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# A decision support procedure for a Price-Taker producer operating on Day-Ahead and Physical Derivatives Electricity Markets

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## Abstract

In this paper we develop a decision support procedure for a Price-Taker producer operating on Day-Ahead and Physical Derivatives Electricity Markets. The management of the electricity generation companies and their operation in the liberalized electricity market on a short-term horizon is an interesting problem in continuous evolution. Specifically, the incorporation of the Electricity Derivatives Market is the natural improvement in the Electricity Day-Ahead Markets in most countries in the world. Therefore, the inclusion of the management of derivatives products in generation company models is also a natural improvement of them. In this work, the derivatives products studied are the futures contracts. This work is based on Vespucci and Innorta [2008] where a decision support procedure was developed for the short-term hydro-thermal resource scheduling problem of a power producer who operates in the liberalized electricity market and aims at maximizing his own profit. The generation company is supposed to be price-taker and the resources owned by the producer are hydroelectric plants and thermal units. In the short-term horizon (from one to ten days), the generation company has to solve the unit commitment problem for the thermal units and the economic dispatch problem for the available hydro plants and the committed thermal units. Its decisions must be compatible with both technical constraints and market constraints. This decision support procedure has been improved modelling the settlement of the futures contracts, see Corchero and Heredia [2007], controlling the quantity that each committed thermal unit or hydro plant has to produce to fulfill these contracts. The settlement of the futures contracts could change all the decisions in the model, both the unit commitment and the economic dispatch optimal solution. The model is a mixed integer linear programming model, where the objective function represents the total profit determined on the bases of price forecast and the constraints describe the hydro system, the thermal system, the futures contracts settlement and the market.

## 1 Introduction

The transformation that in the last decade has changed the electric power industry is starting a new phase in some countries as Spain or Italy. After the liberalization of the electricity sector with the start-up of the Electricity Markets, the next step is being the creation of Derivatives Electricity Markets. In some Electricity Markets these two phases took place at once and nowadays they have a lot of experience in electricity derivatives trading, whereas in other cases the Derivatives Market is just starting-up or in phase of market design. In this work we deal with the participation of a producer in both markets, in the Day-Ahead and in the Derivatives Electricity Market.

In the Day-Ahead Market each power producer, in competition with other producers, aims at maximizing his own profit. Each market participant sends his sell or buy bids to the Market Operator for each hour

of the following day. On the basis of the aggregated supply and demand curves, the Market Operator determines the equilibrium point of supply and demand. The electricity price is then the market clearing price resulting from the interactions among all market participants. Simultaneously, power generators can sell their production directly to consumers, on the basis of bilateral contracts.

This new framework, in which the price at which electricity will be sold is unknown until the clearing process, has increased the risk factors. One of the techniques for hedging against market-price risk is the participation in futures markets and, for this reason, the creation of Derivatives Electricity Markets has been the next natural step after the electricity sector deregulation. The Derivatives Electricity Market gives to the participants the opportunity to use financial tools in order to reduce uncertainty and hedge risk. The derivatives products considered in this work are the futures contracts. An electricity futures contract is an exchange-traded derivative that represents agreements to sell electricity in the future for a specified price. The main characteristics of a futures contract are:

- *procurement*: futures contracts can have either physical or financial settlement. Physical futures contracts have cash settlement and physical delivery, whereas financial ones have cash settlement only.
- *delivery period*: it defines the contract duration. In the most common products the delivery period is a year, a quarter, a month, a week or a day.
- *load*: futures contracts can be either base or peak load. In base load futures contracts the quantity to be procured is constant in all hours of the delivery period. In peak load futures contracts there is procurement only in peak hours (from 8 am to 24 pm, Monday to Friday).

Nowadays, in the electricity market framework, futures contracts are traded at organized Derivatives Markets. As in the Day-Ahead Market, producers and other participants send their bids for futures contracts to the Market Operator who does the clearing process. Usually, these contracts can be physical or financial. The financial ones are not taken into account in this work because they are only an exchange of money that does not affect the producer short-term operation. By contrast, a physical futures contract entails a quantity of energy that has to be produced mandatorily by the power producer so it changes the daily operation of the units; those are the contracts we will include in the model. The participation in the Day-Ahead Market and in the Derivatives Market has been studied independently but the inclusion of physical contracts in the electricity markets affects directly the unit commitment and the technical operation of the units so it is needed a jointly approach. This work focuses on a price-taker producer, that is without enough market share to have capability to alter the market price through its bidding strategy: therefore prices do not depend on his own production decisions and are exogenous to the decision model. This kind of producer must take into account the price at which electricity will be sold. In this first approximation we suppose this price known: this is not a strong hypothesis as the market price can be forecasted. Thus, in this paper we develop a model for a producer where the optimal schedule is determined on the basis of price forecasts.

Different time horizons are considered in the producer's scheduling problem. A time horizon of at least one year (medium-term scheduling) is considered when determining the optimal maintenance plans of hydro and thermal plants and the optimal weekly discharge of seasonal basins. A time horizon of a week is considered for determining the unit commitment of thermal groups, i.e. the start-up and shut-down manoeuvres of the available thermal groups as well as the production levels of the committed thermal units and of the available hydro plants in each hour. In this work we develop a decision support procedure for the short-term hydro-thermal resource scheduling problem of a price-taker power producer who participates in the Day-Ahead and in the Derivatives Electricity Market. The decision support procedure, based on mixed integer LP models, determines the unit commitment of thermal units, the production levels of committed thermal units and available hydro plants in each hour and the economic dispatch of the futures contracts the producer has committed in the Derivatives Electricity Market, while satisfying constraints

describing the hydro system, the thermal system, the futures contract settlement and the market. The annual scheduling decisions (optimal maintenance plans of hydro and thermal plants and optimal weekly discharge of seasonal basins) are given, as well as forecasts on basin natural inflows.

In the following sections the mathematical model is introduced: in sections 2 and 3 we introduce the models of the hydroelectric system and of the thermal system respectively; in section 4 the market constraints and the constraints for determining the futures contracts dispatch are discussed; finally in section 5 the objective function is presented. The planning horizon is short-term with a discretization period of 1 hour. Let  $T$  denote the number of periods considered and let  $t$ ,  $0 \leq t \leq T$ , denote the period index, where  $t = 0$  denotes the last hour of the planning horizon immediately preceding the one in consideration.

## 2 Model of the hydroelectric system

The hydroelectric system consists of a number of cascades, i.e. sets of hydraulically interconnected hydro plants, pumped-storage hydro plants and basins. It is mathematically represented by a directed multi-graph, where nodes represent water storages (basins) and arcs represent water flows (either power generation, or pumping, or spillage). Let  $J$  denote the set of nodes and  $I$  denote the set of arcs. The arc-node incidence matrix, whose  $(i, j)$ -entry is denoted by  $A_{i,j}$ , represents the interconnections among water storages and water flows in the hydroelectric system ( $A_{i,j} = -1$ , if arc  $i$  leaves node  $j$ ;  $A_{i,j} = 1$ , if arc  $i$  enters node  $j$ ;  $A_{i,j} = 0$ , otherwise). For each arc  $i$  and for each node  $j$  the following data are relevant

- $k_i [MWh/10^3m^3]$ : energy coefficient ( $k_i > 0$ , if arc  $i$  represents generation;  $k_i < 0$ , if arc  $i$  represents pumping;  $k_i = 0$ , if arc  $i$  represents spillage)
- $\bar{q}_i [10^3m^3/h]$ : maximum water flow in arc  $i$
- $\rho_i [h]$ : delay on arc  $i$
- $B_{j,t} [10^3m^3/h]$ : natural inflow in basin  $j$  in hour  $t$
- $\bar{v}_j [10^3m^3]$ : maximum storage volume in basin  $j$
- $v_{j,0} [10^3m^3]$ : initial storage volume in basin  $j$
- $\underline{v}_{j,T} [10^3m^3]$ : minimum storage volume in basin  $j$  at the end of hour  $t$ , determined by the medium-term resource scheduling.

The power producer must schedule the hourly production of each hydro plant, which is expressed as the product of the hydro plant energy coefficient times the turbined volume in hour  $t$ , as well as the hourly pumped and spilled volumes. The decision variables of the hydro scheduling problem are

- $q_{i,t} [10^3m^3/h]$ : water flow on arc  $i$  in hour  $t$  (turbined volume, if arc  $i$  represents generation; pumped volume, if arc  $i$  represents pumping; spilled volume, if arc  $i$  represents spillage)
- $v_{j,t} [10^3m^3]$ : storage volume in basin  $j$  at the end of hour  $t$ .

The values assigned to the decision variables must satisfy the following constraints that describe the hydroelectric system:

- flow on arc  $i$  in hour  $t$  is nonnegative and bounded above by the maximum volume that can be either turbined, or pumped, or spilled

$$0 \leq q_{i,t} \leq \bar{q}_i \quad i \in I, \quad 1 \leq t \leq T \quad (1)$$

- the storage volume in basin  $j$  at the end of hour  $t$  is nonnegative and bounded above by the maximum storage volume

$$0 \leq v_{j,t} \leq \bar{v}_j \quad j \in J, \quad 1 \leq t \leq T' \quad (2)$$

- at the end of hour  $T$  the storage volume in basin  $j$  is bounded below by the minimum storage volume required at the end of the current planning period, so as to provide the required initial storage volume at the beginning of the following planning period

$$\underline{v}_{j,T} \leq v_{j,t} \quad j \in J \quad (3)$$

- the storage volume in basin  $j$  at the end of hour  $t$  must be equal to the basin storage volume at the end of hour  $t - 1$  plus inflows in hour  $t$  (taking into account the delays on arcs entering node  $j$ ) minus outflows in hour  $t$

$$v_{j,t} = v_{j,t-1} + B_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t-\rho_i} \quad j \in J, \quad 1 \leq t \leq T \quad (4)$$

Basin inflows are natural inflows, turbine discharge from upstream hydro plants, pumped volumes from downstream hydro plants, spilled volumes from upstream basins. Basin outflows are turbine discharge to downstream hydro plants, pumped volumes to upstream hydro plants and spilled volumes to downstream basins.

### 3 Model of the thermal system

Let  $K$  denote the set of thermal units owned by the power producer. For every unit  $k \in K$  the producer must decide the unit commitment, taking into account minimum up-time and minimum down-time constraints, and the hourly production of each committed unit, taking into account lower and upper bounds on production levels and ramping constraints. The decision variables in the thermal system problem are

- $p_{k,t}$  [MWh]: production level of unit  $k$  in hour  $t$ , for  $k \in K$  and  $1 \leq t \leq T$

and

$$\alpha_{k,t}, \beta_{k,t}, \gamma_{k,t} \in \{0, 1\} \quad k \in K, \quad 1 \leq t \leq T \quad (5)$$

where

- $\alpha_{k,t} = 1$  [0]: start-up [no start-up] of unit  $k$  in hour  $t$
- $\beta_{k,t} = 1$  [0]: shut-down [no shut-down] of unit  $k$  in hour  $t$

- $\gamma_{k,t} = 1$  [0] : unit  $k$  is *on* [*off*] in hour  $t$ .

In order to take into account minimum up-time and minimum down-time constraints, the following data are needed for every thermal unit  $k \in K$  :

- $ta_k$  [h] : minimum number of hours unit  $k$  must be *on* after start-up
- $ts_k$  [h] : minimum number of hours unit  $k$  must be *off* after shut-down
- $\gamma_{k,0}$  [0/1] : state of unit  $k$  in the last hour of the previous planning period
- $nh_k$  [h] : number of hours in which unit  $k$  has been in state  $\gamma_{k,0}$  since the last manoeuvre in the scheduling period preceding the current one.

In order to take into account constraints on hourly production levels, the following data are needed for every thermal unit  $k \in K$  :

- $\underline{p}_k$  [MWh] : minimum production level when unit  $k$  is *on*
- $\bar{p}_k$  [MWh] : maximum production level when unit  $k$  is *off*
- $vsu_k$  [MWh] : maximum production of unit  $k$  at start-up
- $vsd_k$  [MWh] : maximum production of unit  $k$  at shut-down
- $\delta u_k$  [MWh] : maximum production increase per hour of unit  $k$
- $\delta d_k$  [MWh] : maximum production decrease per hour of unit  $k$
- $p_{k,0}$  [MWh] : production of unit  $k$  at the beginning of scheduling period

The thermal system is modelled by the following constraints:

- values of binary variables representing manoeuvres in hour  $t$  and states in hours  $t - 1$  and  $t$  must be coherent

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \quad k \in K, \quad 1 \leq t \leq T \quad (6)$$

- *minimum up-time* constraints: if unit  $k$  was *on* in the last hour of the previous scheduling period, it must be *on* at least for the first  $ta_k - nh_k$  hours of the current scheduling period

$$\text{if } \gamma_{k,0} = 1 \quad \text{then } \gamma_{k,t} = 1 \quad \text{for } 1 \leq t \leq ta_k - nh_k; \quad (7)$$

moreover, when a thermal unit is started-up, it must be *on* for at least  $ta_k$  hours, i.e. if a start-up manoeuvre takes place in hour  $t$ , then unit  $k$  must be *on* either for  $ta_k - 1$  subsequent hours, if  $ta_k - 1 \leq T - t$ , or for  $T - t$  subsequent hours

$$\sum_{\tau=t+1}^{\min(t+ta_k-1, T)} \gamma_{k,\tau} \geq \alpha_{k,t} \cdot \min(ta_k - 1, T - t) \quad k \in K, \quad 1 \leq t \leq T \quad (8)$$

- *minimum down-time* constraints: if unit  $k$  was *off* in the last hour of the previous scheduling period, it must be *off* at least for the first  $ts_k - nh_k$  hours of the current scheduling period

$$\text{if } \gamma_{k,0} = 0 \quad \text{then } \gamma_{k,t} = 0 \quad \text{for } 1 \leq t \leq ts_k - nh_k \quad (9)$$

moreover, when a thermal unit is shut-down, it must be *off* for at least  $ts_k$  hours, i.e. if a shut-down manoeuvre takes place in hour  $t$ , then unit  $k$  must be *off* either for  $ts_k - 1$  subsequent hours, if  $ts_k - 1 \leq T - t$ , or for  $T - t$  subsequent hours

$$\sum_{\tau=t+1}^{\min(t+ts_k-1, T)} \gamma_{k,\tau} \leq (1 - \beta_{k,t}) \cdot \min(ts_k - 1, T - t) \quad k \in K, \quad 1 \leq t \leq T \quad (10)$$

The hourly production levels  $p_{k,t}$  are subject to the following constraints:

- if unit  $k$  is *on* in hour  $t$ , the hourly production  $p_{k,t}$  must be neither less than the minimum level  $\underline{p}_k$  nor greater than the maximum level  $\bar{p}_k$ ; if unit  $k$  is *off* in hour  $t$ , the hourly production must be zero

$$\gamma_{k,t} \cdot \underline{p}_k \leq p_{k,t} \leq \gamma_{k,t} \cdot \bar{p}_k \quad k \in K, \quad 1 \leq t \leq T \quad (11)$$

- *ramp-up* constraint: if unit  $k$  is started-up in hour  $t$ , the hourly production  $p_{k,t}$  cannot be greater than  $vsu_k$ ; moreover, if the production levels in two subsequent hours  $t - 1$  and  $t$  are such that  $p_{k,t-1} \leq p_{k,t}$ , the production variation is bounded above by  $\delta u_k$

$$p_{k,t} - p_{k,t-1} \leq \delta u_k + \alpha_{k,t} \cdot (vsu_k - \delta u_k) \quad k \in K, \quad 1 \leq t \leq T \quad (12)$$

- *ramp-down* constraint: if unit  $k$  is shut-down in hour  $t$ , the hourly production  $p_{k,t}$  cannot be greater than  $vsd_k$  and if the production levels in two subsequent hours  $t - 1$  and  $t$  are such that  $p_{k,t-1} \geq p_{k,t}$ , the production variation is bounded above by  $\delta d_k$

$$p_{k,t} - p_{k,t-1} \geq -\delta d_k + \beta_{k,t} \cdot (-vsd_k + \delta d_k) \quad k \in K, \quad 1 \leq t \leq T \quad (13)$$

## 4 Market constraints and futures contract dispatch

In every hour  $t$  of the planning period the power producer must satisfy the load  $car_t$  that derives from his bilateral contracts. If his total production exceeds the load from bilateral contracts, the excess quantity,  $sell_t$ , is sold on the spot market; if his total production is less than the load from bilateral contracts, the amount of energy,  $buy_t$ , necessary to meet the load  $car_t$ , must be bought on the market. The market constraints are therefore

$$\sum_{i \in I} k_i \cdot q_{i,t} + \sum_{k \in K} p_{k,t} + buy_t - sell_t = car_t \quad 1 \leq t \leq T \quad (14)$$

A future contract is defined as a pair price-quantity,  $(\lambda_f, L_f)$ ,  $f \in F$ , where  $F$  is the producer futures contracts portfolio resulting from the Derivatives Market clearing process. The futures contracts included



in this study are physical and base load, meaning an agreement to sell some constant quantity of electricity at some price with physical delivery and cash settlement. The futures contracts are settled by differences, i.e., each futures contract has daily cash settlement of the price differences between the spot reference price and the futures settlement price.

Let  $F$  denote the set of futures contracts and let  $f$  be the index of futures contracts. The following subsets are defined:

- $I_g \subseteq I$  : subset of arcs representing power generation (i.e. arcs with energy coefficient  $k_i > 0$ )
- $I_f \subseteq I_g, f \in F$  : subset of hydro plants that participate in futures contract  $f$
- $K_f \subseteq K, f \in F$  : subset of thermal units that participate in futures contract  $f$
- $F_i \subseteq F, \text{ for } i \in I_g$  : subset of futures contracts in which hydro plant  $i$  participates
- $F_k \subseteq F, \text{ for } k \in K$  : subset of futures contracts in which thermal unit  $k$  participates

The decision variables related to the futures contract dispatch problem are

- $gh_{i,t,f}$  [MWh] : energy to be produced by hydro plant  $i$  in hour  $t$  for serving future contract  $f$
- $gt_{k,t,f}$  [MWh] : energy to be produced by thermal unit  $k$  in hour  $t$  for serving future contract  $f$

The futures contract dispatch problem is subject to the following restrictions :

- the total energy produced by hydro plant  $i$  in hour  $t$  for serving future contracts in which hydro plant  $i$  participates cannot exceed the total energy produced by hydro plant  $i$  in hour  $t$ : this is expressed by the following constraints

$$0 \leq \sum_{f \in F_i} gh_{i,t,f} \leq k_i \cdot q_{i,t} \quad i \in I_g, \quad 1 \leq t \leq T \quad (15)$$

- the total energy produced by thermal unit  $k$  in hour  $t$  for serving future contracts in which thermal unit  $k$  participates cannot exceed the total energy produced by thermal unit  $k$  in hour  $t$ : this is expressed by the following constraints

$$\gamma_{k,t} \cdot \underline{p}_k \leq \sum_{f \in F_k} gt_{k,t,f} \leq p_{k,t} \quad k \in K, \quad 1 \leq t \leq T \quad (16)$$

- the energy quantity  $L_f$  for serving future contract  $f$  in every hour  $t$  must be produced by the hydro units and the thermal units

$$\sum_{i \in I_f} gh_{i,t,f} + \sum_{k \in K_f} gt_{k,t,f} = L_f \quad f \in F, \quad 1 \leq t \leq T \quad (17)$$

## 5 The objective function

The power producer is assumed to be a Price Taker, i.e. not able to influence the market price, which is therefore exogenous to the model. It is assumed that the hourly sell price  $\lambda_t$  [Euro/MWh] can be forecasted as well as the hourly purchase price  $\mu_t$ ,  $\mu_t \geq \lambda_t$ .

Two types of costs are associated to thermal production: costs of manouvres and generation costs. For every unit  $k$  costs  $csu_k$  [Euro] and  $csd_k$  [Euro] are associated to every start-up and shut-down manouvre respectively. The thermal generation cost  $G_{k,t}$  of unit  $k$  in hour  $t$  is assumed to be a convex quadratic function of the production level  $p_{k,t}$

$$G_{k,t}(p_{k,t}) = g_{2,k} \cdot p_{k,t}^2 + g_{1,k} \cdot p_{k,t} + g_{0,k}$$

where  $g_{2,k}$  [Euro/MWh<sup>2</sup>],  $g_{1,k}$  [Euro/MWh] and  $g_{0,k}$  [Euro] are the quadratic generation cost coefficients for unit  $k$ . Because of the unit commitment binary decision variables, the model we develop is of Mixed Integer type: the generation cost functions are therefore linearized, so as to obtain a Mixed Integer Linear Programming model. For every unit  $k$  the interval  $[p_k, \bar{p}_k]$  is divided in  $H$  subintervals of width  $\bar{p}_{k,h}$ ,  $1 \leq h \leq H$ . Let  $p_{k,t,h-1}$  and  $p_{k,t,h}$  denote the extreme points of subinterval  $h$ , let  $gl_{k,h}$  denote the slope of the straight line segment passing through points  $(p_{k,t,h-1}, G_{k,t}(p_{k,t,h-1}))$  and  $(p_{k,t,h}, G_{k,t}(p_{k,t,h}))$  and let  $pl_{k,t,h}$  denote the real variable associated to subinterval  $h$ . The linearized generation costs of thermal unit  $k$  in hour  $t$  are then given by

$$G_{k,t}^{lin}(p_{k,t}) = \left( g_{2,k} \cdot \underline{p}_k^2 + g_{1,k} \cdot \underline{p}_k + g_{0,k} \right) \cdot \gamma_{k,t} + \sum_{h=1}^H gl_{k,h} \cdot pl_{k,t,h}$$

where variables  $pl_{k,t,h}$  are subject to the constraints

$$p_{k,t} = \underline{p}_k \cdot \gamma_{k,t} + \sum_{h=1}^H pl_{k,t,h} \quad k \in K, \quad 1 \leq t \leq T \quad (18)$$

$$0 \leq pl_{k,t,h} \leq \bar{p}_{k,h} \quad k \in K, \quad 1 \leq t \leq T, \quad 1 \leq h \leq H \quad (19)$$

See Vespucci et al. [2007] for a detailed description of the linearization procedure. The following objective function represents the power producer profits

$$\max \sum_{t=1}^T \left[ \lambda_t \cdot sell_t - \mu_t \cdot buy_t - \sum_{k \in K} (csu_k \cdot \alpha_{k,t} + csd_k \cdot \beta_{k,t} + (g_{2,k} \cdot \underline{p}_k^2 + g_{1,k} \cdot \underline{p}_k + g_{0,k}) \cdot \gamma_{k,t} + \sum_{h=1}^H gl_{k,h} \cdot pl_{k,t,h}) \right] \quad (20)$$

## 6 The complete model

The complete model for the short-term hydro-thermal resource scheduling problem with futures contract dispatch is as follows.

For  $1 \leq t \leq T$ ,  $i \in I$ ,  $j \in J$ ,  $k \in K$  and  $1 \leq h \leq H$ , find values of decision variables  $q_{i,t}$ ,  $v_{j,t}$ ,  $p_{k,t}$ ,  $\alpha_{k,t}$ ,  $\beta_{k,t}$ ,  $\gamma_{k,t}$ ,  $sell_t$ ,  $buy_t$ ,  $gh_{i,t,f}$ ,  $gt_{k,t,f}$  and  $pl_{k,t,h}$  so as to

$$\max \sum_{t=1}^T \left[ \lambda_t \cdot sell_t - \mu_t \cdot buy_t - \sum_{k \in K} (csu_k \cdot \alpha_{k,t} + csd_k \cdot \beta_{k,t} + (g_{2,k} \cdot \underline{p}_k^2 + g_{1,k} \cdot \underline{p}_k + g_{0,k}) \cdot \gamma_{k,t} + \sum_{h=1}^H gl_{k,h} \cdot pl_{k,t,h}) \right]$$

subject to

$$0 \leq q_{i,t} \leq \bar{q}_i \quad i \in I, \quad 1 \leq t \leq T$$

$$0 \leq v_{j,t} \leq \bar{v}_j \quad j \in J, \quad 1 \leq t \leq T$$

$$\underline{v}_{j,T} \leq v_{j,t} \quad j \in J$$

$$v_{j,t} = v_{j,t-1} + B_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t-\rho_i} \quad j \in J, \quad 1 \leq t \leq T$$

$$\alpha_{k,t}, \beta_{k,t}, \gamma_{k,t} \in \{0, 1\} \quad k \in K, \quad 1 \leq t \leq T$$

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \quad k \in K, \quad 1 \leq t \leq T$$

$$\text{if } \gamma_{k,0} = 1 \quad \text{then} \quad \gamma_{k,t} = 1 \quad k \in K, \quad 1 \leq t \leq ta_k - nh_k$$

$$\sum_{\tau=t+1}^{\min(t+ta_k-1, T)} \gamma_{k,\tau} \geq \alpha_{k,t} \cdot \min(ta_k - 1, T - t) \quad k \in K, \quad 1 \leq t \leq T$$

$$\text{if } \gamma_{k,0} = 0 \quad \text{then} \quad \gamma_{k,t} = 0 \quad k \in K, \quad 1 \leq t \leq ts_k - nh_k$$

$$\sum_{\tau=t+1}^{\min(t+ts_k-1, T)} \gamma_{k,\tau} \leq (1 - \beta_{k,t}) \cdot \min(ts_k - 1, T - t) \quad k \in K, \quad 1 \leq t \leq T$$

$$0 \leq p_{k,t} \leq \gamma_{k,t} \cdot \bar{p}_k \quad k \in K, \quad 1 \leq t \leq T$$

$$p_{k,t} - p_{k,t-1} \leq \delta u_k + \alpha_{k,t} \cdot (vsu_k - \delta u_k) \quad k \in K, \quad 1 \leq t \leq T$$

$$p_{k,t} - p_{k,t-1} \geq -\delta d_k + \beta_{k,t} \cdot (-vsd_k + \delta d_k) \quad k \in K, \quad 1 \leq t \leq T$$

$$\sum_{i \in I} k_i \cdot q_{i,t} + \sum_{k \in K} p_{k,t} + buy_t - sell_t = car_t \quad 1 \leq t \leq T$$

$$0 \leq \sum_{f \in F_i} gh_{i,t,f} \leq k_i \cdot q_{i,t} \quad i \in I_g, \quad 1 \leq t \leq T$$

$$\gamma_{k,t} \cdot \underline{p}_k \leq \sum_{f \in F_k} gt_{k,t,f} \leq p_{k,t} \quad k \in K, \quad 1 \leq t \leq T$$

$$\sum_{i \in I_f} gh_{i,t,f} + \sum_{k \in K_f} gt_{k,t,f} = L_f \quad f \in F, \quad 1 \leq t \leq T$$

$$0 \leq pl_{k,t,h} \leq \bar{p}_{k,h} \quad k \in K, \quad 1 \leq t \leq T, \quad 1 \leq h \leq H$$

$$p_{k,t} = \underline{p}_k \cdot \gamma_{k,t} + \sum_{h=1}^H pl_{k,t,h} \quad k \in K, \quad 1 \leq t \leq T$$

## 7 Conclusions

In this paper we have introduced a model for the short-term hydro-thermal resource scheduling problem with futures contract dispatch. The Derivatives Electricity Market modelled is the Spanish one. The model is a mixed-integer linear programming model, obtained by linearizing the quadratic thermal generation costs, so as to allow large dimensional instances to be solved. The model is currently being used for determining the optimal solution of case studies related to the Spanish electricity market.

## References

- Corchero, C. and F. Heredia. A mixed-integer stochastic programming model for the day-ahead and futures energy markets coordination. In *Book of Abstracts of EURO2007 (Prague)*, page 139, 2007.
- Vespucci, M. and M. Innorta. Decision support models for scheduling hydro and thermal power plants in the liberalized electric energy market. In *Proceedings of iEMSs 2008 (Barcelona)*, 2008.
- Vespucci, M., M. Innorta, E. Allevi, and M. Bertocchi. Models for short-term scheduling of hydro-thermal resources in a liberalized electric energy market. Technical Report 3, DMSIA University of Bergamo, 2007.