Solving electricity market quadratic problems by Branch and Fix Coordination methods.

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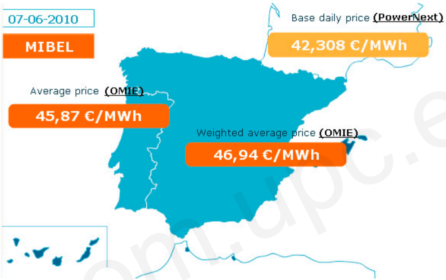
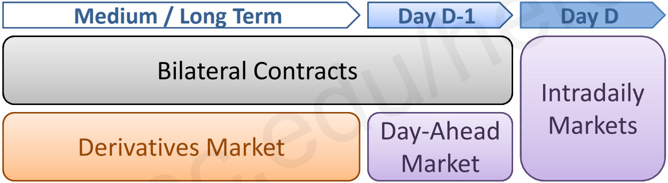
Iberian Electricity Market (MIBEL)



- The MIBEL (created in 2007) joins Spanish and Portuguese electricity system.
- The spot market operator (day-ahead, reserve and intraday markets), called *OMIE*, is located in Spanish.(`www.omel.es`).
- This Derivatives Market has its own market operator called *OMIP*, located in Portugal (`www.omip.pt`)
- The quality, security and reliability of the electric supply is guaranteed by the Independent System Operator (ISO).

Markets in the MIBEL



Derivatives Market

Physical Futures Contracts
Financial and Physical Settlement. Positions are sent to OMEL's Mercado Diario for physical delivery.
Financial Futures Contracts
OMIClear cash settles the differences between the Spot Reference Price and the Final Settlement Price

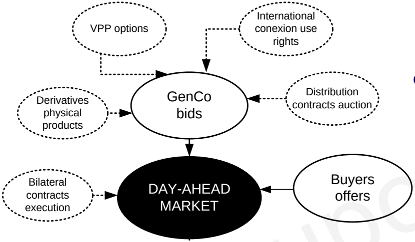
Bilateral Contracts

Organized markets
- Virtual Power Plants auctions (**EPE**) - Distribution auctions (**SD**) - International Capacity Interconnection auctions - International Capacity Interconnection nomination
Non organized markets
- **National** BC **before** the spot market - **International** BC **before** the spot market - **National** BC **after** the spot market

Day-Ahead Market

Day-Ahead Market
Hourly action. The matching procedure takes place 24h before the delivery period. Physical futures contracts are settled through a zero price bid.

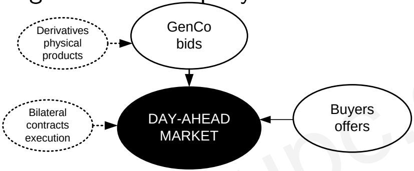
Day-Ahead Market mechanism



- The Day-Ahead Market (DAM) is the most important part of the electricity market (78% of the total system demand traded through DAM on 2009).
- The objective of this market is to carry out the energy transactions for the next day by means of the selling and buying offers presented by the market agents.
- The DAM is formed up by twenty-four hourly auctions that are cleared simultaneously between 10:00 and 10:30am of day D-1.

The GenCo's optimal DAM bid problem

The GenCo's optimal DAM bid problem considers a *Price-Taker* generation company with:



- A set of thermal generation units, I, with quadratic generation costs, start-up and shut-down costs and minimum operation and idle times.
- Each generation unit can submit sell bids to the $T = 24$ auctions of the DAM.
- A set of S scenarios for the spot price of auction t, λ_t^s.
- A set of physical futures contracts, F, of energy L_j^{FC} $j \in F$.
- A pool of bilateral contracts B of energy L_k^{BC}, $k \in B$.

Objectives

The objective of the study is to decide:

- the **optimal economic dispatch of the physical futures and bilateral contract** among the thermal units
- the **optimal bidding at Day-Ahead Market** abiding by the MIBEL rules
- the **optimal unit commitment** (binary on/off state) of the thermal units

maximizing the expected Day-Ahead Market profits taking into account futures and bilateral contracts.

Variables

First stage variables: for each time period $t \in T$ and thermal unit $i \in I$

- Unit commitment: $u_{it} \in \{0, 1\}$, c^u_{it}, c^d_{it}
- Zero price offer bid : q_{it}
- Scheduled energy for futures contract j: f_{itj} $j \in F$
- Scheduled energy for bilaterals contract: b_{it}

Second stage variables: for each $t \in T$, $i \in I$ and scenario $s \in S$

- Matched energy: $p^{M,s}_{it}$
- Total generation: p^s_{it}

Physical Future Contracts modeling

- A base load *Futures Contract* $j \in F$ consists in a pair $(L_j^{FC}, \lambda_j^{FC})$
- L_j^{FC}: amount of energy (MWh) to be procured each interval of the delivery period by the set U_j of generation units.
- λ_j^{FC}: price of the contract (c€/MWh).

Physical future contract contraints:

$$\sum_{i \in U_j} f_{itj} = L_j^{FC} \ , \ j \in F \ , \ t \in T$$

$$f_{itj} \geq 0 \ , \ j \in F \ , \ i \in I \ , \ t \in T$$

Bilateral Contracts modeling

- A base load *Bilateral Contract* $k \in B$ consists in a pair $(L_k^{BC}, \lambda_k^{BC})$
- L_k^{BC}: amount of energy (MWh) to be procured each interval t of the delivery period.
- λ_k^{BC}: price of the contract (c€/MWh).

Bilateral contract constraints:

$$\sum_{i \in I} b_{it} = \sum_{k \in B} L_k^{BC} \; , \; t \in T$$

$$0 \leq b_{it} \leq \overline{P}_i u_{it} \; , \; i \in I \; , \; t \in T$$

Day-Ahead Market bidding model

The energies L_j^{FC} and L_k^{BC} should be integrated in the MIBEL's day-ahead bid respecting the following **bidding rules**:

(BR1) To guarantee its inclusion in the operational programming, any committed unit i ($u_{it} = 1$) would bid its minimum generation level $\underline{P}_i$ at zero price (instrumental price bid q_{it}).

(BR2) If generator i contributes with f_{itj} MWh at period t to the coverage of the FC j, then the energy f_{itj} must be included into the instrumental price bid q_{it}.

(BR3) If generator i contributes with b_{it} MWh at period t to the coverage of the BCs, then the energy b_{it} must be excluded from the bid to the DAM. Unit i can offer its remaining production capacity $\overline{P}_i - b_{it}$ to the pool.

Day-ahead market bidding model: constraints

Matched energy:

(BR3) $p_{it}^{M,s} \leq \overline{P}_i u_{it} - b_{it} \; , \; i \in I \; , \; t \in T \; , \; s \in S$

(BR1) $p_{it}^{M,s} \geq q_{it} \; , \; i \in I \; , \; t \in T \; , \; s \in S$

Instrumental price bid:

(BR1+3) $q_{it} \geq \underline{P}_i u_{it} - b_{it} \; , \; i \in I \; , \; t \in T$

(BR2) $q_{it} \geq \sum_{j | i \in U_j} f_{itj} \; , \; t \in T \; , \; i \in I$

Total energy generation:

$$p_{it}^{s} = b_{it} + p_{it}^{M,s} \; , \; t \in T \; , \; i \in I \; , \; s \in S$$

Start-up/shut-down costs and up/down time constraints

Start-up and shut-down costs constraints:

$$c^u_{it} \geq c^{on}_i [u_{it} - u_{i,(t-1)}] \qquad t \in T \setminus \{1\}\ ,\ i \in I$$
$$c^d_{it} \geq c^{off}_i [u_{i,(t-1)} - u_{it}] \qquad t \in T \setminus \{1\}\ ,\ i \in I$$

Minimum up/down time constraints:

$$u \in X(t^{on}, t^{off})$$

where:

- c^{on}_i and c^{off}_i are the constant start-up and shut-down costs, respectively, of thermal unit i.
- t^{on}_i (resp. t^{off}_i) is minimum operation (iddle) time..

Objective function

Maximization of the day-ahead market clearing's benefits

$$\max_{p,q,f,b} \sum_{t \in T} \sum_{i \in I} \Big(- c^u_{it} - c^d_{it} + \\ + \sum_{s \in S} P^s \Big[\lambda^s_t p^{M,s}_{it} - (c^b_{it} u_{it} + c^l_i p^s_{it} + c^q_i (p^s_{it})^2) \Big] \Big)$$

Incomes from Futures and bilateral contracts (constant):

- **Futures contracts:** $\sum_{t \in T} \sum_{j \in J} \left(\lambda^{FC}_j - \lambda_t \right) L^{FC}_t$
- **Bilateral contracts:** $|T| \sum_{k \in B} \lambda^{BC}_k L^{BC}_k$

Summary of the model

Problem DAM-FBC

(**D**ay-**A**head **M**arket bid with **F**utures and **B**ilateral **C**ontracts)

Max E[Profit from the Day-ahead market]

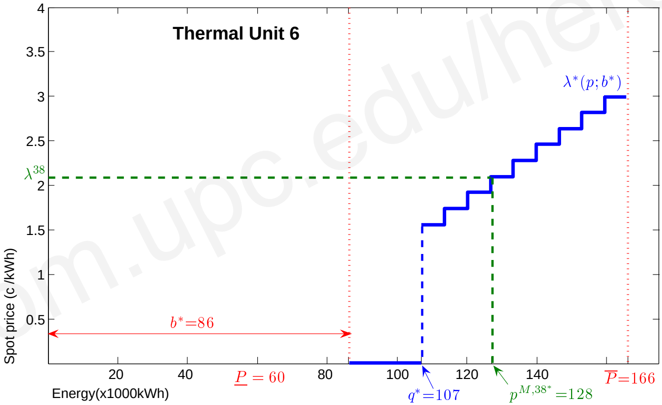
s.t. Physical future and bilateral contracts coverage

Day-ahead market rules

Start-up/shut-down costs and minimum up/down time

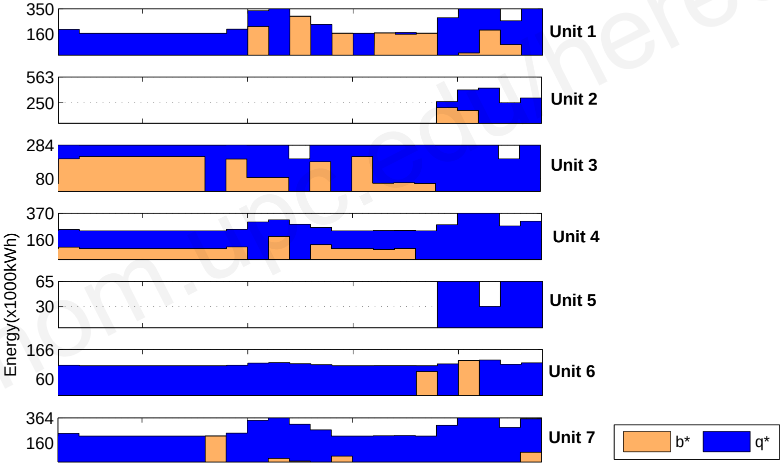
(Two-stage mixed integer quadratic stochastic programming problem)

Results: optimal bidding curve

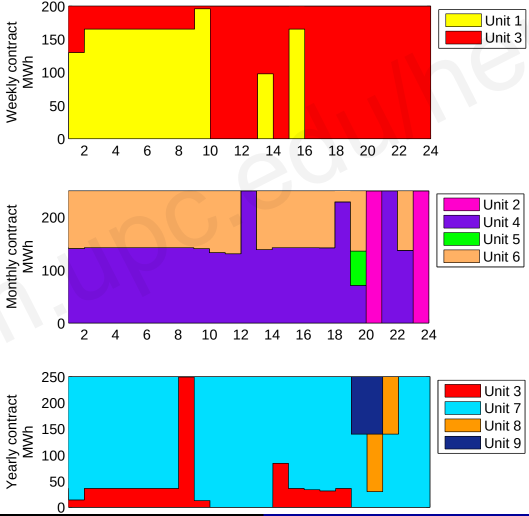


Optimal bidding curve for thermal unit 6 at interval 18

Unit commitment, q_{it}^* and b_{it}^*



Economic dispatch of each futures contracts f^*_{itj}



QBFC method

Model DAM-FBC can be rewritten as the so-called *Deterministic Equivalent Model* (DEM)

$$\begin{aligned}
\text{minimize} \qquad & c^t\delta + \sum_{\omega\in\Omega} p^\omega q^\omega(x, y^\omega) \\
\text{subject to :} \qquad & l_a \le A \begin{bmatrix}\delta \\ x\end{bmatrix} \le u_a, \\
& l_t^\omega \le T^\omega \begin{bmatrix}\delta \\ x \\ y^\omega\end{bmatrix} \le u_t^\omega,\ \omega\in\Omega, \\
& x \ge 0,\ \underline{y} \le y^\omega \le \overline{y},\ \omega\in\Omega, \delta\in\{0,1\}^{n_\delta},
\end{aligned}$$

where $\delta = u$, $x = (c^u, c^d, b, f, q)$, $y = (p^M, p)$, $q(x,y) = b_x^t x + b_y^t y +$ $+y^t Q_{yy} y$, and Q_{yy} being a diagonal matrix.

The compact representation **DEM** can be written as a splitting variable representation (Escudero et al. (2009)).

$$\textbf{(MIQP)} \quad \text{minimize} \quad \sum_{\omega\in\Omega} p^{\omega}(c^t\delta^{\omega} + q^{\omega}(x^{\omega}, y^{\omega}))$$

$$\text{subject to:} \quad l_a \le A \begin{bmatrix} \delta^{\omega} \\ x^{\omega} \end{bmatrix} \le u_a, \ \omega \in \Omega,$$

$$l_t^{\omega} \le T^{\omega} \begin{bmatrix} \delta^{\omega} \\ x^{\omega} \\ y^{\omega} \end{bmatrix} \le u_t^{\omega}, \ \omega \in \Omega,$$

$$x^{\omega} \ge 0, \ \underline{y} \le y^{\omega} \le \overline{y}, \quad \delta^{\omega} \in \{0,1\}^{n_\delta}, \ \omega \in \Omega,$$

Nonanticipativity constraints:

$$(\text{NAC}_\delta) \quad \delta^{\omega} - \delta^{\omega'} = 0, \ \forall \omega, \omega' \in \Omega : \ \omega \ne \omega',$$

$$(\text{NAC}_x) \quad x^{\omega} - x^{\omega'} = 0, \ \forall \omega, \omega' \in \Omega : \ \omega \ne \omega',$$

- In this method **DEM** is solved by using a Branch-and-Fix-Coordination scheme (BFC), an special application of the branch cut algorithm to solve coordinately the problem associated to each scenario $\omega \in \Omega$ in suach a way that the NAC constraints are implicitly satisfied by the Twin-Node-Families concept (TNF) (Alonso-Ayuso et al. (2003)).
- A similar approach to that suggested by Escudero et al. (2009) is used in this method to coordinate the selection of the branching node and branching variable for each scenario-related BF tree.
- More details in the contributed talk "*An algorithm for two-stage stochastic quadratic problems*" (CT7 Stochastic Optimization I, Thursday).

In order to gain computational efficiency we can take scenario clusters; i.e., instead of solving a submodel for each scenario $\omega \in \Omega$ we can solve a submodel for each *scenario cluster* $p = 1, \ldots, \widehat{p}$.

$$
\textbf{(MIQP}^p\textbf{)} \quad \text{minimize} \quad \sum_{\omega \in \Omega^p} p^\omega (c^t \delta^p + q^\omega(x^p, y^\omega))
$$

$$
\text{subject to :} \quad l_a \leq A \begin{bmatrix} \delta^p \\ x^p \end{bmatrix} \leq u_a,
$$

$$
l_t^\omega \leq T^\omega \begin{bmatrix} \delta^p \\ x^p \\ y^\omega \end{bmatrix} \leq u_t^\omega, \omega \in \Omega^p
$$

$$
x^p \geq 0, \; \underline{y} \leq y^\omega \leq \overline{y}, \; \omega \in \Omega^p, \quad \delta^p \in \{0,1\}^{n_\delta}
$$

Submodels are linked by NAC $\delta^p - \delta^{p'} = 0$ and $x^p - x^{p'} = 0$, $\forall \; p, p' \in \{1, \ldots, \widehat{p}\}, \;\; p \neq p'$.

Perspective Cuts

- Problem (MIQP^p) is a Mixed-Integer Quadratic Program (MIQP), which is difficult to solve efficiently, especially for large-scale instances.
- A possibility is to approximate the quadratic functions $q^\omega(x^p, y^\omega)$ by the outer polyhedral approximation defined by the so called perspective cuts (Frangioni and Gentile, 2006), so that this problem can be solved as a Mixed-Integer Linear Program (MILP) by general-purpose MILP solvers.

The PC formulation of problems $\mathbf{MIQP}^p$ are:

$$(\mathbf{MILP}^p) \quad \min \quad \sum_{\omega \in \Omega^p} p^\omega \left\{ b_x^t x + \left(\sum_{i=1}^{n} v_i^\omega \right) \right\}$$

s.t.:

$$v_i^\omega \geq \alpha y_i^\omega + \beta \delta_i^\omega, \; (\alpha, \beta) \in C_i^\omega, \; i \in \{1, \dots, n\}, \; \omega \in \Omega^p \quad (*)$$

$$l_a \leq A \begin{bmatrix} \delta^p \\ x^p \end{bmatrix} \leq u_a,$$

$$l_t^\omega \leq T^\omega \begin{bmatrix} \delta^p \\ x^p \\ y^\omega \end{bmatrix} \leq u_t^\omega, \; \omega \in \Omega^p$$

$$x^p \geq 0, \; y^\omega \in [\underline{y}, \overline{y}], \; \omega \in \Omega^p, \text{ and } \delta^p \in \{0,1\}^{n_\delta}$$

where the linear inequalities $(*)$ define the perspective cuts approximation to $q^\omega(x^p, y^\omega)$.

QBFC's implementation

- These two algorithmic alternatives have been considered:
  - QBFC: coordination of δ in the TNF of the BF trees for clusters $p \in \{1, \ldots, \widehat{p}\}$ without using PCs.
  - QBFC-PC: coordination of δ in the TNF of the BF trees for clusters $p \in \{1, \ldots, \widehat{p}\}$ using PCs.
- Both algorithmic alternatives has been compared using a set of 15 instances of the DAM-FBC problem.
- These methods have been implemented in C++ with the help of Cplex 12.1 to solve both the (MIQP^p) and (MILP^p) subproblems.
- The tests have been performed on HP with Intel(R) Core(TM)2 Quad CPU Q8300 2.50GHz 4 CPU under SUSE Linux Enterprise Desktop 11 (x86_64).

Prob.	$\|S\|$	$\|T\|$	# var_{QBFC}	# $var_{QBFC-PC}$	# bin	# constr
P01	10	12	1296	1776	48	1788
P02	20	12	2256	3216	48	3228
P03	30	12	3216	4656	48	4668
P04	40	12	4176	6096	48	6108
P05	50	12	5136	7536	48	7548
P11	10	24	2592	3552	96	3600
P12	20	24	4512	6432	96	6480
P13	30	24	6432	9312	96	9360
P14	40	24	8352	12192	96	12240
P15	50	24	10272	15072	96	15120

- For every problem $|F| = |BC| = 2$ and $|I| = 4$.
- If we use PCF, the problem increases the number of variables in $m = |T| \cdot |I| \cdot |S|$ and the number of constraints in $2 \cdot m$.

Prob.	$\widehat{p}$	QBFC (sec.)	QBFC-PC (sec.)	$\frac{QBFC-PC}{QBFC}$ (∗)	# PC
P01	2	10.1	3.4	0.34	280
P02	4	18.7	8.7	0.47	825
P03	5	2153.0	39.8	0.02	1685
P04	5	50.0	45.1	0.90	1491
P05	5	113.7	19.5	0.17	1276
P11	2	86.8	27.4	0.32	513
P12	4	469.7	50.3	0.11	1821
P13	5	687.3	176.6	0.26	3454
P14	5	1198.0	276.7	0.23	4239
P15	5	1190.9	246.3	0.21	2592

(∗) the running time with PC is a 30% of the running time without PC (average).

Conclusions

- We have presented an Optimal Bidding Model for a price-taker generation company operating both in the MIBEL Derivatives and Day-Ahead Electricity Market (DAM-FBC).
- The model developed gives the producer:
  - The optimal bid for the spot market.
  - The optimal allocation of the physical futures and bilateral contracts among the thermal units
  - The unit commitment

  following in detail the MIBEL rules.
- The (DAM-FBC) has been solved both with the standard BFC method and with a PC variation which reduces the running time to a 30% on the average.

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