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The gap between the theoretical running time and the practical performance of interior-point methods (IPMs) is one of their most noticeable features. State-of-the-art implementations, which are usually based on an infeasible IPM of complexity $O(n^2|\log \epsilon|)$, rarely need more than 100 iterations to obtain a solution, independently of the problem dimension. For instance, some difficult linear multicommodity problems of up to 1000000 variables and more than 300000 constraints are solved in Castro (2003a) with an IPM in 200 iterations and about one day of execution (the dual simplex of CPLEX required more than 35 days of execution, and the IPM of CPLEX exhausted the 1Gb of memory of the computer). Even more dramatic is the number of iterations required by an IPM for the solution of some large convex separable quadratic multicommodity problems (i.e., 1000000 variables and 30000 constraints) arising in the field of statistical data protection: solutions are obtained in less than 10 iterations, Castro (2003b).

The excellent survey of Salahi, Sotirov and Terlaky discusses self-regular IPMs (SR-IPMs), a recent family of methods that reduce the theoretical complexity of current large-update—or long-step—feasible IPMs, from $O(n \log \frac{n}{\epsilon})$ to $O(\sqrt{n} \log n \log \frac{n}{\epsilon})$. For infeasible IPMs, which are used in practice, they provide a similar running time. That suggests some questions and comments, which are not addressed in the paper.

- The authors don't provide computational experience comparing infeasible SR-IPMs with current implementations (although some references are given). Current implementations of standard IPMs are based on Mehrotra's heuristic (Mehrotra (1992)) or the higher-order Gondzio corrections (Gondzio (1996)). Is there an equivalent heuristic for SR-IPMs to reduce the number of iterations performed?
- Even if a heuristic as the above mentioned is not available, I wonder if, from the better theoretical running time for the feasible case, we can expect a less number of iterations for implementations based on SR-IPMs instead of standard IPMs. And what about the execution times: are they comparable?
- For some very large-scale problems, we can not rely on Cholesky fac-

torizations of the normal equations, and must use preconditioned conjugate gradients (PCG). Efficient preconditioners have been mainly devised for IPMs for network optimization problems (Castro (2000), Frangioni and Gentile (2004), Resende and Veiga (1993)). In these situations heuristic directions as Mehrotra's one are not effective, since they force the solution of two systems with the same matrix. The reduction in number of iterations is not worthwhile since the PCG must be applied twice (Castro (2000)). SR-IPMs may be a better alternative for this very large-scale problems that require PCG and pure (i.e., without heuristic directions) primal-dual IPMs. It seems to be worth to explore the efficiency of SR-IPMs in these situations.

References

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Since the 1970's, one of the most intriguing research question in linear optimization (LO) has been the the following: is there an algorithm which