Optimum Long-Term Hydrothermal Coordination with Fuel Limits

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Abstract—**Optimizing the thermal production of electricity in the long term, once the maintenance schedules have been decided, means optimizing both the fuel procurement policies and the use of fuels for generation in each thermal unit throughout the time period under study. A fundamental constraint to be satisfied at each interval into which the long time period is subdivided is the covering of its load duration curve with thermal and the stochastic hydrogeneration. A new procedure to optimize this problem is proposed. It is based on the use of a power-energy function for each interval, which changes with the deterministic and stochastic hydrogeneration and with thermal generation. Through this function the generation duration curves that come from the load duration curves of all intervals, are matched.**

Keywords—Hydrothermal Scheduling, Long-Term Operating Planning, Fuel Budgeting, Electricity Generation, Nonlinear Optimization

I. Introduction

The solution to the long-term hydrothermal coordination indicates how to distribute the hydroelectric generation (cost-free) in each reservoir of the reservoir system over a long period of time (e.g. one year) , so that the fuel expenditure during the period is minimized. When some thermal units can use more than one fuel or share the same fuel contract with other units, and there are fuel limits for one or more units over the whole period or parts of it, fuel acquisition and usage must also be optimized in coordination with hydrogeneration, which leads to a bigger problem. As usual, the long time period or horizon under consideration (e.g. one year) will be subdivided into several time intervals of shorter duration (e.g. one month) for which optimal values of decision variables are to be found.

The fundamental difference between long term and short term hydrothermal optimization, aside from the length of the time period studied, lies in the fact that the availability of thermal plant, the demand for electricity and the water inflows in the reservoirs are not deterministic, but only known as probability density functions.

The literature on long-term hydrothermal coordination is rich. However, only a few papers on this subject describe methods that deal with stochastic inflows and that balance thermal and hydro-generation through the load duration curve (l.d.c.) and not just with the peak load or through the total energy demand of the interval. Sherkat et al. [10] consider at each interval a staircase l.d.c. with a few load segments. These l.d.c's are peak shaved with the expected values of the generations corresponding to the releases of all reservoirs but one in turn, and optimize with dynamic programming the releases of the remaining reservoir, considering the thermal cost curves of the load segments of the l.d.c. for a series of river inflow sequences. Contaxis and Kavatza [4] optimize the stored volumes of the reservoirs with dynamic programming, obtaining for each reservoir in each interval a probability distribution function of hydrogeneration from the stochastic water inflows (while satisfying the reservoir balance equations with expected values of inflows and outflows) and convolve [11] the probability distribution functions of hydrogeneration, replacing the most expensive thermal units to cover the l.d.c. of each interval. Neither the method of Sherkat et al [10] nor that of Contaxis and Kavatza [4] deals with fuel limits.

Ranjit Kumar et al. [9] optimize the long-term fuel procurement and use with fuel limits. They use probabilistic production costing methods [1,11] with a given priority (loading order) list to determine the maximum limits on the energies generated by each unit in each interval, and then a network flow solution [5] for the entire period is used to generate a new priority list for each interval, correcting priorities according to capacity factors in the network solution. System and unit fuel limits are modified to correct the mismatches between generation and the l.d.c. This method does not consider hydrogeneration.

The work presented here describes a new model for long-term hydrothermal coordination with fuel limits. This model is to be used in hydro scheduling and fuel budgeting to minimize the cost of fuels acquired plus that of unsupplied energy over a long time period (e.g. one or two years), optimizing for each interval,

- the expected fuel supply requirements of each possible type
- the emergency energy imports to cover the uncovered load
- the quantities of fuel used for generation by each thermal unit and stored
- the volumes of water stored and discharged for generation at each reservoir (in terms of water inflow availability).

The constraints to be satisfied include the balances of fuels acquired and spent, and those of water inflows and discharges, and covering the l.d.c of each interval. The computational results described include the solution details of a real case.

This work is an extension of a former one on long-term hydrogeneration optimization [8], in which thermal generation and load covering with hydro was simplified through precalculated functions of variation of expected thermal production cost with hydrogeneration, and no fuel limits were considered. The hydro model used here is the same as in [8] regarding stochastic water inflow and stochastic hydrogeneration representation, and not much detail will be given

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here on this aspect, as these topics are fully described in [8].

It is not easy to carry out a comparison between the performances of the method proposed and that of other existing methods mentioned, due to their different characteristics. Evaluating and comparing long-term policies, obtained with different procedures, over a long-term period with stochastic events is an open problem, out of the scope of this work. The method proposed is more accurate than existing methods in the consideration of the long-term stochasticity of hydrogeneration and in the covering of the load duration curve of the intervals, without requiring excessive computational resources. Thus it can be assumed, though it cannot be proved, that the results obtained are closer to the real optimum of the long-term hydrothermal coordination than with other existing methods.

II. The L.D.C. and the G.D.C. of each Interval

There are well known procedures [1,11] to build up the probabilistic generation duration curve (g.d.c.) from the l.d.c. when only thermal units —with unlimited fuel supply— are employed. The generating capability, the forced outage rate of each unit, and a loading order list are required. From the g.d.c. we get the expected energies E_j ($j=1,\ldots,N_u$, N_u being the number of units) that the j^{th} unit will most probably generate when contributing to cover the load. We also get the non negligeable —and expensive— emergency energy E_X that will have to be imported (see figs. 1a) and 1b), where $N_u=6$).

Given that the long term time period (e.g. one year) will be subdivided into shorter time intervals (e.g. one month) and we will have a predicted l.d.c. for each interval "i", we can obtain through the g.d.c.'s the energies E_j^i ($i=1,\ldots,N_i$, N_i being the number of intervals) that the j^{th} unit will most probably generate over the i^{th} interval. Thus $\Sigma_i^{N_i} E_j^i$ will represent the expected energy to be generated over the long term period by the j^{th} unit.

Should the available fuel for the j^{th} unit be less than that necessary to generate $\Sigma_i^{N_i} E_j^i$ MWh, the g.d.c. of one or several intervals ought to be modified. The modification of the g.d.c. of one such interval "i" can take either of two forms:

• a change in the loading order list of one or several intervals so that the j^{th} unit is further down in the list and generates less [9], or

• a reduction in the power output P_j^i of the j^{th} unit below its rated capability $\overline{P}_j$, over one or several intervals "i".

Fuel limits occur naturally even when there are no fuel restrictions, but there are different contracts affecting limited amounts of the same fuel. Deciding how much to buy of each fuel under which contract (together with the use of hydropower) and allocating limited amounts of such fuel to different intervals is an optimization problem where the fuel limits described will have to be considered. Thus, in order to duly account for interval load, one of the types of g.d.c. modification will have to be used. The second one, that of the power level reduction, has been adopted here.

Optimizing hydrogeneration over the long term period means allocating hydroenergies to each interval helping to cover its l.d.c.s. The g.d.c. of each interval will thus include an "optimized" amount of hydrogeneration (together with an "optimized" amount of fuel-limited thermal generation).

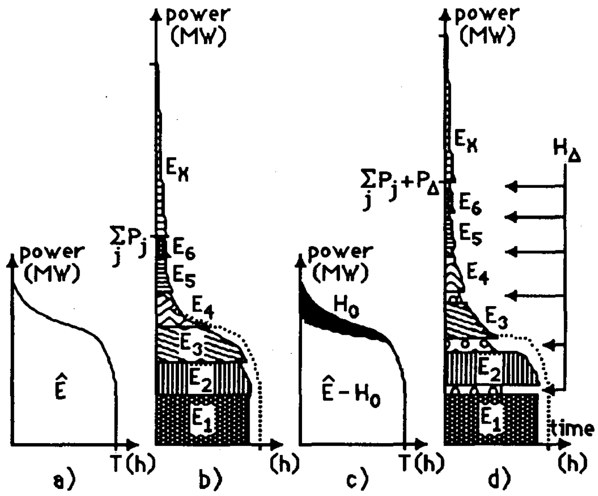

Fig. 1 a) Original load-duration curve (l.d.c.)
b) Generation-duration curve (g.d.c.) and l.d.c. (dotted)
c) L.d.c. peak-shaved with deterministic hydrogeneration H0
d) G.d.c. corresponding to peak-shaved l.d.c., including stochastic hydrogeneration HD

III. Hydrogeneration Contribution to each G.D.C.

As developed in [8] the hydrogeneration to be optimized at each long term interval can be described by a block probability distribution such as that in fig. 2b) if a multicommodity long term hydro optimization model of a multireservoir system is used. The block probability areas $p_1, p_2,\ldots, p_K$. (see fig. 2) are fixed beforehand. (K being the number of probability blocks with which the stochastic water inflows (see fig. 2a)) are approximated [8], (K=3 in fig. 2). The hydrogenerations $H_0, H_1,\ldots, H_K$ for each reservoir and each interval are part of the optimization results. Its probability distribution suggests that there are two basic types of hydrogeneration: the deterministic hydrogeneration H_0 MWh (with 100% availability) and an stochastic hydrogeneration H_Δ MWh — whose availability is less than 100%— which comes from H_1, $H_2,\ldots, H_K$ and $p_1, p_2,\ldots, p_K$. The values of H_k^i ($k=0,1,\ldots,K$ and $i=1,\ldots,N_i$) for each reservoir will be optimized together with fuel and thermal generation.

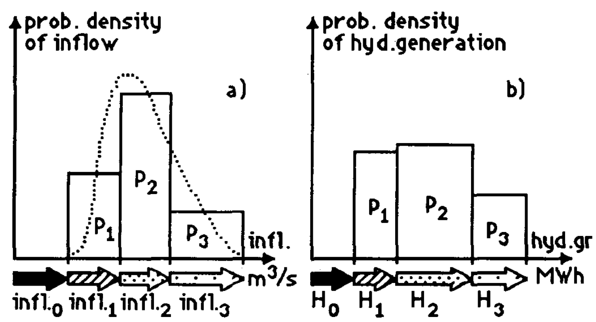

Fig. 2 a) Probability density function of inflow in a reservoir over a given interval and approximation by 3 blocks
b) Resulting approximated probability density function of hydrogeneration in a reservoir over a given interval

When an elaborated long-term hydrogeneration model — such as the multicommodity one— is available, there is no need to take the simplifying assumption that all hydrogeneration in a given interval is loaded on its g.d.c. as a single thermal unit. The deterministic hydroenergy H_0 and the stochastic hydroenergy H_Δ must influence the covering of the l.d.c. in different ways. The deterministic part H_0 is used to peak-shave the l.d.c., whereas the stochastic H_Δ contributes to the covering of the l.d.c. just as any other thermal unit, whose availability is less than 100%. Thus for each interval "i" first its l.d.c., whose area is $\widehat{E^i}$, is peak-shaved with $H_0{}^i$ (see fig. 1c)) and then thermal units plus the stochastic hydrogeneration part $H_\Delta{}^i$ cover the peak-shaved l.d.c. (see fig. 1d)).

For the stochastic hydrogeneration to behave as a thermal unit in the g.d.c. we would need to know about it the same parameters that characterize thermal units in the g.d.c., i.e. power output, forced outage rate and position in loading order list. None of these is determined, and instead we have that the area (energy) in the g.d.c. of interval "i" corresponding to the expected stochastic hydrogeneration should be:

$$H_\Delta{}^i = \sum_{k=1}^{K} q_k H_k{}^i \tag{1}$$

where q_k is the availability rate corresponding to a pseudo unit that could generate at most $H_k{}^i$ MWh over interval "i" (see fig. 2b)). It is simple to prove that for a multiblock probability density function such as that in fig. 2b), the availability rate q_k is:

$$q_k = \frac{p_k}{2} + \sum_{j=k+1}^{K} p_j \tag{2}$$

The integration of the stochastic hydroenergy into the g.d.c. will be made under the following assumptions:

• The area (expected energy) in the g.d.c. will be $H_\Delta{}^i$ (see fig. 1d))

• Instead of considering that the stochastic hydroenergy comes from a single pseudo-unit, it will be assumed that it comes from N_u pseudo-units, each one generating $H_{\Delta j}{}^i$ and placed in the loading order list just after each of the N_u thermal units (see fig. 1d)).

• The power output of the ensemble of stochastic hydro pseudo-units must be such that together with the mean power output of the deterministic hydro production ($H_0{}^i$ MWh) it is less than or equal to the maximum rated hydro-power capability $\overline{P}_H$. Since the mean deterministic hydropower is clearly $H_0{}^i/T^i$ (T^i being the interval's duration), we will have that the total power corresponding to stochastic hydro-energy $P_\Delta{}^i$ and the power of the j^{th} pseudo-unit $P_{\Delta j}{}^i$ will satisfy at the i^{th} interval:

$$P_\Delta{}^i = \sum_{j=1}^{N_u} P_{\Delta j}{}^i \leq \overline{P}_H - \frac{H_0{}^i}{T^i} \tag{3}$$

$P_\Delta{}^i$ changes with $H_\Delta{}^i$. Its value depends on the relative values of $H_{\Delta j}{}^i$ ($j=1,\ldots,N_u$), as these energies are placed in different positions in the loading order list.

• The availability of pseudo-units could be approximated by:

$$\sum_{k=1}^{K} q_k H_k{}^i \Big/ \sum_{k=1}^{K} H_k{}^i \tag{4}$$

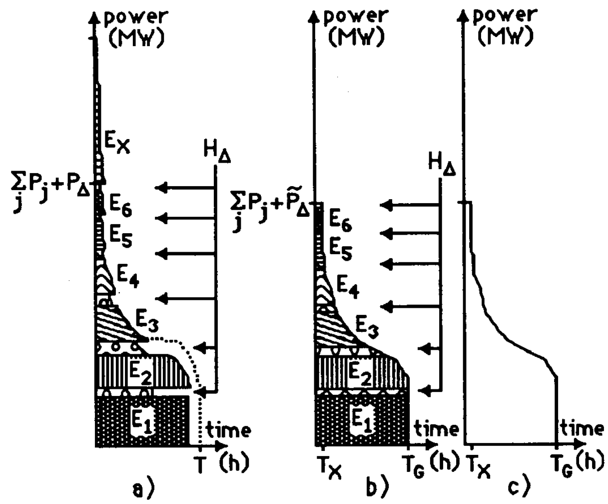

Fig. 3 a) Generation-duration curve with stochastic hydro
b) Smothed generation-duration curve (s.g.d.c.) showing modified stochastic hydro H_Δ
c) S.g.d.c

• For the i^{th} interval the emergency import $E_X{}^i$, associated to the g.d.c., changes with $H_0{}^i$, with $H_\Delta{}^i$ and with the differences $\sum_j^{N_u} \left(\overline{P}_j - P_j{}^i\right)$. Simulations made with several g.d.c. changing $H_0{}^i$, $H_\Delta{}^i$ and $P_j{}^i$ show that this variation can be approximated by an expression such as:

$$E_X{}^i = \frac{a^i (Y)^2}{H_0{}^i - b^i} + c^i \tag{5}$$

with

$$Y = \overline{P}_H - \frac{H_0{}^i}{T^i} - P_\Delta{}^i + \sum_{j=1}^{N_u} \left(\overline{P}_j - P_j{}^i\right) \tag{6}$$

where the parameters a^i, b^i and c^i must be estimated for the l.d.c. of each interval

• The maximum shortage duration $T_X{}^i$ with unsupplied energy is a function of the same type and of the same variables as those of the emergency energy, and in the i^{th} interval could be estimated with an expression of the type (5) with different parameters:

$$T_X{}^i = \frac{d^i (Y)^2}{H_0{}^i - e^i} + f^i \tag{7}$$

where d^i, e^i and f^i must also be estimated for each interval

The g.d.c. of the i^{th} interval could look like that shown in fig. 1d).

IV. Approximating the Generation Duration Curve

The covering of the g.d.c. of each interval by the contributions of thermal and hydro generation are constraints that must be taken into account by the long term coordination optimization, but in order to make optimization possible and computationally efficient, the discontinuities in the g.d.c. must be smoothed out.

Figs. 3b) and 3c) show a smoothed g.d.c. (s.g.d.c.) equivalent to the g.d.c. of a given interval. The main

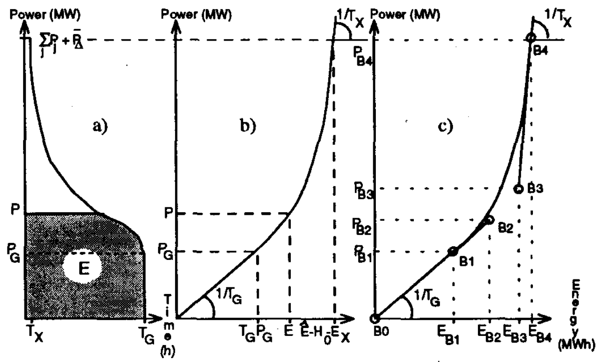

Fig. 4 a) Inverted smoothed generation duration curve
b) Power-energy function (PE(.))
c) Approximation of PE(.) function by a straight segment connected to a four point Bézier curve

difference is in the shape of the slices corresponding to stochastic hydrogeneration of pseudo-units. The availability of these pseudo-units is much less than that of a normal thermal unit (e.g.: from expressions (2) and (4), taking the simplifying assumption that $H_1=H_2=...=H_K$ and that $p_1=p_2=...=p_K=1/K$ one gets that the availability rate would be 0.5). This brings about indentations in the g.d.c. Smoothing out these indentations while keeping the expected hydrogeneration area is equivalent to increasing the availability of these units to that of a thermal unit (i.e. increasing the duration of its generation —see fig. 3b)—) while reducing the power output of the stochastic hydrogeneration, which is not an unrealistic practice.

By integrating the s.g.d.c. (see fig. 4a)) along the power axis up to the power at which the emergency energy steps in, we obtain the energy corresponding to each power level, so that we could plot, as in fig. 4b), the power level w.r.t. energy for a given s.g.d.c.. This function will be referred to as Power-Energy (PE) function, and it is important to point out that there is a bijective correspondence between the PE function and the s.g.d.c. from which it has been obtained. The PE function of a given interval "i" (see fig. 4b)) has several interesting features:

- its left part is a straight line of slope $1/T_G^i$ through the origin, where T_G^i is the expected base generation duration of the s.g.d.c.. The value of T_G^i depends on T^i and the forced outage rates of the first units in the loading order list, and can be estimated without difficulty
- the point at which the straight part of the PE function starts an upward bend has power P_G^i and its energy is obviously $P_G^i T_G^i$. P_G^i corresponds approximately to the base power of the l.d.c. of the interval with corrections that depend on the capacities and forced outage rates of the first thermal units in the loading order list
- the durations of the s.g.d.c. are the inverse of the derivative of the PE function: $t = 1/(dPE/dE)$
- the upper right part of the function has slope $1/T_X^i$, where T_X^i is the maximum shortage duration (with emergency energy imports) in the s.g.d.c.; it can be calculated with (7)
- the coordinates of its extreme upper-right point are

$$(P_\Delta^i + \sum_{j=1}^{N_u} P_j^i\ ,\ \widehat{E^i} - H_0^i - E_X^i\ ) \tag{8}$$

where E_X^i can be calculated with expression (5) (see Fig. 3).

The PE function of each interval thus changes with H_0^i, H_Δ^i and E_j^i (j=1,...,Nu) and will be used when optimizing the long term hydrothermal coordination to ensure that the generations of each interval conform to their s.g.d.c.s.

V. Representing the S.G.D.C. of the Intervals

One way of including the s.g.d.c. covering constraint in the optimization process is through an analytic expression of the PE function. The analytic expression proposed for the PE function has two distinct parts as in a former work by the authors [7]:

- a straight line segment (B_0,B_1) through the origin and with slope $1/T_G^i$ up to the point (P_G^i,$P_G^i T_G^i$), and
- a Bézier curve [2] generated with four points: B_1, B_2, B_3 and B_4 (see fig. 4c)).

The line uniting B_3 and B_4 has slope $1/T_X^i$ and its final point B_4 has the coordinates (8). Points B_0,B_1 and B_2 are situated on the same straight line (with slope $1/T_G^i$), as this ensures —in Bézier curves— that there is continuity in the first derivative of the curve at the linking point B_1.

Point B_2 is placed at a fixed proportion of the distance between B_1 and the intersection point of the straight lines from the origin with slope $1/T_G^i$ and from B4 with slope $1/T_X^i$. The same is true for point B_3 between B4 and the same intersection point.

Assuming that the coordinates (power and energy) of points B_m, m=1,...,4, in fig. 5.b) are (P_{Bm} ,E_{Bm}) m=1,...,4, the PE curve can be expressed as follows: for energies between 0 and E_{B1}

$$P = E/T_G^i \tag{9}$$

and for energies between E_{B1} and E_{B4} (or power between P_{B1} and P_{B4}) through the Bézier curve, expressed by:

$$\begin{aligned} P &= P_{B1}(1-\beta)^3 + 3P_{B2}\beta(1-\beta)^2 + 3P_{B3}\beta^2(1-\beta) + P_{B4}\beta^3 \\ E &= E_{B1}(1-\beta)^3 + 3E_{B2}\beta(1-\beta)^2 + 3E_{B3}\beta^2(1-\beta) + E_{B4}\beta^3 \\ &0 \leq \beta \leq 1 \end{aligned} \tag{10}$$

which is a parametric curve in β.

Finding either P or E corresponding to a given E or P in curve (10) means finding a root (between 0 and 1) of a third degree polynomial. In the programs developed this is done using Cardano's method [3].

The coordinates (P_{Bm} , E_{Bm}) m=1,...,4, of the representation of the PE function of the s.g.d.c. of the i^{th} interval, can all be expressed in terms of either:

- the parameters $\overline{P}_H$, $\overline{P}_j$ (j=1,...,N_u), p_K (k=1,...,K), $\widehat{E^i}$ and T^i,
- or the predetermined constants T_G^i, P_G^i, a^i, b^i, c^i, d^i, e^i, f^i ,
- or the optimization variables H_k^i (k=0,1,...,K) and E_j^i (j=1,...,N_u),

thus the PE curve (9,10) of each interval and its derivatives can be employed in hydrothermal optimization. It should be pointed out that there is no need to have explicitely E_X^i in the formulation as this energy will always be the balance: $\widehat{E^i}$-H_0^i-$\Sigma_j(E_j^i+H_{\Delta j}^i)$ and that the coincidence of power at point B_4 between that calculated through the PE curve and P_Δ^i+ $\Sigma_j P_j^i$ can be imposed through an equality constraint as (13) considered later. Thus the PE curve (9,10) self-adapts to changes in H_k^i (k=0,1,...,K) and E_j^i (j=1,...,N_u).

VI. Power and Generation Constraints of the S.G.D.C.

The PE curve defined in the former Section for any interval "i" $P=PE^i(E)$ is instrumental in ensuring that thermal generation, stochastic hydro-generation and power output conform to the s.g.d.c. of their interval.

As shown in fig. 4c), from the PE curve we can obtain the expression of the power generated by the j^{th} unit in the i^{th} interval as a function of thermal and stochastic hydro-generation:

$$P_j^i = PE^i\left(E_j^i + \sum_{m=1}^{j-1}\left(E_m^i + H_{\Delta m}^i\right)\right) - PE^i\left(\sum_{m=1}^{j-1}\left(E_m^i + H_{\Delta m}^i\right)\right) \tag{11}$$

Through (11) we can enforce that no thermal unit generates beyond its power capability:

$$P_j^i \le \overline{P}_j \quad (j=1,...,N_u) \tag{12}$$

The power level corresponding to total thermal generation plus stochastic hydro-generation (see fig. 5) must be:

$$PE^i\left(\sum_{j=1}^{N_u}\left(E_j^i + H_{\Delta j}^i\right)\right) = P_\Delta^i + \sum_{j=1}^{N_u} P_j^i \tag{13}$$

P_Δ^i thus behaves as the power of an energy limited thermal unit. In terms of the energies of the N_u hydro pseudo-units, and taking into acount (1) and (2) H_Δ^i is:

$$H_\Delta^i = \sum_{j=1}^{N_u} H_{\Delta j}^i = \sum_{k=1}^{K}\left(\frac{p_k}{2} + \sum_{j=k+1}^{K} p_j\right) H_k^i \tag{14}$$

VII. Hydro-Constraints and Hydro-Generation

H_0^i and H_K^i (k=1,...,K) of (14) are not directly optimized as they are intermediate variables. The hydro-variables really optimized are the multicommodity stored volumes and discharges. The multicommodity constraints and generation function that relate multicommodity hydro-generation to multicommodity stored volumes and discharges is fully elaborated on in [8].

Let v_{nk}^i and d_{nk}^i be the stored volume and discharge of the n^{th} reservoir in the i^{th} interval of water corresponding to commodity "k", and let $\overline{V}_n$ and $\overline{D}_n$ be the maximum volume and the maximum discharge of the n^{th} reservoir. In the sample replicated hydro-network in fig. 5a), considering that the n^{th} reservoir is downstream of the $(n-1)^{th}$ reservoir, the balance equations for the n^{th} reservoir in the i^{th} interval would be:

$$\sum_{j=0}^{k} v_{nj}^{i-1} + \sum_{j=0}^{k} d_{(n-1)j}^i = \sum_{j=0}^{k} v_{nj}^i + \sum_{j=0}^{k} d_{nj}^i \quad k=0,1,...,K \tag{15}$$

and the bounds and mutual capacity constraints for stored volumes and discharges are respectively:

$$0 \le v_{nk}^i \le \overline{V}_n, \quad 0 \le d_{nk}^i \le \overline{D}_n \quad k=0,1,...,K \tag{16}$$

$$\sum_{k=0}^{K} v_{nk}^i \le \overline{V}_n, \quad \sum_{k=0}^{K} d_{nk}^i \le \overline{D}_n \tag{17}$$

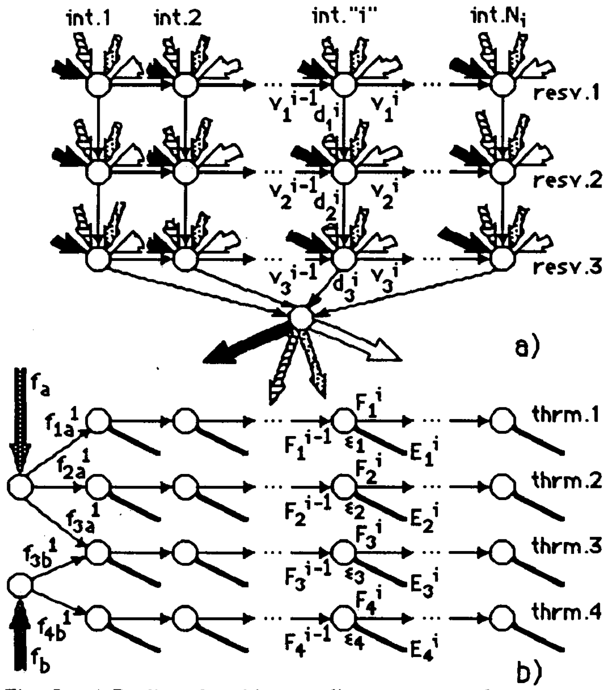

Fig. 5 a) Replicated multicommodity water network
b) Replicated generalized fuels' network

A hydrogeneration function h(.) for a reservoir computes the generated power in terms of the initial and final volume and the discharge over a given interval. The function employed will give the cumulative generation up to commodity "k" in terms of initial and final volumes, and discharges up to commodity "k":

$$\sum_{j=0}^{k} H_{nj}^i = h\left(\sum_{j=0}^{k} v_{nj}^{i-1}, \sum_{j=0}^{k} v_{nj}^i, \sum_{j=0}^{k} d_{nj}^i\right) \tag{18}$$

thus,

$$H_{nk}^i = h\left(\sum_{j=0}^{k} v_{nj}^{i-1}, \sum_{j=0}^{k} v_{nj}^i, \sum_{j=0}^{k} d_{nj}^i\right) - h\left(\sum_{j=0}^{k-1} v_{nj}^{i-1}, \sum_{j=0}^{k-1} v_{nj}^i, \sum_{j=0}^{k-1} d_{nj}^i\right) \tag{19}$$

If there are N_r reservoirs,

$$H_k^i = \sum_{n=1}^{N_r} H_{nk}^i \quad k=0,1,...,K \tag{20}$$

to be employed in (14) and for H_0^i.

VIII. Fuel-Constraints and Thermal Generation Efficiency

The sample replicated generalized fuel-network in fig. 5b) shows the fuel flows from contract sources to generation and stockpile in successive intervals. An efficiency and units transformation coefficient ε_{jl} has been introduced for the j^{th} thermal unit and the l^{th} fuel. Through this coefficient the fuel remainder F_{jl}^i in fuel units (e.g. tons of coal) at the end of the i^{th} interval in unit "j" stockpile or tank, and the possible fuel suply f_{jl}^i over the i^{th} interval can be related to its thermal generation E_{jl}^i (in MWh) in the balance equations:

$$F_{jl}^{i-1} + f_{jl}^{i} = F_{jl}^{i} + \varepsilon_{jl}E_{jl}^{i} \quad \begin{cases} j=1,\dots,N_u \\ l=1,\dots,N_f \\ i=1,\dots,N_i \end{cases} \tag{21}$$

The bounds and mutual capacity constraints for stored fuels and energies generated are respectively:

$$0 \le F_{jl}^{i} \le \overline{F}_j \ , \quad 0 \le E_{jl}^{i} \le \overline{E}_j^{i} \quad \begin{cases} j=1,\dots,N_u \\ l=1,\dots,N_f \\ i=1,\dots,N_i \end{cases} \tag{22}$$

$$\sum_{l=1}^{N_f} F_{jl}^{i} \le \overline{F}_j \ , \quad E_j^{i} = \sum_{l=1}^{N_f} E_{jl}^{i} \le \overline{E}_j^{i} \quad \begin{cases} j=1,\dots,N_u \\ i=1,\dots,N_i \end{cases} \tag{23}$$

where $\overline{F}_j$ is the stockpile or tank capacity at the j^{th} substation and $\overline{E}_j^{i}$ is the maximum energy that the j^{th} unit can generate over the i^{th} interval, whose length may be different from that of other intervals.

The left hand side of the equality in (23) gives the total generation of the j^{th} unit over the i^{th} interval.

Fuel supplies of the l^{th} fuel for different units in the same or different intervals can be associated through simple network constraints as in the left side of fig. 5b), to a specific single contract with a global fuel limit. Take-or-pay contracts can be modeled placing lower limits $f_{jl}^{i}>0$ to fuel supplies. The same fuel under a different contract (including spot contracts) is considered as a different fuel in the formulation presented.

It is important to point out that although many multicommodity and generalized network flow concepts and terminology [5] have been used in the description of the hydro-network and the fuels' network, no specialised network code can be applied to the solution of this problem, as a specialised code that could solve multicommodity generalized network flows with nonlinear side constrants (11,13,18,19) does not exist at this time.

IX. Objective Function

The objective function to be minimized is the cost of fuels supplied plus the payments for emergency imports:

$$\sum_{i=1}^{N_i} \left(\sum_{j}^{N_u} \sum_{l}^{N_f} \left[\pi_{jl}^{i} f_{jl}^{i} \right] + \pi_X^{i} E_X^{i} \right) \tag{24}$$

where π_{jl}^{i} are the prices of the supply to the j^{th} unit of the l^{th} fuel over the i^{th} interval and π_X^{i} is the price of emergency imports E_X^{i} —defined by (5,6)— in the i^{th} interval.

The objective function (24) must be minimized subject to three groups of constraints:

- g.d.c. covering constraints for each interval expressed through (12-14) employing the PE^i functions defined through (5-10)
- hydro-network multicommodity constraints (15-17) and hydrogeneration function (18-20), and
- fuels network multicommodity generalized constraints (21-23).

X. Computational Results and Real Case Exemple

The model put forward can be solved with a general purpose constrained nonlinear optimization package, and many tests have been carried out using the Minos package version 5.3 [6]. Some relevant points about the programs developed are:

- Subsidiary programs for preparing the predetermined constants of PE curves from l.d.c.s and thermal and hydro parameters must be used. These programs have also been developed.
- A quite long subsidiary program to generate data for Minos' MPS file and the user's FUNOBJ and FUNCON routines implementing the hydrothermal model described, had to be developed. The use of a sparse Jacobian (matrix of derivatives of nonlinear constraints) and correct user-supplied derivatives is essential to ensure convergence and that program size is within reasonable limits
- Provided that correct analytical derivatives (no program generated finite differences) are employed, convergence to the solution was reasonable, despite the nonlinearities in the constraints and objective function.

A sample of required computation times using the Minos 5.3 package is given in Table I. The computer used is a SUN Sparc 10/41 workstation. Case a) of Table I corresponds to the solution represented in fig. 6, which is a real life exemple. Its reservoir system consists of 3 cascaded reservoirs, which will be referred to as "upper", "middle" and "lower" reservoir, with characteristics detailed in Table II. The thermal system has 11 units described in Table III using five fuels whose prices (in Spanish currency: Pts) and availability are in Table IV. Units #5 and #7 can use either fuel-oil or gas. Emergency energy, dearer than any available fuel, can be employed with no limit.

The time period is one year starting in May 1st (beginning

TABLE I. Sample of Problems Solved and Computational Results.

case	intervals	reservoirs	K+1 commod.	thermal units	fuel limits	variables	linear constraints	nonlinear constraints	feasibility iterations	optimiz. iterations	CPU seconds
a)	12	3	4	11	9	1122	464	192	450	2860	196.4
b)	14	7	4	10	-	1789	866	210	1110	3948	242.0
c)	24	4	4	10	2	2323	1009	360	765	4047	295.4
d)	36	7	4	13	5	5062	2338	648	2295	5836	1080.5
e)	36	11	4	23	8	8368	3738	1008	3450	3882	1553.0

TABLE II. Characteristics of Reservoirs [head = sb + sl (volume) + sq (volume)^2 + sc (volume)^3]

reserv.	max. volume (Hm^3)	min. volume (Hm^3)	#disch.	max. dischrg (m^3/s)	efficiency (p.u.)	#spill.	s_b (m)	s_l (m/Hm^3)	s_q (m/Hm^6)	s_c (m/Hm^9)
upper	265	26	1	140	0.9	1	166.4	0.407	-0.0108	$0.14*10^{-5}$
middle	119	21	2	100+60	0.9+0.8	1	71.63	0.478	-0.016	$0.25*10^{-5}$
lower	38	25	1	400	0.9	1	28.2	0.436	-0.0416	$0.5*10^{-4}$

TABLE III. CHARACTERISTICS OF THERMAL UNITS.

Unit	Power (MW)	Outage rate %	Fuels used	Fuel type "k"	Efficiency %	Maxm. purchase (GWh)	Maxm. storage (GWh)
Th1	540	3 .0	1	Nuclear	60.0	—	—
Th2	320	3.0	1	Nuclear	60.0	—	—
Th3	160	1.0	1	Coal	33.9	—	796.7
Th4	520	7.0	1	Gas	35.0	4095.0	—
Th5	350	12.0	2	Fuel-oil (2)	34.4	—	—
				Gas	33.6	1095.0	—
Th6	300	7.0	1	Gas	35.0	4095.0	—
Th7	350	19.0	2	Fuel-oil (2)	34.4	—	—
				Gas	33.6	1095.0	—
Th8	350	6.0	1	Fuel-oil (2)	34.4	—	—
Th9	300	7.0	1	Gas	35.0	4095.0	—
Th10	172	10.0	1	Fuel-oil (1)	34.4	—	975.0
Th11	172	10.0	1	Fuel-oil (1)	34.4	—	975.0

of the dry season), and has been subdivided into 12 one month intervals. Table V gives the main characteristics (peak load, duration and energy) of the l.d.c.s of each interval. Table VI shows the natural inflows considered, in expected value and standard deviation, for each reservoir over the total time period. These inflows, obtained from historical data in the way described in [8], are fed into the algorithm described as multicommodity inflows (as in fig. 2a)) with three probability blocks of p_1=0.3 , p_2=0.4 and p_3=0.3; (e.g.: for the October inflow in the upper reservoir we have infl.$_0$=8m^3/s, infl.$_1$=7m^3/s, infl.$_2$=5m^3/s and infl.$_3$=10m^3/s).

At the top of fig. 6 are the optimum trajectories of stored volume, discharge and generation for each reservoir and multicommodity water (four commodities as in the inflows are considered). The graphical output related to reservoir management is self-explanatory. It should only be pointed out that:

- Initial and final volumes are user-fixed and coincide
- The upper reservoir is the main responsible for the regulation
- There is a loose relation among the optimal policies of each multicommodity water (and that is why it is sensible to employ a multicommodity model for long-term hydro optimization) [8].

The bottom part of fig. 6 shows the s.g.d.c. of each interval with the slices corresponding the 11 thermal units and to stochastic hydrogeneration (shown with a pattern of bubbles).

The graphical output of the s.g.d.c.'s requires some comments:

TABLE IV. FUELS EMPLOYED BY THERMAL UNITS.

Name	Price (Pts/MWh)	Maximum fuel avalaible (GWh)
Coal	1631.64	911.383
Fuel-oil (contract 1)	1306.96	—
Fuel-oil (contract 2)	1306.96	1083.334
Gas	1504.72	—
Nuclear	980.0	—

- Given the interval times (e.g.: 744 h for October and 720 h for November) thermal units' generation duration is always less (due to forced outage rates).
- The s.g.d.c. represented is truncated where emergency energy would start. At the top left of each s.g.d.c. the power reached by thermal units and stochastic hydrogeneration is indicated. This is not constant as there are fuel limitations, which preclude some thermal units in some intervals from generating at their rated capability, besides the stochastic hydrogeneration H_Δ^i changes at each interval, thus giving rise to different P_Δ^i.
- At the top right of each s.g.d.c. the optimal emergency imports E_X^i, the optimum deterministic hydrogeneration H_0^i and the stochastic hydrogeneration H_Δ^i are written .
- Slices of part of H_Δ^i may appear none, one or many times after any thermal unit (e.g.: in April it appears over Th.2 and Th.6).
- The area of the slices is proportional to the generation of each unit (e.g.: in August the generations in GWh are: Th.1 388; Th.2 213; $H_{\Delta 2}$ 77; Th.6 58; Th.9 25; Th.10 7; Th.11 5 and $H_{\Delta 11}$ 2)
- Only Th.1 and Th.2 (both shared nuclear stations) have no limitations in their fuel supply and generate at their rated power. Thermal units #6, #9, #10 and #11 have limits on their fuels but these are not active. Units #3, #4, #5, #7 and #8 have an active fuel limit and thus at some interval they may not generate at all (e.g.: #3, #4 and # 5 do not generate in June) or generate at less than their rated power (e.g.: #4 in December).

XI. CONCLUSIONS

A model for long-term hydro-thermal coordination based on the use of the PE curves to satisfy power and energy constraints has been presented. This model is based in the joint optimization of hydrogeneration and thermal unit generation linked through the covering of the l.d.c.'s of the intervals and on the differentiation of the role of the deterministic

TABLE V. PEAK LOAD, DURATION AND ENERGY OF THE LOAD DURATION CURVE OF EACH INTEVAL.

Month	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April
Peak (MW)	1701	1666	1768	1390	1754	1910	2232	2571	2442	2319	2169	1897
Duration (h)	744	720	744	744	720	744	720	744	744	672	744	720
Energy (GWh)	914.7	909.9	997.8	815.1	958.6	1067.4	1219.1	1434.2	1432.0	1170.6	1209.9	939.9

TABLE VI. EXPECTATION AND STANDARD DEVIATION OF THE PROBABILITY DISTRIBUTIONS OF NATURAL INFLOWS (m3/sec).

	month	May	June	July	August	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April
Upper reserv.	expect.	125	95	89.5	64.25	45.75	17.95	12.5	10	29.85	45.2	80.55	107
	st.dev.	33.39	32.99	35.91	21.67	19.61	5.66	4.13	2.89	24.40	29.64	34.16	25.38
Middle reserv.	expect.	111.5	48.25	43.25	32.5	25	9.15	6.35	5	15.05	23.85	40.5	55
	st.dev.	43.33	17.54	17.80	10.31	8.27	2.74	2.28	1.65	11.29	14.75	18.86	16.98
Lower reserv.	expect.	46.5	26.75	20.75	17.2	12.5	4.95	3	2.5	7.7	11.05	21.2	25
	st.dev.	14.87	7.43	9.37	4.57	4.13	1.98	1.23	0.83	5.53	6.90	8.79	8.27

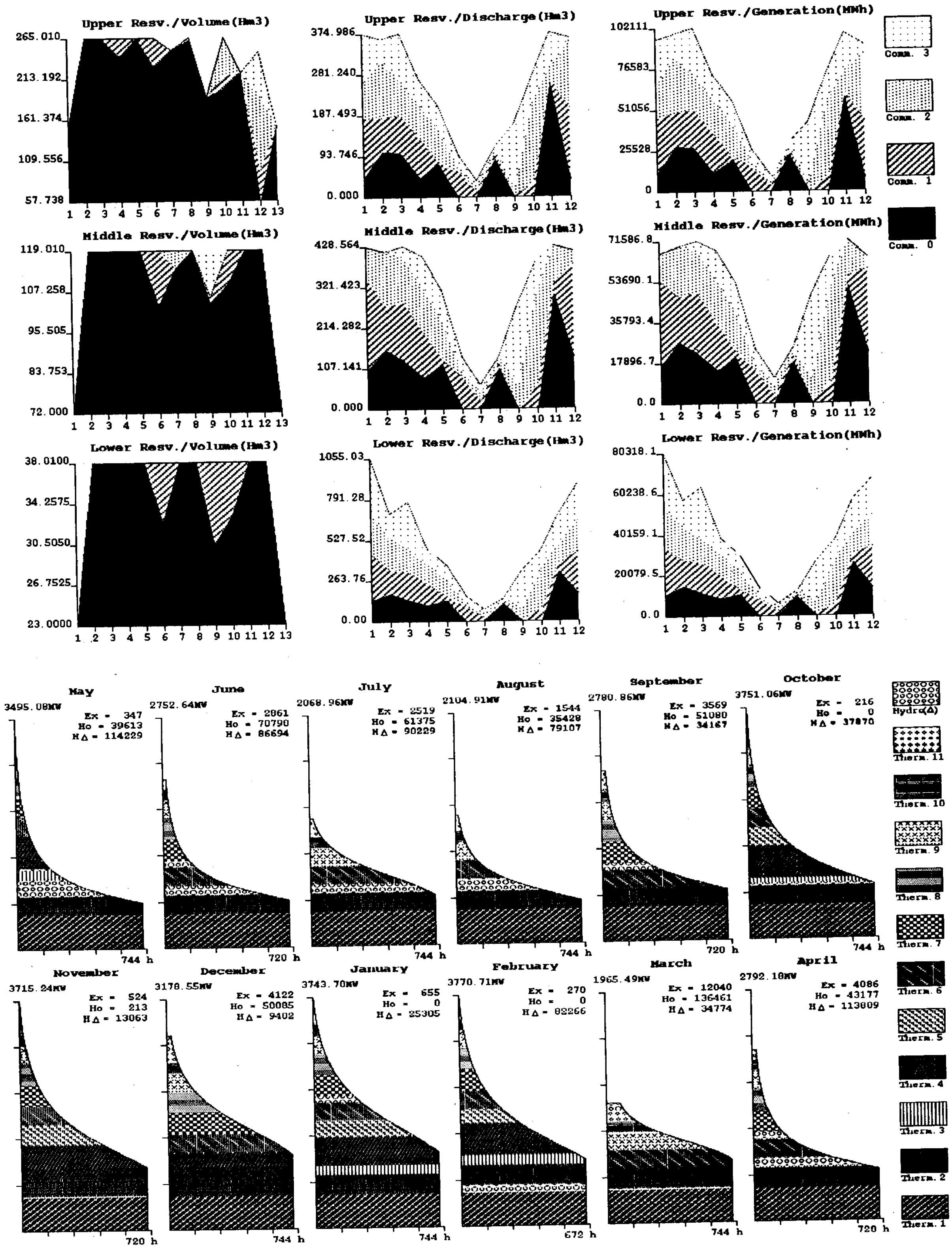

Fig. 6 Graphical output of solution to problem a) showing optimum hydrovariables and thermal and stochastic hydro-generations in the s.g.d.c.s of the intervals

hydrogeneration and the stochastic one, with a procedure to include the stochastic hydrogeneration in the g.d.c., which has been introduced and implemented. A multicommodity network model has been employed to take into account in detail the reservoir system.

The results obtained using the model put forward are consistent with the operating experience of hydrothermal systems and meet the expectations. The computational requirements are moderate. Many other extensions and refinements of the methodology proposed are possible and some are being pursued.

XII. Glossary of Symbols

a^i, b^i, c^i	parameters to estimate the emergency energy import over the i^{th} interval
B_l	Bézier points (l=1,...,4)
$d_{nl}{}^i$	discharge of water corresponding to commodity "l" at reservoir "n" over the i^{th} interval
d^i, e^i, f^i	parameters to estimate the emergency energy duration over the i^{th} interval
$\overline{D}_n$	maximum discharge at reservoir "n"
$\widehat{E}^i$	total energy of l.d.c. in the i^{th} interval
$E_j{}^i, E_X{}^i$	energy generated by thermal unit "j" and emergency energy imported over the i^{th} interval
$f_{jl}{}^i$	fuel "l" supplied to unit "j" over the i^{th} interval
$F_{jl}{}^i$	fuel "l" remaining at the stockpile of unit "j" at the end of the i^{th} interval
g.d.c.	generation duration curve
$H_k{}^i$	hydroenergy of water commodity "k" generated over the i^{th} interval, k=0,1,...,K
$H_0{}^i, H_\Delta{}^i$	deterministic hydroenergy and expected value corresponding to the ensemble of stochastic hydroenergies $H_1{}^i,..., H_K{}^i$, over the i^{th} interval
i	(superscript) indicates interval "i"
j	(subscript) indicates thermal unit "j" in merit order
k, K	indicates water commodity, k=0,1,...,K
l.d.c.	load duration curve
n	(subscript) indicates one of the hydro-reservoirs
N_f, N_i, N_u	number of fuels, of intervals and of thermal units
	minimum power of e.l.d.c.
PE(.)	power-energy function
$P_G{}^i$	base power of s.g.d.c. of interval "i"
p_k	probability of the k^{th} water commodity (k=1,...,K)
$\overline{P}_H$	maximum hydropower capability
$P_j{}^i$	power of thermal unit "j" over the i^{th} interval
$\overline{P}_j$	rated power capability of unit "j"
$P_\Delta{}^i$	power of stochastic hydrogeneration in interval "i"
q_k	availability rate of the k^{th} hydrogeneration
s.g.d.c.	smoothed generation duration curve
$T^i, T_G{}^i, T_X{}^i$	total duration of l.d.c., duration of base power of s.g.d.c. and duration of emergency energy over the i^{th} interval
$v_{nl}{}^i$	stored volume at reservoir "n" of water commodity "l" at the end of the i^{th} interval
$\overline{V}_n$	maximum stored volume at reservoir "n"
β	parameter of Bézier curve
ε_{jl}	efficiency in power generation of unit "j" for fuel "l"
$\pi_{jl}{}^i, \pi_X{}^i$	prices of supply of fuel "l" to unit "j" and of emergency energy over the i^{th} interval

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XIV. Biographies

Narcís Nabona (S'72-M'74) received the Energy Engineering degree from Universitat Politècnica de Catalunya (UPC), Barcelona, in 1968, the Certificate AEA in Control Systems from Université Paul Sabatier, Toulouse (France), in 1969, the M.Sc. and the Ph.D. in Electrical Engineering from the University of Alberta, Edmonton (Canada), in 1970 and from Imperial College of Science and Technology, London (G.B.) in 1974. He developed power system analysis and production planning software and was Consulting Engineer to several Spanish electric utilities until 1984. Then he joined the Statistics and Operations Research Department at UPC where he is currently Professor of Optimization.

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