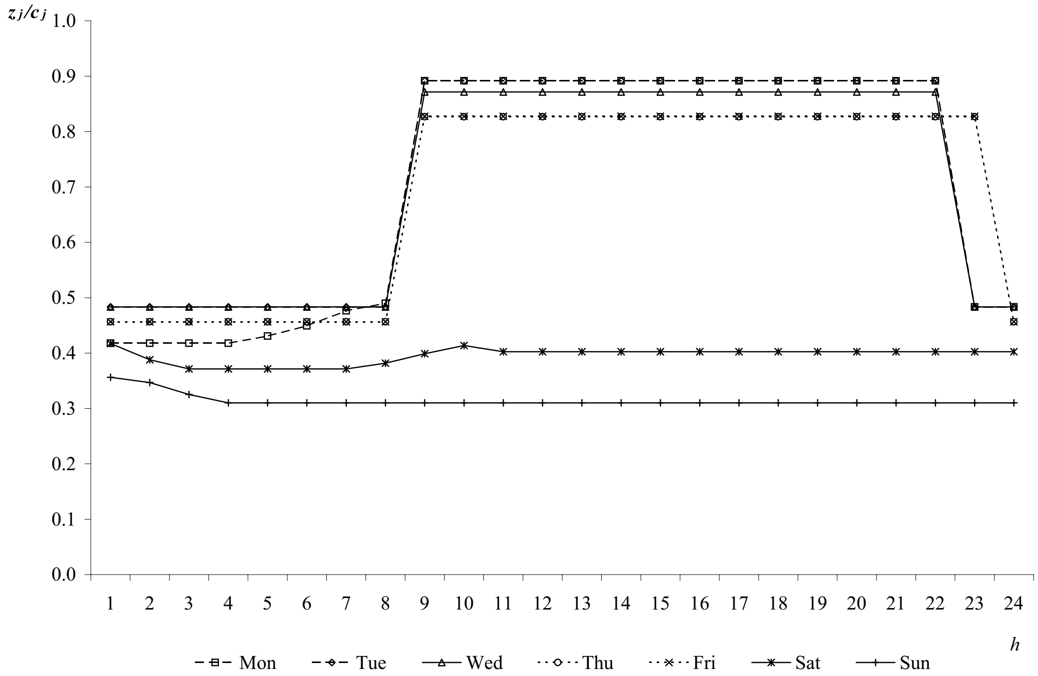

Fig. 3. Proportion of zero-priced bids with respect to maximum capacity for a coal unit in the days of a week

- *peak* hours (9-23h working days), denoted by the superscript ^{P}:
  - $z_j^i = \chi_j^P \overline{c}_j$: where j is not a hydro unit and is committed
  - $z_j^i = \lambda_j^P l^i$: where j is a hydro unit
- *base* hours (24th hour, 0-8h in working days, plus weekend), denoted by the superscript ^{B}:
  - $z_j^i = \chi_j^B \overline{c}_j$: where j is not a hydro unit and is committed
  - $z_j^i = \lambda_j^B l^i$: where j is a hydro unit.

Therefore, the following sets of parameters are to be previously determined:

- C , Λ separation of units into two subsets regarding their bid of zero-priced energy. Units in set C bid their zero-priced energy as a fixed proportion of their capacity $\overline{c}_j$, whereas units in set Λ bid theirs as a fixed proportion of the interval load l^i.
- B , P: separation of intervals into two subsets, those with low demand (*base*) and those with high demand (*peak*), given that units in subset C have different proportionality coefficients, χ_j^B and χ_j^P , with respect to capacity $\overline{c}_j$ for zero-priced bids in base and in peak intervals.
- Coefficients of contribution to zero-priced energy to be determined through queries to a data base including records of zero-priced energy bids of units:
  - λ_j^B , λ_j^P with respect to system load l^i for units in subset Λ in base and peak intervals, and

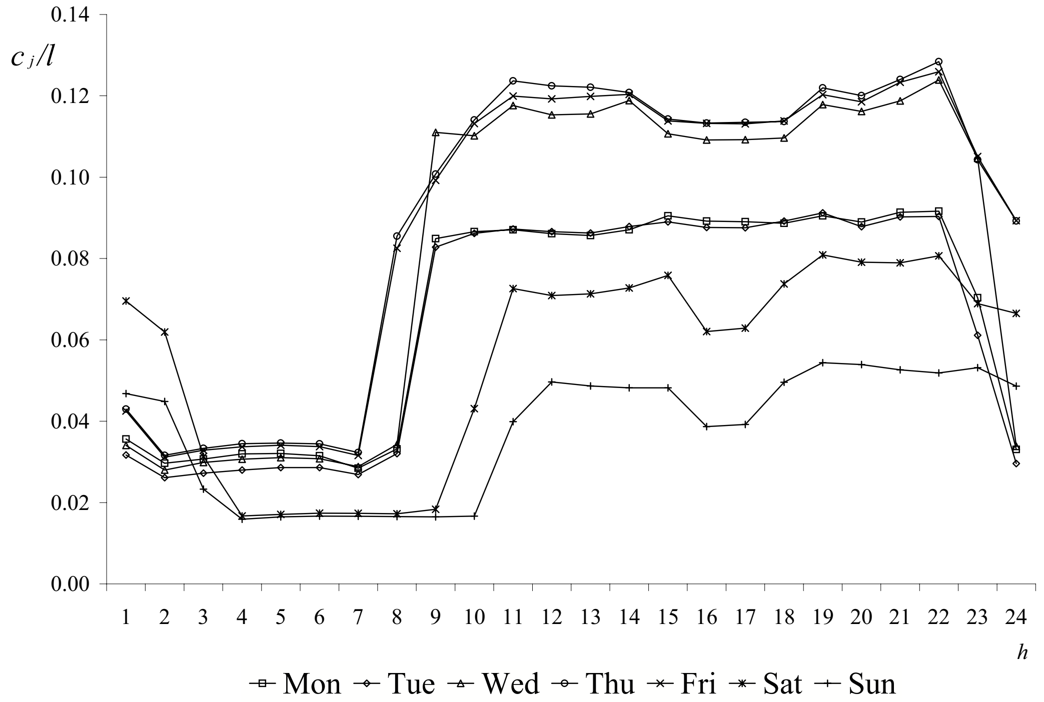

Fig. 4. Proportion of zero-priced bids with respect to the load, for a hydro unit in the days of a week

· χ_j^B , χ_j^P with respect to unit capacity $\overline{c}_j$ for units in subset C in base and peak intervals.

5.3 Parameters for the third stage

The 3rd stage is a nonlinear optimal scheduling leading to a market-price estimate $\widetilde{m}^i$ and optimal generations g_j^i and h_k^i. Again, no market-price estimation is employed. Instead, a nonlinear supply-bid function is used with a linear term that is a linear function of the available generations.

The parameters required are:

- $\widetilde{b}^i$, γ_q^i, γ_c^i, γ_t^i: linear, quadratic, cubic and quartic coefficients of polynomial fit to supply-bid function determined through nonlinear regression for a number of supply-bid curves of the same hour on similar former days.
- It is necessary to refer next to the set U^i of available units at interval i (from the unit commitment results of the 2nd stage).
- β_0^i: has been taken to be a fraction between $\frac{1}{2}\widetilde{b}^i$ in the intervals, if there are any, where all units are committed ($\sum_{j=1}^{n_u} \overline{c}_j = \sum_{j \in U^i} \overline{c}_j = \overline{C}_{\text{All}}$), and $\frac{3}{4}\widetilde{b}^i$ in the interval where $\sum_{j \in U^i} \overline{c}_j$ is a minimum: $\underline{C}_{\min} = \min_i \sum_{j \in U^i} \overline{c}_j$. For each hourly

interval, we have considered:

$$\beta_0^i = \widetilde{b}^i \left(\frac{1}{2} + \frac{\overline{C}_{\text{All}} - \sum_{j \in U^i} \overline{c}_j}{\overline{C}_{\text{All}} - \underline{C}_{\min}} \times \frac{1}{4} \right) \qquad \forall\, i \tag{8}$$

- β_j^i: indicates the contribution to the estimated slope $\widetilde{b}^i$ of the generation of the $j^{\underline{\text{th}}}$ unit above $\widetilde{z}_j^i$. The β_j^i employed take into account the unit cost f_j and are as follows:

$$\begin{cases} \beta_j^i = \alpha_i f_j \quad \text{with} \quad \alpha_i = \dfrac{1.1 \times (\widetilde{b}^i - \beta_0^i)}{\sum\limits_{j \in U^i} (\overline{c}_j - \widetilde{z}_j^i) f_j} & j \in U^i \\ \beta_j^i = 0 & j \notin U^i \end{cases} \qquad \forall\, i\,. \tag{9}$$

Using expression (9) the summation of the contribution of the available units to the estimated slope of the supply-bid function (7) will be:

$$\sum_{j \in U^i} \beta_j^i (g_j^i - \widetilde{z}_j^i) = 1.1 \times (\widetilde{b}^i - \beta_0^i) \frac{\sum\limits_{j \in U^i} (g_j^i - \widetilde{z}_j^i) f_j}{\sum\limits_{j \in U^i} (\overline{c}_j - \widetilde{z}_j^i) f_j}$$

which is reasonable, because, for all available units generating at their maximum capacity, we would have that the generation contribution to the slope would be $1.1 \times (\widetilde{b}^i - \beta_0^i)$.

Given that the slope in the supply-bid function has a direct influence in the overall profit (being maximized), in order to prevent the effect of collusion between participants, it is necessary to place a constraint on the changes in the slopes with respect to the original ones $\widetilde{b}^i$, which are a forecast deduced from available data. The type of constraint introduced is

$$-\sigma \le \sum_{i=1}^{n_i} \left\{ \widetilde{b}^i - \beta_0^i - \sum_{j \in U^i} \beta_j^i (g_j^i - \widetilde{z}_j^i) \right\} \le \sigma$$

where σ is a small tolerance.

6 The unit commitment stage

6.1 Binary variables for unit commitment and associated constraints

The formulation in this subsection uses the same type of binary variables and constraints as in [7].

Let Φ be the subset of units to be committed (started up and shut down). Let $u_j^i \in \mathbb{B}$ ($\mathbb{B}=\{0,1\}$), $j \in \Phi$ be a binary variable expressing the off–on operating status of the $j^{\underline{\text{th}}}$ thermal unit over the $i^{\underline{\text{th}}}$ interval. The generation g_j^i of the $j^{\underline{\text{th}}}$ thermal unit in the $i^{\underline{\text{th}}}$ interval will satisfy

$$\underline{c}_j\, u_j^i \le g_j^i \le \overline{c}_j\, u_j^i \qquad \forall j \quad \forall i \tag{10}$$

and the load balance equation

$$\sum_{j=1}^{n_u} g_j^i + h^i = l^i \qquad \forall i \tag{11}$$

Note that all variables u_j^i, which form the binary vector u, take part linearly (with constant coefficients) in the equations in (10).

Values of u_j^i and u_j^{i+1} must obey certain operating rules to take into account the constraints of the minimum in service time of $\underline{\text{Non}}_j$ hours and minimum idle time of $\underline{\text{Noff}}_j$ hours. It is necessary to introduce two extra binary variables e_j^i and a_j^i ($e_j^i, a_j^i \in \mathbb{B}$) for each u_j^i.

e_j^i is a start-up indicator for the $j^{\underline{\text{th}}}$ thermal unit. It is zero for $i=1,\ldots,n_i$ except when the $j^{\underline{\text{th}}}$ unit has changed from $u_j^{i-1}=0$ to $u_j^i=1$. Similarly a_j^i is a shut-down indicator for the $j^{\underline{\text{th}}}$ thermal unit. It is zero for $i=1,\ldots,n_i$ except when the $j^{\underline{\text{th}}}$ unit has changed from $u_j^{i-1}=1$ to $u_j^i=0$. With e_j^i and a_j^i it is then easy to model the minimum up and down times of unit j.

The following set of constraints:

$$\begin{cases} u_j^i - u_j^{i-1} - e_j^i + a_j^i = 0 & \forall j \in \Phi \quad \forall i \\ e_j^i + \sum_{k=i}^{\min(i+\underline{\text{Non}}_j, n_i)} a_j^{(k} \le 1 & \forall j \in \Phi \quad \forall i \\ a_j^i + \sum_{k=i+1}^{\min(i+\underline{\text{Noff}}_j, n_i)} e_j^{(k} \le 1 & \forall j \in \Phi \quad \forall i \end{cases} \tag{12}$$

uniquely define the binary variables, and force the service and the idle times to be, at least, Non_j and Noff_j.

6.2 Determination of the zero-priced energy $\widehat{z}^{\,i}$ of the $i^{\underline{\text{th}}}$ interval

As indicated in subsection 5.2, in order to properly determine $\widehat{z}^{\,i}$, we need to define several sets and subsets of thermal units and two sets of intervals. Moreover, some of the units are not subject to being committed because they are either permanently in service (e.g., a nuclear unit) or they can be started and stopped at any time and have zero minimum capacity (e.g., the hydro-generation of the competitor companies). Let A define the set of units and pseudo-units always available (Φ is the set of units to be committed).

The full definition of $\widehat{z}^{\,i}$ taking into account the binary variables u_j^i to be optimized is then:

$$\begin{cases} \widehat{z}^{\,i} = \sum\limits_{j\in(\Lambda\cap A)} l^i\,\lambda_j + \sum\limits_{j\in(C\cap A)} \overline{c}_j\,\chi_j^P + \sum\limits_{j\in(\Lambda\cap\Phi)} l^i\,\lambda_j\,u_j^i + \sum\limits_{j\in(C\cap\Phi)} \overline{c}_j\,\chi_j^P\,u_j^i & \forall\, i\in P \\ \\ \widehat{z}^{\,i} = \sum\limits_{j\in(\Lambda\cap A)} l^i\,\lambda_j + \sum\limits_{j\in(C\cap A)} \overline{c}_j\,\chi_j^B + \sum\limits_{j\in(\Lambda\cap\Phi)} l^i\,\lambda_j\,u_j^i + \sum\limits_{j\in(C\cap\Phi)} \overline{c}_j\,\chi_j^B\,u_j^i & \forall\, i\in B \end{cases} \tag{13}$$

6.3 Objective function of unit commitment and associated constraints

In order to obtain a unit commitment that maximizes profit, market price minus costs should be maximized

$$\underset{u_j^i,e_j^i}{\text{maximize}} \quad \sum_{i=1}^{n_i}\Big\{l^i\,\widehat{m}^{\,i} - \sum_{j=1}^{n_u}(f_j\,g_j + q_j\,e_j^i)\Big\} \tag{14}$$

$$\text{subject to} \quad \widehat{m}^{\,i} = y^i + b^i\,(l^i - \widehat{z}^{\,i}) \quad \forall\, i \tag{15}$$

where, from (15) and (13), the objective function (14) is linear in the binary variables u_j^i and e_j^i.

Another important constraint to be taken into account is that of coordination with long-term results (5), as explained in Subsection 2.4.

The fact that we are maximizing the overall profit of all SGCs participating in the pool, and not just the one of the SGC we are dealing with in detail, comes from the fact that we assume that all participating SGCs are trying to maximize their profit at the same time. The market-share of our SGC in the short term is dictated by constraints (5), whose E_j are a long-term result, which is where market shares are to be taken into account [16,15].

A similar form of reasoning can be applied to the effect of hydro-generation or any type of thermal generation whose fuel availability is subject to a long-term constraint (such as a take-or-pay contract). The seasonal use of hydro or this specific thermal is a long-term affair, that is rationally to be solved in long-term planning [16,15], and constraints (5) and the E_j from long-term results for the short-term weekly period take care of their best use.

It should be noticed that for pseudo-units corresponding to hydro-generation basins of competing companies E_j will be the production of these basins over the whole short-term period. For the SGC hydro-production, the long-term planning may have detailed each reservoir [15], and the long-term with short-term coordination is via the final volumes in reservoirs $v_k^{n_i}\ \forall\, k$, which were obtained as a result of the long-term planning solution.

The complete linearized unit-commitment problem can be recast as:

$$
\begin{array}{rl}
\underset{u^i_j,a^i_j,e^i_j,\widehat{z}^{\,i}_j,g^i_j,d^i_k,p^i_k,v^i_k}{\text{maximize}} & \sum_{i=1}^{n_i}\Big\{ l^i\,\widehat{m}^{\,i} - \sum_{j=1}^{n_u}(f_j\,g^i_j + q_j\,e^i_j)\Big\} \\
\text{subject to} & \widehat{m}^{\,i} = y^i + b^i\,(l^i - \widehat{z}^{\,i}) \quad \forall\, i \\
& \widehat{z}^{\,i}_j = l^i\,\lambda^P_j\ \forall\, i\in P \quad \widehat{z}^{\,i}_j = l^i\,\lambda^B_j\ \forall\, i\in B \quad \forall\, j\in(\Lambda\cap A) \\
& \widehat{z}^{\,i}_j = \overline{c}_j\,\chi^P_j\ \forall\, i\in P \quad \widehat{z}^{\,i}_j = \overline{c}_j\,\chi^B_j\ \forall\, i\in B \quad \forall\, j\in(C\cap A) \\
& \widehat{z}^{\,i}_j = l^i\,\lambda^P_j\,u^i_j \quad \forall\, i\in P \qquad \forall\, j\in(\Lambda\cap\Phi) \\
& \widehat{z}^{\,i}_j = l^i\,\lambda^B_j\,u^i_j \quad \forall\, i\in B \qquad \forall\, j\in(\Lambda\cap\Phi) \\
& \widehat{z}^{\,i}_j = \overline{c}_j\,\chi^P_j\,u^i_j \quad \forall\, i\in P \qquad \forall\, j\in(C\cap\Phi) \\
& \widehat{z}^{\,i}_j = \overline{c}_j\,\chi^B_j\,u^i_j \quad \forall\, i\in B \qquad \forall\, j\in(C\cap\Phi) \\
& \widehat{z}^{\,i} = \sum_{j=1}^{n_u}\widehat{z}^{\,i}_j \qquad \forall\, i \\
& v^{i-1}_k + \mathrm{w}^i_k + \sum_{j\in G_k}(d^i_j + p^i_j) = v^i_k + d^i_k + p^i_k \qquad \forall\, k \quad \forall\, i \\
& h^i = \dfrac{\mathrm{g}}{T^i}\sum_{k=1}^{n_k}\rho_k\widehat{s}^i_k d^i_k \qquad \forall\, i \\
& \sum_{j=1}^{n_u} g^i_j + h^i = l^i \qquad \forall\, i \\
& (1-\delta)E_j \le \sum_{i=1}^{n_i} T^i g^i_j \le (1+\delta)E_j \qquad \forall\, j \\
& e^i_j + \sum_{k=i}^{\min(i+\underline{\mathrm{Non}}_j,n_i)} a^{(k}_j \le 1 \qquad \forall\, j\in\Phi \quad \forall\, i \\
& a^i_j + \sum_{k=i+1}^{\min(i+\underline{\mathrm{Noff}}_j,n_i)} e^{(k}_j \le 1 \qquad \forall\, j\in\Phi \quad \forall\, i \\
& u^i_j - u^{i-1}_j - e^i_j + a^i_j = 0 \qquad \forall\, j\in\Phi \quad \forall\, i \\
& \underline{c}_j\,u^i_j \le g^i_j \le \overline{c}_j\,u^i_j \qquad \forall\, j\in\Phi \quad \forall\, i \\
& \underline{c}_j \le g^i_j \le \overline{c}_j \qquad \forall\, j\in A \quad \forall\, i \\
& g^i_j \ge \widehat{z}^{\,i}_j \qquad \forall\, j \quad \forall\, i \\
& u^i_j, a^i_j, e^i_j \in \mathbb{B} \qquad \forall\, j\,, \quad \forall\, i \\
& 0\le d^i_k \le \overline{d}_k \quad 0\le p^i_k\le\overline{p}_k \quad 0\le v^i_k\le\overline{v}_k \qquad \forall\, k\,, \quad \forall\, i
\end{array}
\tag{16}
$$

which is a mixed-integer linear programming problem. From its solution, only the on-off state variables u^i_j and the zero-priced bids z^i_j will be kept; the rest, including a market-price estimation $\widehat{m}^{\,i}$, and generation levels for each unit and interval, will be discarded.

7 The nonlinear scheduling stage

The nonlinearities considered in this stage come from the water head function and hydro-generation function in the reservoirs of the SGC (1, 3), and from the hourly supply-bid functions employed (7). The zero-priced energy bids $\widehat{z}^{\,i}$ obtained in the linearized unit commitment stage are here taken as a parameter, and so are the availability of units and pseudo-units according to the optimal u_j^i obtained. Let U^i be the set of available units in the $i^{\underline{\text{th}}}$ interval.

$$U^i = \{j \in A, j \in \Phi : u_j^i = 1\}$$

The nonlinear continuous problem to be solved is now

$$\begin{aligned}
\underset{g_j^i, d_k^i, p_k^i, v_k^i}{\text{maximize}} \;& \sum_{i=1}^{n_i} \Big\{ l^i \,\widetilde{m}^{\,i} - \sum_{j \in U^i} f_j \, g_j^i \Big\} \\
\text{subject to } \;& \widetilde{m}^{\,i} = \Big\{ \beta_0^i + \sum_{j=1}^{n_u} \beta_j^i (g_j^i - \widehat{z}_j^i) \Big\} (l^i - \widehat{z}^{\,i}) + \Gamma^i \quad \forall\, i \\
& -\sigma \le \sum_{i=1}^{n_i} \Big\{ \widetilde{b}^i - \beta_0^i - \sum_{j \in U^i} \beta_j^i (g_j^i - \widehat{z}_j^i) \Big\} \le \sigma \\
& v_k^{i-1} + \mathrm{w}_k^i + \sum_{j \in G_k} (d_j^i + p_j^i) = v_k^i + d_k^i + p_k^i \qquad \forall\, k \quad \forall\, i \\
& h^i = \frac{\mathrm{g}}{T^i} \sum_{k=1}^{n_k} \rho_k \Big\{ \mathrm{s}_{\mathrm{b}k} + \frac{\mathrm{s}_{\mathrm{l}k}}{2} (v_k^{i-1} + v_k^i) + \frac{\mathrm{s}_{\mathrm{q}k}}{3} (v_k^i - v_k^{i-1})^2 \\
& \qquad + \mathrm{s}_{\mathrm{q}k} v_k^{i-1} v_k^i + \frac{\mathrm{s}_{\mathrm{c}k}}{4} ({v_k^{i-1}}^2 + {v_k^i}^2)(v_k^{i-1} + v_k^i) \Big\} d_k^i \qquad \forall\, i \qquad (17) \\
& \sum_{j \in U^i} g_j^i + h^i = l^i \qquad \forall\, i \\
& (1-\delta) E_j \le \sum_{i=1}^{n_i} T^i g_j^i \le (1+\delta) E_j \qquad \forall\, j \\
& \underline{c}_j \le g_j^i \le \overline{c}_j \quad g_j^i \ge \widehat{z}_j^i \qquad \forall\, i \quad \forall\, j \in U^i \\
& g_j^i = 0 \qquad \forall\, i \quad \forall\, j \notin U^i \\
& 0 \le d_k^i \le \overline{d}_k \quad 0 \le p_k^i \le \overline{p}_k \quad 0 \le v_k^i \le \overline{v}_k \qquad \forall\, k \quad \forall\, i\,,
\end{aligned}$$

which is a nonlinear continuous optimization problem, but only due to the hydro-generation function (3), given that the objective function and the rest of the constraints are linear. From its solution, a market-price estimation $\widetilde{m}^{\,i}$, generations g_j^i, and optimal reservoir variables are obtained.

The same justification of the overall profit maximization instead of that of the SGC dealt with, as in subsection 6.3, applies here.

The preparation of a generation bid for the SGC from the results of the linearized unit commitment and from the results of the nonlinear scheduling is simple. The results of the linearized unit commitment (16) indicate which units of the SGC will be operated and when. From (13) and the optimized u_j^i we can readily establish the zero-priced bids $\widehat{z}_j^i$ of each unit at all intervals.

From the optimal generations g_j^i and the market prices $\widetilde{m}^i$ of the nonlinear scheduling solution of (17), the generation segment of the $j^{\underline{\text{th}}}$ unit in the $i^{\underline{\text{th}}}$ interval between $\widehat{z}_j^i$ and g_j^i should be bid at a price safely below $\widetilde{m}^i$ (possibly subdivided into sections at increasing price). The power of this unit higher than g_j^i should be bid at prices safely above $\widetilde{m}^i$

8 Computational results

8.1 Test cases

Each test case considers one specific generation company (SGC) in detail, while the rest of the generation units are amalgamated in big pseudo-units so as to limit n_u. Hydro-generation of the SGC considered has also been modeled in detail, using the water balances (2) and either the nonlinear hydrogeneration (3) for the nonlinear scheduling, or the linearized one (4) for the unit commitment, while the rest of hydro-generation is approximated by considering the hydro production of one or several basins as a single thermal unit with a total energy constraint determined in the long-term solution [15].

All problems have n_i=168 corresponding to the hours of a weekly period. These intervals are subdivided into 75 peak-load intervals and 93 base-load intervals.

Table 1
Test cases for AMPL models of short-term electric power planning

case	n_k	n_h	n_{th}	n_{uF}	n_{uC}	n_b	n_{uBP}	n_{uf}	week
stpcm01	4	1340	7392	5	17	8568	13	9	12
stpcm02	6	2010	11088	14	19	9576	28	5	31
stpcm03	20	6700	20496	37	24	12096	45	16	45

n_h is the total number of hydro-variables in the full hydro model. n_h, in a basin with n_k reservoirs (each having one discharge in case stpcm01), comes to $n_h = n_k \times n_i + n_k \times (n_i - 1)$. These n_h variables take part in n_i nonlinear constraints in the nonlinear scheduling refinement.

n_u is $n_u = n_{uF} + n_{uC}$, where n_{uF} are the units generating at all intervals (as nuclear units) or with possibility of generation at any interval (as the pseudo-thermal unit corresponding to a hydro-generation basin) and n_{uC}, which is the number of units to be committed at each interval, either to generation or to be idle. n_u is also decomposed as $n_u{=}n_{uBP}{+}n_{uf}$, where n_{uBP} are the units that were found to have a differentiated pattern of bidding at zero price on peak and on base hours, and n_{uf} were the units whose zero-price bidding was proportional to the load.

The size of the test cases employed is summarized in Table 1. Column $n_{th}{=}2\times n_u \times n_i$ is the number of continuous thermal variables to be determined in the unit commitment solution, and there are also n_h continuous hydro-variables. Column $n_b{=}3\times n_{uC}\times n_i$ holds the number of binary variables in the unit commitment. The last column indicates the week number in the year to which the test case refers.

In the data analysis stage, data-bases with generation bids in preceeding weeks and in weeks of former years similar to that treated in the test case were formed and used to deduce useful information. *AMPL* [8] was employed to perform the calculations of the β coefficients (8,9). The long-term energies E_j of all generation units corresponding to the short-term weekly period (imposed through constraints (5)) were obtained using the long-term planning procedure developed by the authors using an interior point algorithm to maximize a quadratic long-term profit function subject to load matching constraints using the Bloom and Gallant formulation [1]y and other non-load-matching constraints [18].

8.2 Solutions of short-term unit commitment and nonlinear scheduling

The second and third stages of the solution process were modeled using *AMPL*, and *AMPL* data files for the test cases were prepared. The second stage (unit commitment (16)) of all test cases was solved using the mixed-integer solver in the *Cplex 9.0* package [5], while the nonlinear scheduling refinement (17) of the third stage was solved using the nonlinear optimization tool *Minos 5.5* [14].

Calculations have been carried out on a Toshiba Satellite A30-303 notebook, which has a Pentium 4 processor at 3,06 GHz with 512 MB RAM.

Table 2 shows the results for the test cases prepared. The unit commitment columns show the number of nodes of the branch-and-bound tree explored until an all-integer solution was found, the number of the dual-simplex iterations required, and the duality-gap percentage, as calculated when this solution is first obtained. Requiring a lower duality gap than that obtained leads to much longer computation times without finding any different solution, i.e., once the first all-integer solution is found, there is no point waiting for another one

Table 2
Solutions of unit commitment and of nonlinear scheduling refinement

	Unit Commitment using *Cplex 9.0*				Scheduling using *Minos 5.5*		
case	nodes	dual simp. iterations	gap %	CPU (sec.)	iters.	profit ($\widetilde{m}^{\,i}$) (no s-u.) (€)	CPU (sec.)
stpcm01	10620	912010	9.97	5543	10062	93119329	19.4
stpcm02	3410	1302803	3.94	5611	47902	149138015	75.9
stpcm03	1570	616170	3.74	5718	646863	148571949	3166

with a lower duality gap, as it requires an extremely long computation.

The columns of Table 2 corresponding to the nonlinear scheduling refinement, show the iterations required, the optimal overall profit, where start-up costs have not been considered, and the CPU time. It should be noted that the large number of iterations (and CPU time) required for the nonlinear scheduling of case stpcm03 may be due to the large number of reservoirs n_k=20 considered explicitly in the nonlinear hydro constraints of (17).

The shape of the load, zero-priced bids found in the unit commitment stage, and market prices $\widehat{m}^{\,i}$ and $\widetilde{m}^{\,i}$, for each hourly interval, are shown for case stpcm01 in Figs. 5 and 6. It can be observed that the market-price estimation range of the unit commitment is higher than that of the nonlinear scheduling refinement.

It is important to note that feasibility (as enforced by the System Operator) and optimality of profit maximization (as sought by all market agents), which are the only criteria applied in the models proposed, lead to the same type of variability (*volatility*) in the market price as observed in practice.

9 Conclusions

- The solution of the short-term hydrothermal planning of the electricity generation problem in a liberalized market, as addressed by a SGC participating in it, was formulated as a three-stage process: a data analysis and parameter preparation stage using historical records of bids and loads, a unit commitment solution using a linearized supply-bid function estimation, and a nonlinear scheduling refinement using a nonlinear supply-bid function estimation. No market-price predictions are employed.
- The first stage, of data analysis and parameter preparation, calculates parameters for the next two stages from records of historical data. It requires forecasting (of loads), classification of units according to their bid-

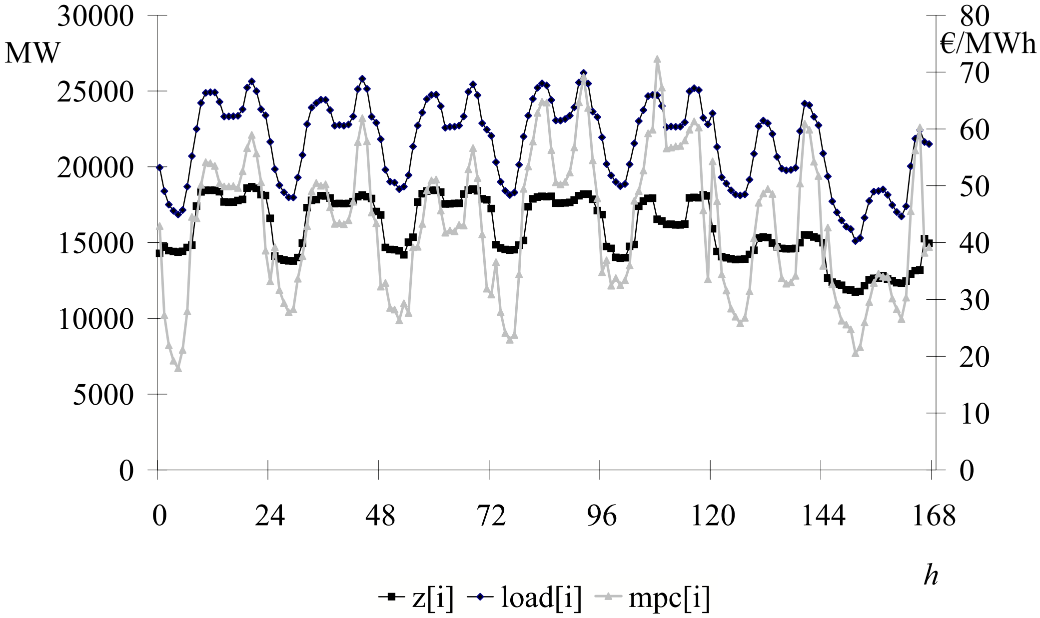

Fig. 5. Results of case stpcm01 showing load, zero-priced bids $\widehat{z}^i$ and market-price estimation $\widehat{m}^i$ (mpc) of unit commitment (grey line with $\triangle$).

ding strategies, data fitting of predicted parameters (such as those of the coefficients of the linear and polynomial approximations to the supply-bid function), and estimation of parameters (such as the γ coefficients). A data base and an *AMPL* calculation model were employed.

- The second stage calculates the unit commitment solution that maximizes the overall profit of all market participants using a linear approximation of the supply-bid function. The units committed, together with the bidding strategies, determine the zero-priced bids of all units.
- The last stage consists of a nonlinear optimization process, in which the commitment to generation and the zero-priced bids obtained in the former stage are considered to be data. The overall profits of a nonlinear supply-bid function, whose slope changes with generations, is maximized. Nonlinear hydro functions are employed.
- The coordination with long-term results is assured in the second and third stages.
- The implementation details of the solution using *AMPL* models and data files were given for the unit commitment using the mixed integer solver of *Cplex 9.0*, and the solver *Minos 5.5* for the nonlinear scheduling optimization.
- The computational experience with several real cases was reported. This includes:

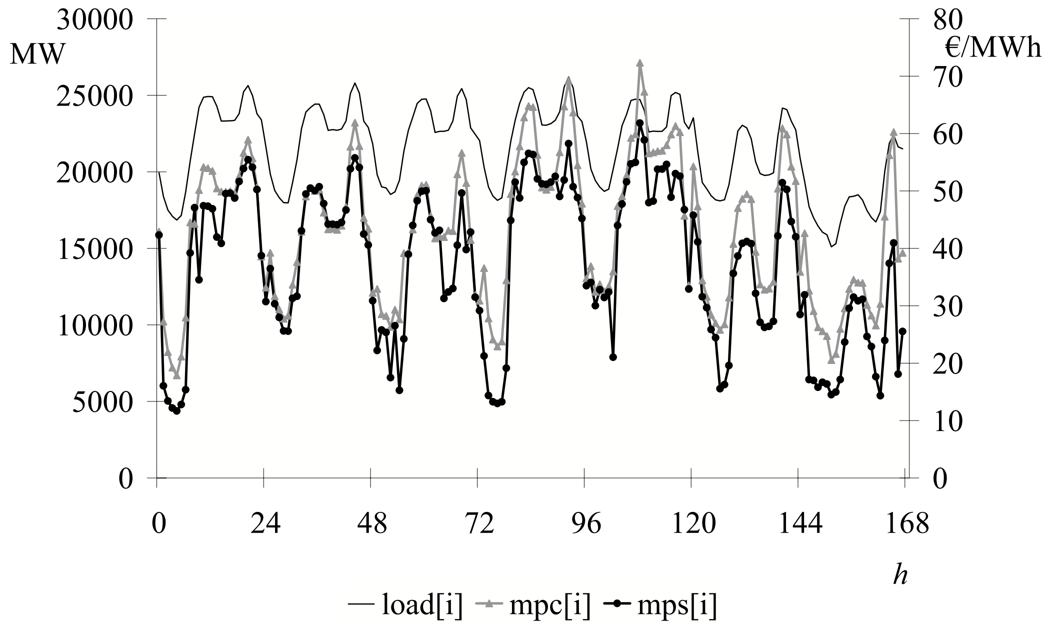

Fig. 6. Results of case stpcm01 showing load, market-price estimation $\widehat{m}^i$ of unit commitment (grey line with △), and market-price estimation $\widetilde{m}^i$ (mps) of nonlinear scheduling (black thick line with •).

  · The solution of the second-stage unit commitments giving details of branch-and-bound nodes explored, iterations, and gaps, optimal overall profit, CPU requirements, and a full graphical output of one case.
  · The solution of the third-stage nonlinear scheduling refinement giving details of iterations, optimal overall profit, CPU requirements, and a full graphical output of one case.

- The procedure presented, though complex, has a rational structure, and makes the most of the available optimization tools (linear mixed-integer programming and nonlinear continuous optimization).
- Lagrangian relaxation procedures could also be employed to solve the mixed-integer problem of the second stage in order to reduce the CPU time required. They have not been tried yet.
- The size of the resulting problems and the CPU requirements allow the practical use by an SGC of the procedure described for short-term planning.
- The preparation of the weekly generation bids follows immediately from the second and third-stage results. Shorter short-term periods (e.g., 24 or 48 h long) can also be considered.
- Some of the procedures presented are specific to the Spanish power pool, e.g., the zero-priced bidding types of units could not occur in different power pools. However, the general idea of the three-stage procedure could

be adapted to SGCs participating in any power pool.

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